Referee 1:

Dear Editor-in-Chief,

We take this opportunity to thank you, and Referee 1 for your thoughtful comments on our manuscript which helped us in improving the manuscript. We hope that the answer of each major and minor comment will meet your expectations. The comments of the reviewers and their replies are listed here one by one. The line number mentioned in reply of each comment is correspond to revised plain manuscript and it may vary from line number of track change revised manuscript. The comments of referee are in red color text and answer of author is in normal font and black color.

Major Problems:

Comment 1. The authors applied two methods to measure the fractal dimensions. They should simply describe the methods and clearly explain the parameters. For example, the authors should explain the definitions of 'length' and 'k' in Figure 1.

Answer 1. We have revised the methodological section and incorporated the sentences and equations which describe the methods more clarity and explain the parameters involved in both the methods. The revisions also include the definition of 'length' and 'k' used in Figure 1. The revised methodological section is incorporated in the revised manuscript accordingly (line number 128-175).

Comment 2. The authors must use a testing example to describe the way applied to estimate the values of multifractal spectrum, i.e., h_w, and to explain whether or not the estimated values are reliable. This will help me to accept the results.

Answer 2: For the testing example, we have taken the 128 data samples of vertical component of geomagnetic field on 13 May, 2019 and 01:00:00 to 01:02:08 hrs (figure 1 f) to explain the way multifractal spectrum values (hw) is estimated. The estimation of multifractal spectrum using wavelet leader technique comprises of following four steps:

(i) In the first step we applied the discrete wavelet transform and decomposed the signal at five levels and restored the values of detail and approximation wavelet coefficients (Figure 1 a-f).

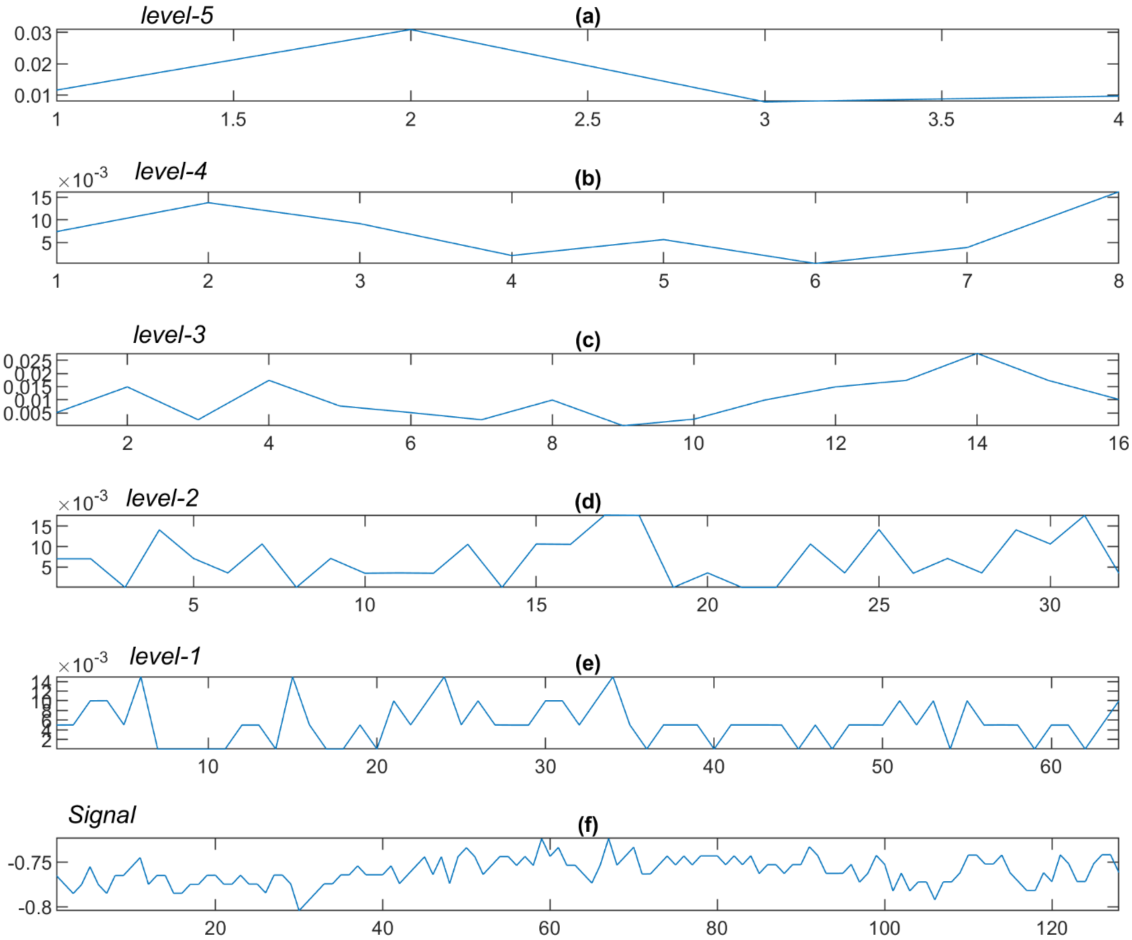

Figure 1. the test signal (f) and its decomposition at level 1 to 5 (e to a) using DWT transform.

(ii) The detail wavelet coefficient is used for computation of wavelet leaders (w_l) from each scale shown in figure 2.

