Response to Referee #2

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[May 29, 2025]

1 General Comments

Comment 1.
The governing equations for sea ice floes do not appear to include the Coriolis force. Please clarify whether this was omitted intentionally and, if so, provide a justification.

Response: We thank the reviewer for this insightful remark. Indeed, the Coriolis force is not explicitly included in the governing equations for the sea ice floes presented here. This omission was intentional, as our current model primarily focuses on resolving short-term, localized ice floe dynamics dominated by collision and contact interactions. At these smaller spatial and temporal scales, the effects of the Coriolis force are considerably weaker compared to other forces, such as drag, collision, and contact forces (see, e.g., [Thorndike and Colony, 1982, Steele et al., 1997]; also Chapter 9, "Ice Dynamics," in [Thorndike, 1986]). Nonetheless, we acknowledge the potential significance of the Coriolis effect for larger-scale and longer-term ice floe motions, and incorporating this force explicitly will be considered in future studies to assess its impact comprehensively.

In the revised manuscript, we have added the following in the end of Section 2.2.1:

> *While the current coupling model does not include the Coriolis force on ice floe movement, it is still a valid assumption as this research focuses on short-term, localized ice floe dynamics where collision and drag forces are dominant [Thorndike and Colony, 1982, Steele et al., 1997, Thorndike, 1986].*

Comment 2.
Are the background velocities of ocean and atmosphere considered for the computation of drag forces and torques on the floes?

Response: We thank the reviewer for raising this question. Yes, the background zonal velocities U^a and U^o from the atmospheric and oceanic models are implicitly considered in calculating the drag forces and torques acting on the ice floes. Specifically, the QG models used to generate atmospheric and oceanic flows incorporate these imposed background zonal velocities U^a and U^o, thereby implicitly influencing the resulting flow fields. Consequently, these background velocities directly affect the dynamic responses and trajectories of the ice floes in our simulations.

To clarify the role of background velocities, we have added Remark 2.1 to the manuscript:

Remark 2.1 *The background zonal velocities U^a and U^o in the atmospheric and oceanic QG models are implicitly included in the computation of drag forces and torques on the ice floes. These background zonal velocities affect the flow fields, thereby influencing the ice floe dynamics and trajectories in the coupled system.*

Comment 3.
There isn't any coupling between atmosphere and ocean and the coupling between ice floes and ocean is only one-way. Can you justify this choice? This may have a significant impact on the accuracy of ocean state estimation, which is shown to perform relatively poorly in the data assimilation experiments.

Response: We thank the reviewer for this important comment. Indeed, in the current study, the coupling is limited: the atmosphere and ocean are not directly coupled, and the interaction between ocean and ice floes is one-way—from ocean to ice. This modeling choice was made to isolate and examine the impact of precipitation and its effects on the coupled system, particularly the response of sea ice floes to atmospheric and oceanic forcing under uncertain moisture conditions.

As an investigation of data assimilation in a coupled atmosphere–ice–ocean framework with precipitation, our goal is to build foundational understanding while keeping the system tractable. We acknowledge that omitting full two-way coupling—especially between ocean and ice—may affect the accuracy of ocean state estimation. However, our results still provide important insights into how observational uncertainty from moisture and precipitation propagates through the system.

Extending the current model to include two-way feedbacks between ice and ocean, as well as a fully coupled atmosphere–ocean interaction, is a natural and important direction for future work and will likely improve ocean state estimation in data assimilation settings. Therefore, we have added the following two modifications in the draft:

- We have added Remark 2.2:

  > *In the presence of sea ice floes, the direct coupling between the atmosphere and ocean is relatively weak compared to other components of the system, such as the atmosphere–ice floe interaction. Moreover, the atmospheric and oceanic models have different time scales—specifically, the ocean evolves on a much slower time scale than the atmosphere—and the current study focuses on short-term simulations. As a simplification, we choose to neglect the atmosphere–ocean coupling, which remains a reasonable assumption within the scope of the present modeling framework.*

- In the Conclusion section, we have added the following statement:

  > *As a future direction, we aim to extend the current framework by incorporating two-way coupling between the ocean and ice components, as well as a fully coupled atmosphere–ocean interaction, to better capture feedback mechanisms and improve the accuracy of ocean state estimation in data assimilation settings.*

Comment 4.
Line 229: it is assumed that precipitation causes an increase in ice floe thickness. This may be a reasonable assumption if precipitation is snow, but this is not guaranteed especially during summer and fall and in the MIZ [Boisvert et al. 2023]. Furthermore, solar insolation is set to 1361 W/m2 (Table 4), which is very unrealistic for polar winters.

