

We have addressed the valuable comments of the reviewer's line by line and made corresponding modifications to the manuscript. The comments are displayed in red color, the author's response in black color.

Comment 1. In the introductory text, considering fractal analysis as a mathematical tool to deal with the main problem must be more elaborated. I think also that several works have been done in; literature in this direction. Some missed references are:

Geophys. J. Int. 167, 1215–1219 (2006); Pure Appl. Geophys. 162, 73–90 (2005); J. Earth Syst. Sci. 128, 22 (2019); Acta Mech 233, 2107–2122 (2022); Int. J. GeoInform. 9, 384 (2020)

Answer 1. We have revised the introduction section and elaborated the fractal's efficacy to deal with the problem. We have also incorporated the suggested references and few additional references also (line number 90-173). In the revised part of introduction, we have added two paragraph. In the first paragraph, we trace the evolution of fractals from their initial use in assessing natural geometries (such as clouds, coastlines, and mountain surfaces) to their diverse applications across scientific domains. These include medical science, material science, telecommunications, environmental science, computer graphics, and Earth sciences—with a focus on seismology and earthquake precursor studies. The second paragraph delves into fractal methods, starting from traditional approaches like box counting and Hausdorff method, and progressing to advanced techniques such as power spectrum analysis, Detrended Fluctuation Analysis (DFA), and the Higuchi method. We also address the limitations of these methods when applied to multifractal geometries. Finally, we briefly explore multifractal methods and their applications in geosciences.

Comment 2. line 92: Fractal method aid the study of the complex nature...Which method?

Answer 2. We have revised the sentence which address the details about the common methods of fractal used in study of complex nature of tectonics (line number 112-118). The revised sentence highlights popular fractal methods such as Hausdorff dimension, box counting, correlation dimension,

Higuchi fractal dimension, used in study of complex nature of Earth's system. These methods help to decipher the seismicity patterns in various tectonic environments as well as before and after the main earthquake event.

Comment 3. What we learn from Figure 1?

Answer 3: Figure 1 depict the area of our study, the spatial distribution of earthquakes surrounding the geomagnetic station, geometry of major fault systems and bathymetry.

Comment 4. Equations must be numerated 1, 2,... After each equation, please put a dot or a comma, depending on the subsequent sentence, e.g. after equation (i), Where must be where, etc... Please check the whole manuscript. By the way, (i) is not an equation.

Answer 4. The equations are corrected throughout manuscript (line number 342-408).

Comment 5. Origin of data in Figure 2 must be mentioned by a reference.

Answer 5. Figure 2 is an example of geomagnetic data showing power law and self-affine nature, based on our own data.

Comment 6. Most of the equations addressed must be clarified and discussed. They fall from nowhere.

Answer 6. We have revised the second part of section 2 and elaborated all the equations used for computation of multifractal spectrum from wavelet coefficients and wavelet leaders (line number 295-376). In the revised section, we described the limitation of Holder exponent in formulation of multifractal spectrum (using the increment in spatial or time domain) due to band limited and discrete time sampled data. Further, introduced the role of structure function to resolve the issue. Again we have discussed the multifractal formulation from wavelet coefficient (using DWT) by highlighting its advantage over increment function. We again discussed the limitation of wavelet coefficients in formulation of multifractal spectrum (does not support holder exponent for $q < 0$). Finally, we discussed the wavelet leader techniques to overcome the limitation of wavelet coefficients for

multifractal formulation. The multifractal formulation using wavelet leader techniques supports the holder exponent for both $q > 0$ and $q < 0$.

Comment 7. Fractal dimension is mentioned in line 219. However, the basic idea of fractal dimension and its properties must be addressed in the introductory text. Various definitions are found in literature. This is an important notion that deserves to be mentioned clearly. Please check also the missing references: *Commun Earth Environ* 5, 146 (2024); *Nat. Hazards* 77, 33–49 (2013); *J. Asian Earth Sci.* 58, 98–107 (2012); *Bulletin of the Seismological Society of America* 92(8):3318-3320 (2002); *Fractal Fract.* 8(5), 252 (2024); *Earthquake Science* 37, 107-121 (2024); *Pure Appl. Geophys.* 176, 2739–2750 (2019); *J Geol Soc India* 78, 226–232 (2011)

Answer 7. We have incorporated a paragraph in Introduction, which comprises the details of basics of fractal dimension methods such as box counting and Hausdorff. In addition, we have also incorporated the details about common fractal and multifractal methods used in analysis of geomagnetic signal. We have also incorporated the suggested references relevant to present study. (line number 153-172)

Comment 8. Discussion section is well-done. However, I would like to see more discussion concerning the confrontation of the fractal methodology used with observations. How fractal dimensions have been used in the present work? any estimation about their values? how they are correlated to the geometry of the region studied?