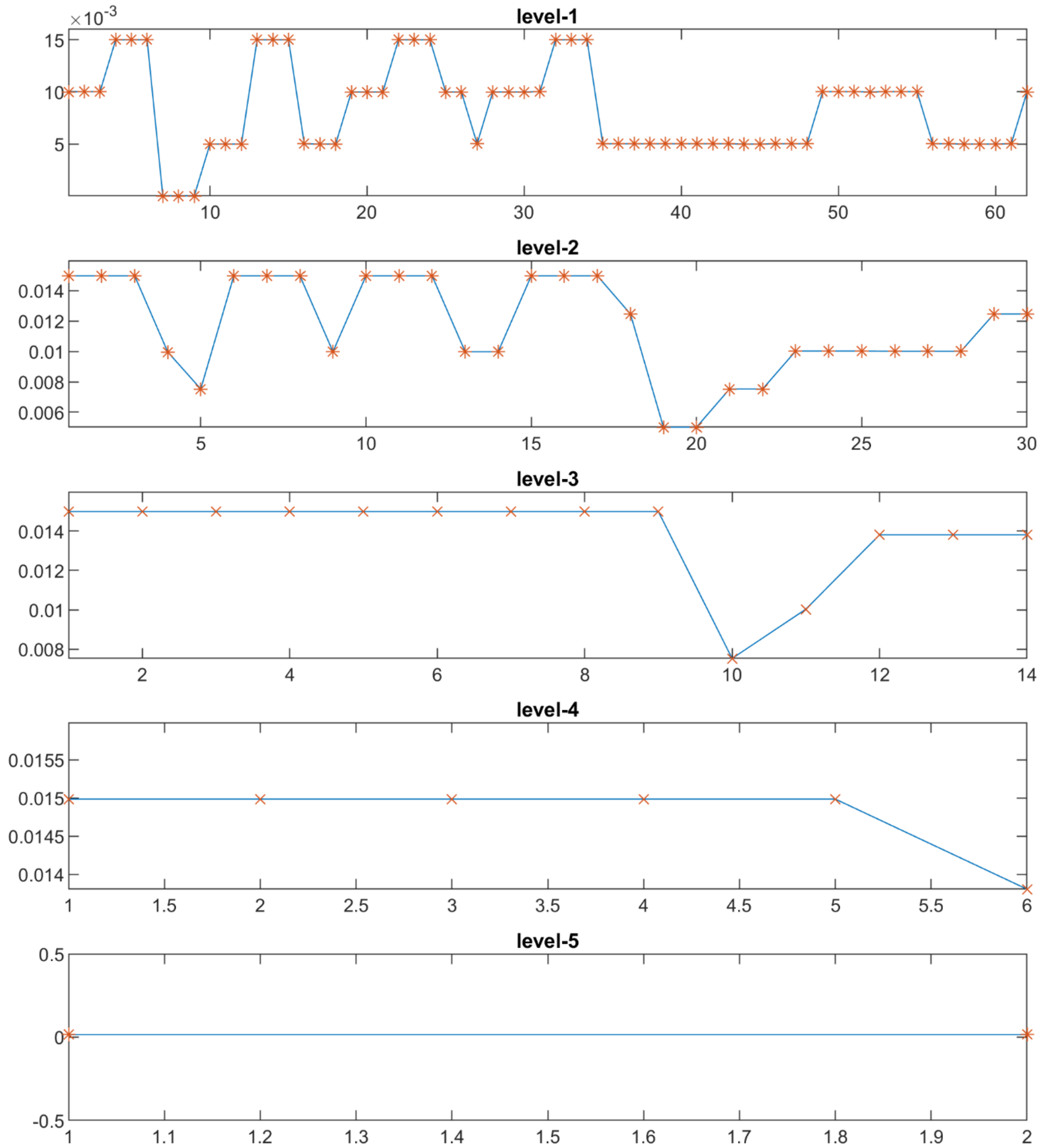

Figure 2. Wavelet leader selected from detail wavelet coefficients at level 1 to 5 (from top).

(iii) The w_l estimated at each scale is used to compute multiresolution structure function of multifractal parameter $\varphi_q, D_q, H_{q,}$ and C_p at linearly space moment order (q=-5 to +5), in which $D_q, and\ H_q$ are the parameters of the multifractal spectrum. The equations involved to compute these parameters are explained clearly by Jaffard et al. (2007) and Serano and Figliola (2009).

(iv) The variation of $D_q, and\ H_q$ from scale 2 to 5 at moment order q is shown in figure 3a and b respectively.