Response: We thank the reviewer for this helpful comment. The assumption that precipitation increases ice floe thickness is based on the assumption of our model, where precipitation is interpreted as snowfall. We acknowledge that this assumption is most valid during colder months and in regions where snow dominates, and may not hold during summer or fall in the Marginal Ice Zone (MIZ), as highlighted in [Boisvert et al., 2023]. As a first step toward investigating the influence of precipitation on ice floe height, we adopt this assumption; in addition, the ice floe thickness changes does not significantly impact the overall dynamics of the coupled system. In the revised manuscript, we have added the following statement in the end of Section 2.5.1:

> *It is noted that the assumption that precipitation increases ice floe thickness is based on the idealization that all precipitation is interpreted as snowfall. This assumption is most valid during colder months and in regions where snow predominates, and may not hold during summer or fall in the Marginal Ice Zone (MIZ), as highlighted in [Boisvert et al., 2023]. As a first step toward investigating the influence of precipitation on ice floes, we adopt this assumption to explore its impact within an idealized modeling framework.*

Regarding the solar insolation value of 1361 W/m^2, we note that the chosen value corresponds to the solar constant and is used as a simplified representation of solar radiation to mimic an idealized melting process for ice floes. While this value is not realistic for polar winters, the simplification is reasonable given that our current study focuses on a short simulation period of approximately two months. It enables us to isolate and investigate key coupled dynamics in a controlled setting. In the manuscript, we have revised the following sentence in Section 2.5.2:

> *Here, E_s represents the solar insolation, known as the solar constant. While this value may differ in polar winter conditions, it is treated as constant in this study as a simplification appropriate for the short simulation period.*

Comment 5.
Line 464: Clearly state that the "truth" is obtained from the full coupled model and describe the numerical scheme used to solve it.

Response: We thank the reviewer for this helpful suggestion. In the revised manuscript, we have explicitly stated in Line 464 that the "truth" trajectory is obtained by running the full coupled atmosphere–ice–ocean model using high-resolution numerical integration.

The numerical schemes employed for each component of the model are as follows: the atmospheric and oceanic equations are spatially discretized using a spectral method, and temporally integrated using an adaptive third-order Runge–Kutta scheme, following the approach in [Qi and Majda, 2016, Edwards et al., 2020a, Edwards et al., 2020b]. The sea ice component is simulated using the discrete element method (DEM), with time integration performed via a forward Euler scheme.

In the beginning of Section 4, we have added the following in the revised manuscript:

> *The coupled atmosphere–ocean-ice model employs different numerical schemes for each component. The atmospheric and oceanic equations are discretized in space using a spectral method and integrated in time with an adaptive third-order Runge–Kutta scheme, following [Qi and Majda, 2016, Edwards et al., 2020a, Edwards et al., 2020b]. The sea ice component is simulated using the discrete element method (DEM), with time integration using a forward Euler scheme to resolve floe dynamics dominated by contact and drag forces.*

Comment 6.
Line 470: The definition of plentiful and sparse observations is not clear. Have you applied Equation (54) with different values of the threshold?

Response: We thank the reviewer for pointing out this. Equation (54) determines whether the position of the l-th floe is observed at an observation time t_k:

$$\begin{cases} q_{t,m}\big(\mathbf{x}_l^{\mathrm{obs}}(t_k)\big) < q_\epsilon & \Longrightarrow \text{ floe is } \textit{observed}, \\ q_{t,m}\big(\mathbf{x}_l^{\mathrm{obs}}(t_k)\big) \geq q_\epsilon & \Longrightarrow \text{ floe is } \textit{not} \text{ observed.} \end{cases} \tag{54}$$

Here $q_{t,m}$ is the total water that is from the atmospheric model at mth floe location at time t and q_ϵ is a tunable threshold. By increasing q_ϵ we expect more floes are observed and thus have plentiful observations, whereas a lower q_ϵ admits less observed ice floes and yields sparse observations. Based on this principle, we choose two different regimes of q_ϵ which yield two different percentage of floes that are observed:

- **Plentiful observations**: at least 70% of the floes satisfy "observed condition" in Eq. (54) and therefore are observed;
- **Sparse observations**: only about 30% of the floes satisfy "observed condition" in Eq. (54) and thus are observed.

