Response to Referee #2

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We thank the reviewer for the positive feedback and are glad that most of our revisions were well received. Below we provide a point-by-point response to the remaining comment.We hope this revision addresses the reviewer's concern.

1 General Comments

Comment 1.
In the response to comment 9, the authors state that "Floes that are not observed at a given cycle are not excluded; instead, their observation-error variance is inflated according to Eq. (50), thereby reducing their influence in the analysis". This seems to conflict with Eq. (54) which sets out a different criterion, based on the local total water content and specifies conditions under which a floe cannot be observed at all. Moreover, if "not observed" refers to an inflated observation error (as per Eq. 50), it appears that the same equation also determines the observation level (plentiful or sparse), based on the mean total water content [qt(x,t)]. I would appreciate clarification on how observational availability and uncertainty are operationally determined. In this context, I think that it would be beneficial to merge paragraphs 3.2 and 3.3.1 and to make a clear distinction between true floes' coordinates and observations, adding an equation that links the two.

Response: We thank the reviewer for identifying the potential ambiguity and for the helpful structural suggestion. We agree that confusion may arise. Indeed, in Eq.(54)), "not observed" denotes a floe whose observation–error variance is inflated according to Eq.(50)) and Adjusting the threshold in Eq.(50)) therefore changes the observation level. Here, Eq (54). also reminds readers that, in the data assimilation, LETKF scheme, a floe is treated as unobserved and therefore assigned large uncertainty—whenever q_t exceeds this threshold. We follow the reviewer's suggestion to merge paragraphs related to observability in Sec 3.3.1 to Sec 3.2. The revised part in Sec 3.2 yields the following:

> *To represent observational uncertainty in DA, we use the total water content $q_t(\mathbf{x}, t)$ as a controlling factor. Above each floe, we calculate the mean total water content, $[q_t(\mathbf{x}, t)]$ at time t:*
>
> $$[q_t(\mathbf{x}, t)] = \frac{1}{|\Omega_l|} \int_{\Omega_l} q_t(\mathbf{x}, t) d\mathbf{x}. \tag{1}$$
>
> *This mean value, $[q_t(\mathbf{x}, t)]$, encapsulates the spatial distribution of water content above each ice floe, serving as an approximation for the uncertainty in observations. Variations in $[q_t(\mathbf{x}, t)]$ from one floe to another can indicate the degree of uncertainty inherent*

in the observational data, as it reflects the heterogeneity in the physical characteristics of the ice. In particular, we set a threshold, $\widetilde{q}_t$*, such that the observational uncertainty* σ_l^{obs} *is given by the following: for l-th floe,*

$$\sigma_l^{\text{obs}}\big(\mathbf{x}_l(t)\big) = \begin{cases} 5\times 10^2\, m, & \text{if } [q_t(\mathbf{x},t)] < \widetilde{q}_t \text{ (small observation uncertainty)}, \\ 2r_l, & \text{if } [q_t(\mathbf{x},t)] \geq \widetilde{q}_t \text{ (large observation uncertainty)}. \end{cases} \tag{2}$$

where r_l *denotes the radius of the l-th floe and* $\mathbf{x}_l$ *its trajectory. It is important to note that when the mean total water content over the floe,* $[q_t(\mathbf{x},t)]$*, is high, which indicates significant cloud cover and can be classified as* unobserved*, its position can still be approximated. However, these estimates are often highly inaccurate. Consequently, in the data-assimilation setting, we assign floes classified as* unobserved *a markedly inflated observational uncertainty, taken here as twice their radius.*

Comment 2.
An additional comment regarding Eq. 50: is there a defined lower bound on the observational uncertainty in the case of significant cloud cover? According to the current formulation, it seems possible that a small floe with high mean total water content could yield a lower observational uncertainty than a floe under clear conditions. This seems counterintuitive and may require a justification.

Response: We thank the reviewer for raising this question. In our scheme a floe classified *unobserved* (i.e. $[q_t] \geq \widetilde{q}_t$) is always assigned

$$\sigma_l^{\text{obs}} = 2r_l.$$

Because this uncertainty is on the order of the floe's diameter—even for the smallest floes—the observation contributes negligibly to the analysis. Also, it it is worthwhile to note that in all test cases the minimum floe radius is 8×10^3 m; hence a floe beneath thick cloud is assigned at least $2r_l = 1.6 \times 10^4$ m of uncertainty, far exceeding lower bound of uncertainty when the floe is under clear sky of 5×10^2 m.

To clarify this point for readers, we have added the following remark in the end of Sec. 3.2. in revised manuscript:

> ***Remark.*** *An ice floe is classified as "*unobserved*" whenever the mean total water content over the floe exceeds the threshold,* $[q_t] \geq \widetilde{q}_t$ *(cf. Eq. (50)). In that case we inflate the observation–error standard deviation to* $\sigma_l^{\text{obs}} = 2r_l$*, with* r_l *the floe radius. Because this value is of the same order as the floe's diameter, the associated observation exerts negligible influence on the analysis. Among all test cases the smallest floe radius is* $r_l^{\min} = 8\times 10^3$ *m, so an unobserved floe takes at least* $2r_l = 1.6\times 10^4$ *m of uncertainty, clearly distinguished from the observed case with* $\sigma_l^{obs} = 5 \times 10^2$ *m.*

2 Minor Points

Comment 1.
- Line 309 (line 345 in the revised ms) : Apologies for not being clear earlier — I was referring to the tilde accent, which appears to be a typographical error.

Response: We thank the reviewer for pointing this out and apology for this typo. We have revised the manuscript and correct it to $\widehat{\psi}_{bt,\mathbf{k}}$.