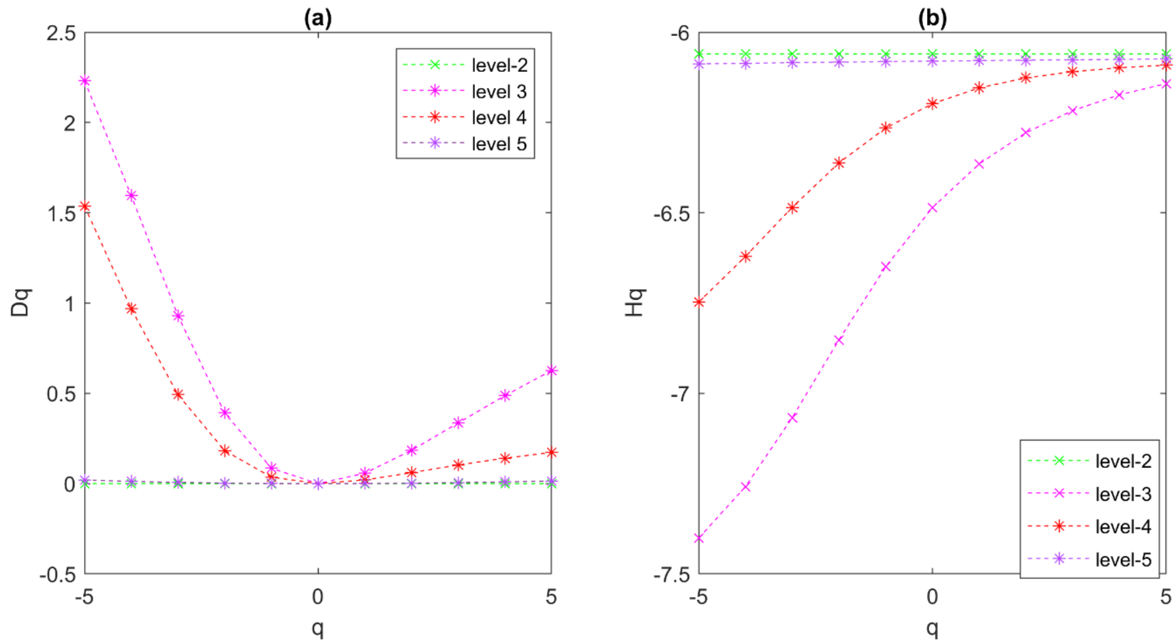

Figure 3. The variation in multifractal parameter (a) D_q and (b) H_q with moment order q at level 2 to 5.

(v) At this stage, we have the values of multifractal parameters at scale one to five and moment order q. The final values of multifractal parameters correspond to q (-5 to +5) is the slope of linear regression of multifractal parameters measured at different scales verses log of scales. Thus, each value of multifractal parameters ($\varphi_q, D_q, H_{q,}$ and C_p) are now available with respect to moment order q(-5 to +5). The variation of $\varphi_q, D_q, and\ H_{q,}$ with respect to q is shown in figure 4 a-c respectively, and multifractal spectrum ($H_{q,}$ vs D_q) shown in figure 4d.

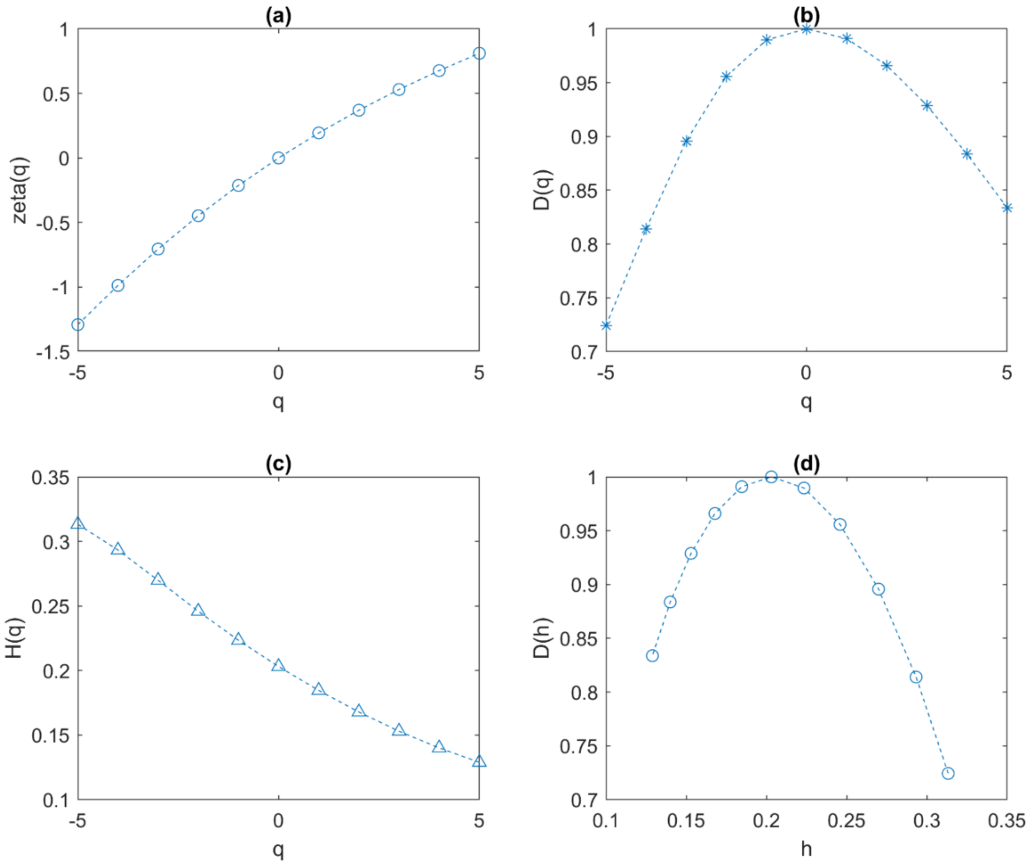

Figure 4. The variation in final multifractal parameters (a) φ , (b) D, (c) H with respect to moment order q and the spectrum of multifarctal parameter (D $vs. h$) is shown in (d).