Answer 8. We have revised and incorporated the discussion about fractal methodology and their confrontation with observation several previous study, and also discussed about challenges and ambiguity in the interpretation of earthquake precursor studies. The previous studies in these domain supports and motivates to conduct such studies. The methodological section incorporates the steps involve in computation of fractal dimension. The changes in fractal and multifractal parameters of geomagnetic data indicates the different characteristics of SEM signatures to link the lithospheric processes in the region of earthquake preparation zones (explained briefly in discussion section).

Comment 9. The statistical self-affine fractal yielded from the power spectrum deserves also to be more elaborated.

Answer 9. In the present work, we have computed the self-affine fractal dimension of geomagnetic signal using Higuchi method, not from power spectrum method. The previous study (Hattori et al., 2004a; Gotoh et al., 2003; Smirnova et al., 2004) has also mentioned that Higuchi method is more consistent and reliable than other methods, and accordingly we have used Higuchi method in our study. Thus, we believe that the incorporation of self affine fractal dimension from power spectrum method in the manuscript is irrelevant.

Comment 10. Is there any estimate of the evolution of the signal energy with scale, together with the relationships between the features advocated?

Answer 10. In the present study, a notable and significant increase and then decrease to its original value is observed in fractal, multifractal and diurnal ratio parameters. For example, diurnal ratio anomaly increase from 1.21 to 1.26 then reached to 1.21 to 1.31 then return to original value 1.21 prior to earthquake 52 and 61. Similarly, enhancements in f_D and h_w observed from 1.35 and 0.24 and reached to 1.38 and 0.25 then return to its original value prior to earthquake 56 and 49 respectively. A similar observation also marked prior to the clustered of earthquake 46-48 and 53-55. These increments are attributed to the evolution of energy and then decrease prior to earthquake as seen previous analysis (Hayakawa and Ito, 1999). The earthquakes occurred in the study region are in very narrow range of magnitude (M 4.5-5.3) and their associated energies are also too close to incorporate the characteristics of energy in our analysis. However, the more distinguish characteristics of earthquakes such as epicentral distance, focal depth, and associated faults are correlated with enhancements or EM signatures to understand the fractal nature of pre-earthquake processes. Thus, present study restricted on moderate magnitude earthquakes occurred on different faults within relatively short time intervals. Hence, at few instances it becomes difficult to correlate these enhancements or EM signatures with occurrence of specific earthquakes; rather the sequence

of earthquakes would possibly decide the fractal nature, which may be deciphered based on a long time series of data. These limitations restrict us to relate the observed significant changes in geomagnetic signal with the magnitude and energy characteristics of earthquake and underlying processes.

Comment 11. Please one more time, introduce fractal dimensions in a proper way. This is important for readers. Further, multifractal study lead us to understand the spectra of fractal dimension. FDs have been used in various fields of studies. Some well-known references are: *Philosophical Magazine Lett.* 85(1) 33–40 (2005); *Chaos, Solitons & Fractals* 128, 71-82 (2019); *Acta Biotheoretica*, 52(1): 41-56 (2004); *Journal of Thermal Stresses* 44, 899-912 (2021); *Geographical Analysis*, 35(4): 310-328 (2003); *Acta Mechanica* 232, 1413-1424; (2021); *Chaos, Solitons & Fractals*, 35(1): 85-98 (2008); *Advances in Space Research* 74 (5), 2510-2529 (2024); *Discrete Dynamics in Nature and Society*, Vol. 2010, Article ID 194715, 22 pages (2010); *Chaos, Solitons & Fractals*, 45 (2): 115–124 (2012); *Chaos, Solitons & Fractals*, 49(1):47-60 (2013) *Dynamics of Atmospheres and Oceans* 106, 101459 (2024) A revision is required.

Answer 11. We appreciate the reviewer's comment suggesting more elaborate description of fractal dimensions. Accordingly, we have revised the Introduction section. We have first explained the fundamentals of the fractal method and later the common methods used in analysis of time series of geomagnetic signal. At the end, we explained advancement of fractal into spectra of fractal i.e. multifractal and the common multifractal methods used in recent study for analysis of geomagnetic signal. We have incorporated few relevant references from suggested list. (line number 90-173).