Response to Editor

General comments

I thank the authors for this new version of their paper. I consider it can now be accepted from the scientific point of view. But there remain a number of points to be improved as concerns the edition of the paper. Here are examples.

[Reply to this comment]

We thank the Editor for the positive evaluation of our work. We also sincerely appreciate the careful reading of our revised manuscript and the constructive suggestions for improving its clarity, consistency, and accuracy. Below, we provide detailed point-by-point responses to each of the comments.

Specific comments

R1. In response to a comment by Referee #2, the authors now mention how the Nature Run has been produced (ll. 123-124). However, that is done as a passing remark when describing how "observations" have been obtained. And the specific notation of the Run (X_t^{NR}) is introduced only on l. 230.

I think all information on the Nature Run should be introduced in the first place it is mentioned (ll. 115-116).

[Reply to this comment]

Thank you for the suggestion. We agree that the Nature Run (NR) should be introduced at its first mention. Accordingly, we have revised the text at the first occurrence of the NR to provide a concise but complete explanation. Specifically, we now explain that the NR is a long-term trajectory generated by simulating the Lorenz-63 system without any control inputs. It is introduced as a reference representing the true system behavior, serving both as the basis for generating noisy observations and as the benchmark for evaluating control performance. We also introduce the notation X_t^{NR} at this point.

[Revised parts]

The following part has been added to **Section 2.2 on Page 4** of the revised manuscript.

"… To assess the effectiveness of this control strategy, a reference trajectory that represents the "true" system behavior is necessary. This reference, known as the Nature Run (NR), is generated by running a long-term simulation of the uncontrolled Lorenz-63 system. We denote the state of the NR at time t as $X_t^{NR} \in \boldsymbol{R}^3$*, where the three components correspond to the variables x, y, and z. Within the CSE framework, the NR serves as the ground truth. However, its true state variables are not directly observable …"*

R2. Ll. 252-253, *Miyoshi and Sun […] conducted experiments […] for 40 different initial conditions.* Did those 40 different initial conditions correspond to 40 different Nature Runs, or what?

[Reply to this comment]

We appreciate the Editor's careful reading and the important clarification request. Yes, each of the 40 initial conditions used by Miyoshi and Sun (2022) corresponds to a distinct Nature Run. We have clarified this in the manuscript to avoid ambiguity.

[Revised parts]

The following part has been added to **Section 4 on Page 9** of the revised manuscript for clarification.

"… (each initial condition corresponds to a different NR) …"

R3. Similarly, ll. 270-271, *The initial values of the system were set …* This now refers to experiments performed with MPC. I understand that these initial values were not those of the Nature Run. But what were they exactly? How do they differ from the 40 initial values of Miyoshi and Sun (which are again used for MPC, as mentioned on l. 285)

[Reply to this comment]

Thank you for this important question. In short, the mentioned illustrative initial value $(x, y, z) = (8.20747939, 10.0860429, 23.86324441)$ is distinct from, and not part of, the set of 40 initial conditions used in the comparative evaluation.

Specifically, these initial values correspond to a single illustrative trajectory used to visualize the dynamics and control process in Figure 2. This setting was used solely to demonstrate the effectiveness of the proposed MPC approach. For the main control performance evaluation, including Tables 2 and 3 and the associated success rate comparisons, we used the exact same 40 initial conditions as in Miyoshi and Sun (2022), and applied them consistently to both the CSE and MPC experiments.

[Revised parts]

The following part has been added to **Section 4.2 on Page 11** of the revised manuscript.

"The illustrative example above demonstrates the effectiveness of our control method. To further assess its feasibility, we conducted systematic evaluations using a broader set of initial conditions. Specifically, we conducted simulations for 40 different initial conditions to examine the control success rate …"

R4. The value of the observation period T_a (which I understand is always T_a= 8 steps) is not mentioned in Table 1.

[Reply to this comment]

We thank the editor for this point. We have added the observation period $T_a = 8$ to Table 1 under “Parameters common in CSE, MPC, and MPCIL”.

[Revised parts]

In **Table 1 on Page 10** of the revised manuscript we added

“$T_a = 8$”

R5. L. 232, ... *the state transition from time t + k to time t + k + 1.*

[Reply to this comment]

We thank the Editor for the suggestion and revised the wording accordingly.

[Revised parts]

The following part has been revised in **Section 3.3 on Page 8**.

“... the state transition from time t + k to time t + k + 1 ...”

R6. These are only examples. I suggest that the authors check carefully their paper for possible inconsistencies, ambiguities or inaccuracies.

[Reply to this comment]

We appreciate this suggestion and have performed a thorough editorial review of the manuscript. In addition to the revisions above, we identified and corrected several minor inconsistencies and ambiguities. Below is an example:

- In Figure 3, we show results for $m = 1, 5, 10$, but the meaning of m was not explained in the caption. We revised the figure caption to clarify this.

[Revised parts]

The following part has been added in the caption of **Figure 3 on Page 13** of the revised manuscript.

“... Here, m denotes the number of training iterations during optimization.”