To further establish the reliability test of the computed multifractal spectrum values, we have tested this method on four different types of synthetic signals with known scaling exponents h1(0.2), h2(0.4), h3(0.6), and h4 (addition of h1, h2, and h3 in series). The small exponent indicates the less correlated or noisier signal, whereas signal of large exponent indicates high correlated or more smooth (figure 5) data. For multifractal, the disturbed signals are expressed through higher degree of multifractal nature or large spectrum width than the spectrum width of less disturb or smooth signal i.e. spectrum width of h4>h1>h2>h3. Thus, we can say that the values are reliable and can fulfil the objective on application of geomagnetic data.

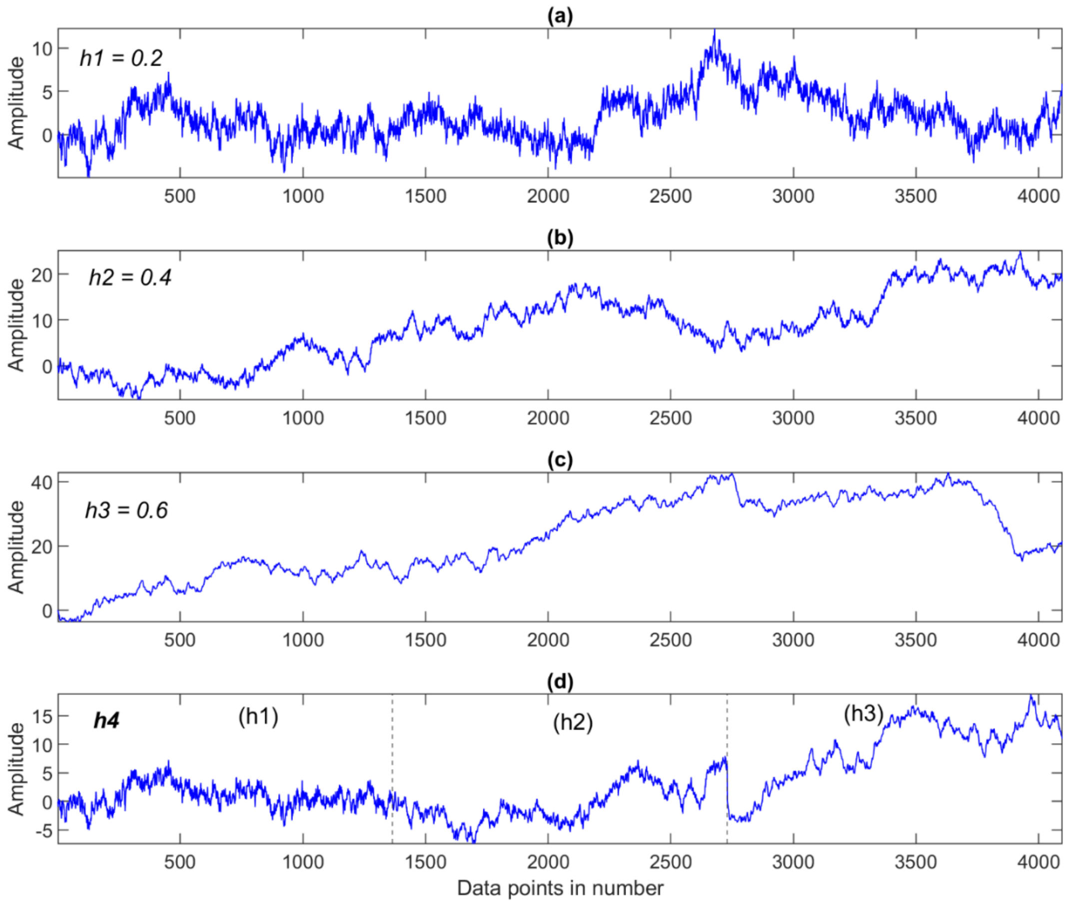

Figure 5. The synthetic signal generated at hurst exponent (a) 0.2, (b) 0.4, (c) 0.5, and (d) combination of all three in series.

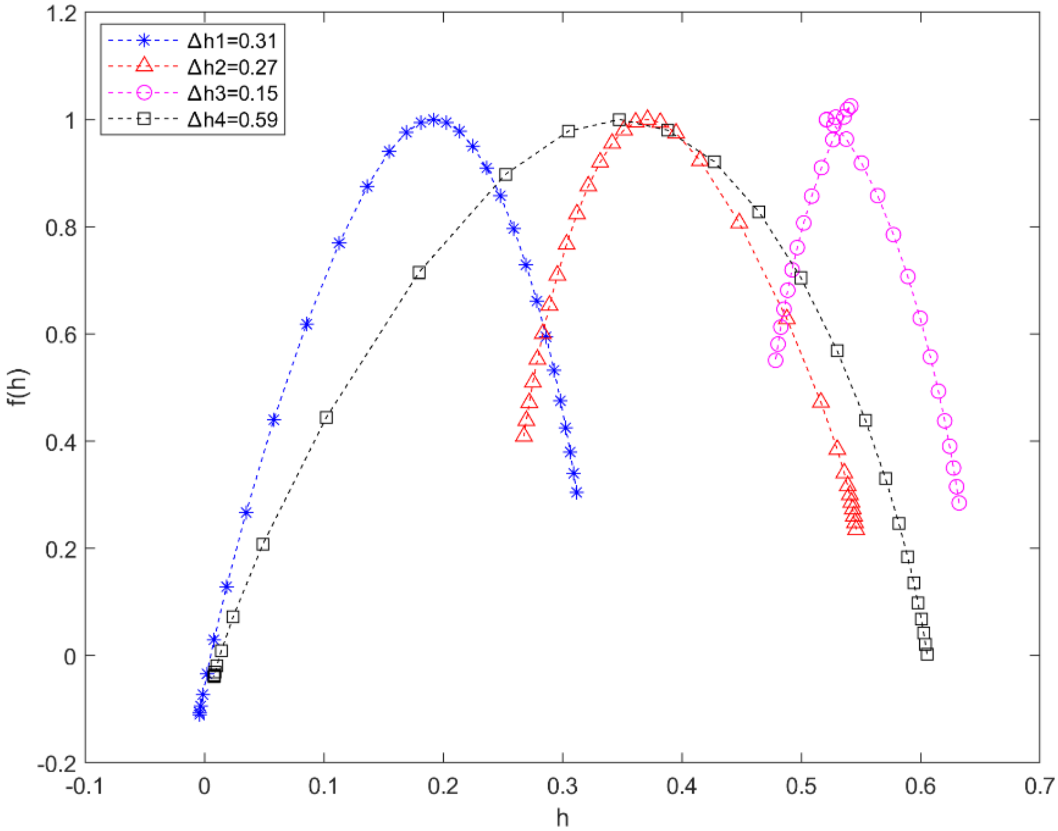

Figure 6. The multfractal spctrum of signal h1, h2, h3, and h4 showing the degree of multifractality**.**

A numerical simulation (synthetic test) of fractal and multifractal on fBm signals performed and has been incorporated to revised manuscript (line 225-233 and line 268-272) and supporting document.

Comment 3. The English writing should be substantially re-written because there are many grammatical and typo errors. Meanwhile, the statements should be re-organized

Answer 3. We have improved English syntax throughout the manuscript.

Comment 4. In Table 1, the authors should replace 'Mod' and 'Large' for Mag (magnitude), 'Mod', 'Shallow', and 'Large' for 'Foc. D.' (Focal Depth),' and 'Mod', 'Small', ad 'Large' for 'Epi. D.' (Epicentral Distance)' by the magnitude range, focal depth range, and epicentral range in numbers.

Answer 4. Table 1 is revised and also the ranges of magnitude, focal depth, and epicentral distances listed in table caption. The revised table is incorporated at the end of this comment and answer section (Page 29).

Minor Problems

Comment 5. The abstract is not concise.

Answer 5. We have re-written the abstract. The revised abstract is now included with 200 words, which also falls under the journal's norm (100-200 words). The modified abstract is included in revised manuscript (line 9-22).

Comment 6. It is better to provide a figure to show an example of observed Z-component seismo-electromagnetic (EM) signatures.

Answer 6. To observe the EM signatures in vertical component of geomagnetic field in night time data (22:00-02:00), we have selected two quite days (25 May and 3 Aug, 2019) in which one (25^{th} May) is interfered by EM field, while second (3 Aug) is not interfered by EM field. Figure 7a, b showing the field on and clearly deciphers the significant fluctuations in the field on 25^{th} May, 2019 even on night time quite data, while field on 3^{rd} Aug, 2019 does not showing such fluctuations on quite day. A significant enhancement in hw (figure 7c) and hwp (figure 7d) also marked on 25^{th} May, 2019, while there in no such enhancements marked in hw and hwp on on 3^{rd} Aug, 2019. This example of observation has been also incorporated in revised manuscript (line 410-415) and supporting document.

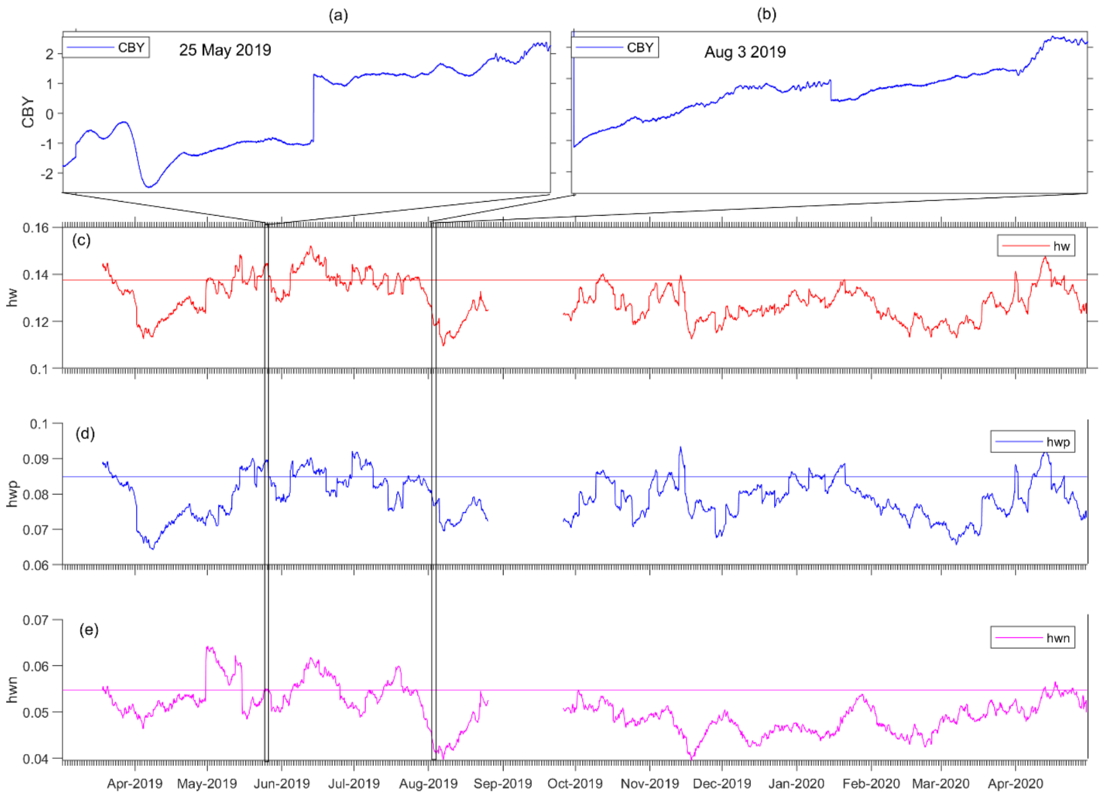

Figure 7. The night time data of vertical component of geomagnetic field on (a) 25th May, 2019 and (b) 3rd Aug, 2019. The multifractal component of (a) hw, (b) hwp, and (c) hwn from Mar, 2019 to April, 2020.

Comment 7. The quality of figures should be improved.

Answer 7. We have replaced all the figures in the revised manuscript with modified high-resolution figures.

Referee 2:

Dear Editor-in-Chief,

We take this opportunity to thank you, and Referee 2 for your thoughtful comments on our manuscript which helped us in improving the manuscript. We hope that the answer of each major and minor comment will meet your expectations. The comments of the reviewers and their replies are listed here one by one. The line number mentioned in reply of each comment is correspond to revised plain manuscript and it may vary from line number of track change revised manuscript. The comments of referee#2 are in red color text and answer of author is in normal font and black color.

Yours sincerely,

Rahul Prajapati, Kusumita Arora

Referee#2. Comment and Answer:

I have checked the present work. The topic addressed is well-known in literature and of particular importance. A series of flaws arise that I would like to ask authors to consider them in their revision. These are listed below:

Comment 1- The importance of fractals must be well-introduced, justified and elaborated. It is applied widely in several fields including seismology and earthquakes sciences. Applications of fractal geometry and fractal dimensions to study various seismic activities have been also explored in details in various studies based on dissimilar methodologies See the present missed references in the field.

Chaos, Solitons & Fractals **14**: 917-928 (2022); Acta Mech. **233**:2107-2122 (2022); Geophys. J. Int. **179**(3): 1787-1799 (2009); Phil. Trans.: Phys. Sci. Eng. **348**(1688): 449-457 (1994); Chaos, Solitons & Fractals **167**: 113000 (2023); Chaos **31**: 043124 (2021); Nat. Haz. Earth Syst. Sci. **23**: 1911-1920 (2023)

Multifractal measures, especially for geophysicist. In: Scholz, CH, Mandelbrot BB (eds) Fractals in geology and geophysics, Birkhäuser Verlag, Basel, pp. 5-42.

Please justify the importance of fractals and multifractals in sciences. See the missing references

The Fractal Dimensionality of Seismic Wave. In: Yuan C, Cui J and Mang HA (eds). Computational Structural Engineering, Springer, Dordrecht.

Fractal models of earthquakes dynamics. Review of Nonlinear Dynamics and Complexity (eds) Schuster HG, pp. 107-158, Wiley-VCH Verlag GmbH & Co. KGaA, Weinheim.

A fractal model of earthquake occurrence: Theory, simulations and comparison with the aftershock data. J. Phys.: Conf. Ser. **319**, 012004.

Fractal Concepts and their Application to Earthquakes in Austria. In: Lehner, F.K., Urai, J.L. (eds) Aspects of Tectonic Faulting. Springer, Berlin, Heidelberg, 2000

Fractal concepts in surface growth. Cambridge University Press., 1995.

Scienze Fisiche Naturali. 31(1):203–9. (2020); Cont. Mech. Therm **34**: 1219-1235 (2022); . Sci. Rep. **10**: 21892 (2020); Remote Sens. **11**, 2112 (2019); Dynamics of Atmospheres and Oceans 106, 101459 (2024); Tectonophysics. 722:154–62 (2017); Pure Appl Geophysics. 172(7):1909–21 (2015); *Pure Appl. Geophys.* **176**, 2739–2750 (2019); Hydrobiologia 851, 2543–2559 (2024); Chaos Solitons and Fractals 178, 114317 (2024); Thermal Science and Engineering Progress 45, 102145 (2024)

Answer 1. We appreciate the refree#2 for this suggestion to include the study of application of fractals and multifractals in field of seismology. The various application of fractals in science as well as infield of earthquake and seismology introduced in the revised version of manuscript (line 76-96). The references also incorporated accordingly.

Comment 2. The methodological schemes addressed in Section 2 requires a careful rewritten. It is not really clear what authors aim to.

Answer 2. We have revised the methodological section and incorporated all relevant sentences and equations to describe the methods with more clarity. All the revised of methodology in incorporated in revised manuscript (line number 128-175).

Comment 3. The analysis done is fine, however, can we improve the numerical simulations? Can we dress a table clarifying data used?

Answer 3. For the numerical simulation of fractal and multifractal analysis in present study, we preferred to simulate four different types of monofractal signals with known scaling exponent h1(0.2), h2(0.4), h3(0.6), and a multifractal signal h4 (addition of h1, h2, and h3 in series). The small exponent indicates the less correlated signal or noisier than signal of large exponent indicates high correlated or smoother (Figure 1). From the theoretical approach, the fractal dimension of more noiser or less correlated signal should be larger than smoother or correlated signal. The fractal dimension of h1, h2, and h3 calculated from Higuchi method is 1.7, 1.6, and 1.4, while for h4 is 1.6 (Figure 2). For multifractal signal h4 the fractal dimension is lower than the h3 even it is more heterogeneous than h3. From the concept of multifractal, the more noisy or heterogeneous signal encompasses through higher degree of multifractal nature and large spectrum width than the spectrum width of less disturb or smooth signal i.e. spectrum width of h4>h1>h2>h3. The spectrum width computed with the same procedure as discussed above is shown in Figure 3, which clearly deciphers that the spectrum width of h4>h1>h2>h3. Thus, the multifractal analysis shows the true and generalised nature of heterogeneity of multifractal signal from width of spectrum. Thus, the testing of synthetic signal using fractal and multifractal approach indicates the efficacy of method to reveal the degree of complexity or heterogeneity or disturbances in signals.

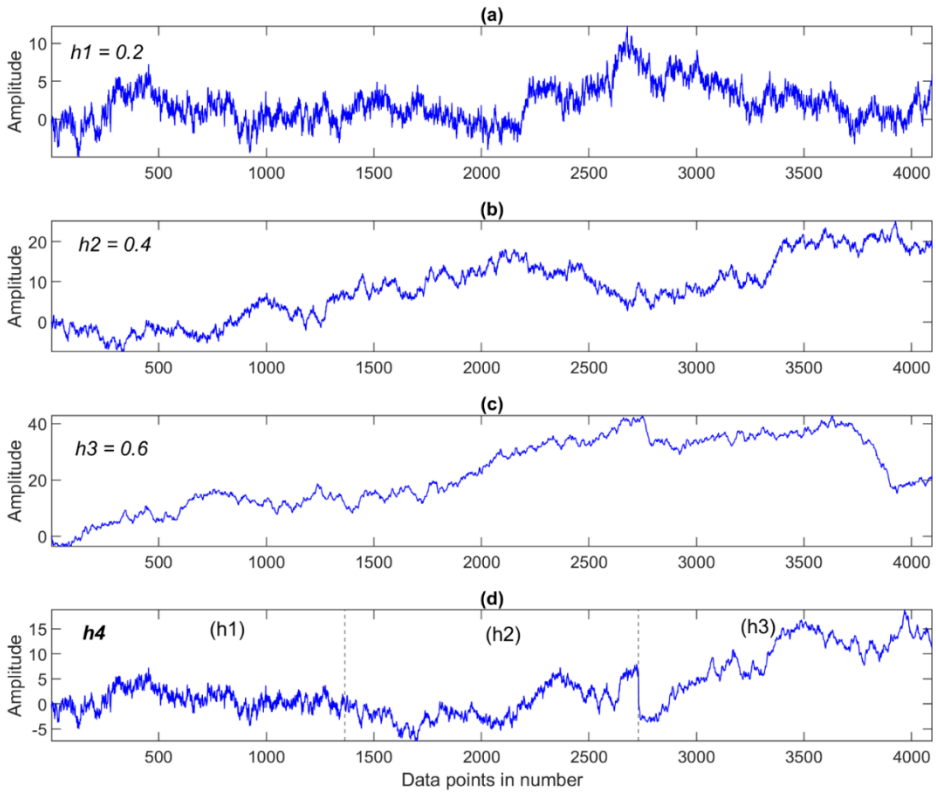

Figure 1. The synthetic signal generated at hurst exponent (a) 0.2, (b) 0.4, (c) 0.5, and (d) combination of all three in series.

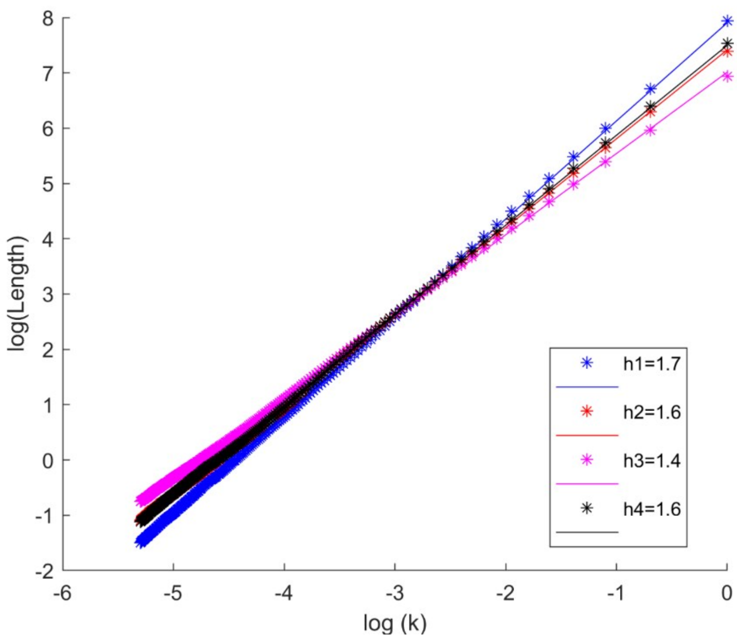

Figure 2. Fractal dimension of synthetic signal h1, h2, h3, and h4 from Higuchi method.

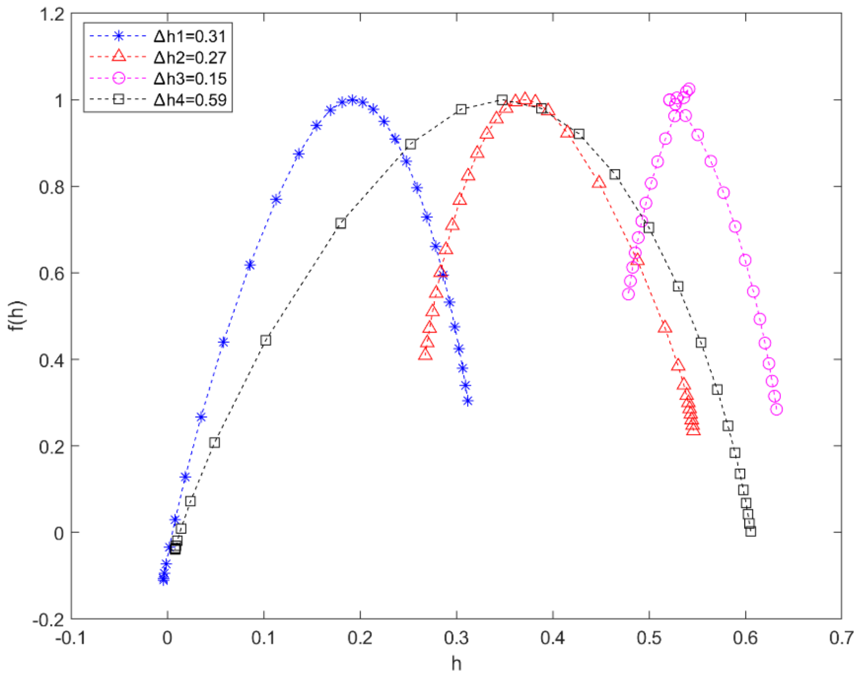

Figure 3. The multifractal spctrum of signal h1, h2, h3, and h4 showing the degree of multifractality**.**

The above discussed numerical simulation of fBm signal and its analysis with fractal and multifractal approach have been incorporated into the revised manuscript (line 225-233 and line 268-272) and supporting document. The earthquake CatLog used in the present study is added in supplementary as T1 and Table T2 – T4 summarises the correlation of enhancements in fractal dimension and each parameter of multifractal component.

Comment 4. Regarding Holder exponent, this is an important factor. The analyses done seem not totally clear. How it is related to fractal dimensions? any estimate for the fractal dimension anyway from observations? What about variations of the Hurst exponent?

Answer 4. The Holder exponent is a set of Hurst exponent i.e. the generalised version of Hurst exponent, which has efficacy to estimate the generalised nature of multifractal signal. The range of variations, maximum and minimum values of Hurst or Holder exponents, contain the information of different characteristics of the signal (discussed in methodology section). In present study we used Holder exponent to analyse all different characteristics of heterogeneity of signal.

From the spectrum method of calculation of fractal dimension, the slope obtained from log-log plot between power of signal and its frequency component is called Hurst exponent, and this Hurst exponent is directly related to fractal dimension from following relation

$$H=5-2D$$

Where H is Hurst exponent and D is fractal dimension.

In the present study, we have estimated the monofractal dimension using Higuchi method because it is more reliable than other methods for time series data (discussed in methodology section). In figure 4 we have observed variations in monofractal dimensions, where the significant enhancements are observed at seven instances. These significant enhancements in fractal dimensions indicate the nature of heterogeneity of high frequency characteristics possibly associated with micro-fracturing processes prior to earthquakes. The variation in Hurst exponents is termed as a holder exponent, which is used for delineation of different characteristics of heterogeneity embedded in the signals.

Comment 5 - Any relation between the energy of earthquake swarm and the Hurst exponent of random variations of the magnetic field of the region studied? Earthquakes represent this change in state of equilibrium which are commonly perceived to occur due to the sudden release of energy in highly stressed zones and they repeatedly occur until the system is once again back to its equilibrium state.

Answer 5 – We appreciate to reviewer the for this comment. To find a relation between energy of earthquake swarm and variation in Hurst exponent, we required a long duration data and the occurrences of earthquake swarms for 5-6 times in the same duration and different magnitude range of earthquakes. In the present study, we have data for duration of 14 months only and range of magnitude in only 4.5-5.3. Thus, we believe that the available data is not enough to establish the relation between earthquake energy and variation in Hurst exponent.

I would like to read the revised version of this work