These explicit percentages—and the corresponding values of q_ϵ—are now stated directly below Table 5 in the revised manuscript, clarifying how the terms "plentiful" and "sparse" are defined.

Comment 7.
Figure 6-7-8: It would be helpful to include the background mean for visual comparison with the analysis.

Response:
We thank the reviewer for this helpful suggestion. To make a more complete visual comparison, we have updated Figures 6–8 to include the background (prior) mean with the analysis (posterior) and truth. This update allows readers to better assess the improvement obtained from data assimilation by comparing the prior and posterior states relative to the true state.

Comment 8.
Line 485: The manuscript states that the ocean field has limited observability. Could you elaborate on this and possibly provide a more quantitative analysis?

Response: We agree that a systematic, quantitative assessment of ocean–state observability would be valuable; however, such an analysis would require dedicated sensitivity experiments (e.g. varying mooring density, tracks, and electromagnetic under-ice soundings) that lies beyond the scope of the present study, whose focus is the feasibility of assimilating floe trajectories under cloud "contamination". A rigorous observability study will be the subject of future work.

To avoid any ambiguity in the current manuscript, we have slightly rephrased the relevant sentence (Line 605) to read

> *"Because existing in-situ and remote sensors sample only limited portions of the stratified upper ocean, its full three-dimensional state remains only partially observable in the present experiments; a quantitative assessment of this limitation is left for future study."*

Comment 9.
Figure 8 shows how DA helps to recover the trajectory of a specific floe under sparse observational conditions. However, it seems that the floe under scrutiny is observed at every assimilation cycle within the period analyzed. If this is not the case, please indicate how many times the floe is observed and show this visually.

Response: We thank the reviewer for pointing this out. In the manuscript, the term "sparse observations" refers to the scenario in which approximately 30% of the floes are observed at each assimilation time, whereas "plentiful observations" corresponds to more than 70%. In both scenarios the assimilation cycle is 24.2 h. Floes that are not observed at a given cycle are not excluded; instead, their observation-error variance is inflated according to Eq. (50), thereby reducing their influence in the analysis. This is why at each assimilation cycle in Figure 8, the floe is always observed; however, the "observed" refers to floe location observed in small uncertainty and the "unobserved" refers to floe location observed in large uncertainty.

Comment 10.
Please add Normalized RMSE values for the prior, in addition to the posterior values, to better assess assimilation effectiveness.

Response: We thank the reviewer for this valuable suggestion. We followed the suggestion, we have added the Normalized Root Mean Square Error (RMSE) values in Tables 6-8 for the prior (background) state in addition to those for the posterior (analysis) state. This inclusion provides a more quantitative assessment for prior and posterior comparison.

2 Minor Comments

Line 181: there is a reference to the background vertical gradients of total water mixing ratio, but Table 4 reports a value for the background vertical gradients of total water. Is there a difference between total water and total water mixing ratio?

Response. We thank the reviewer for pointing out this clarification regarding the terminology and the background vertical gradients.

In this work, we use the term total water to refer to the total water mixing ratio, consistent with the formulation in [Edwards et al., 2020b, Edwards et al., 2020a]. Specifically, the background vertical gradient $\mathrm{d}\tilde{q}_t/\mathrm{d}z$ reported in Table 4 corresponds to the background gradient of the total water mixing ratio. This is also the quantity referenced in Line 181.

To avoid confusion, we have added one sentence in line 435 in the revised manuscript to state explicitly that total water and total water mixing ratio are used interchangeably in our presentation:

> *In addition, in this work, the terms "total water" and "total water mixing ratio" are used interchangeably, following the convention adopted in [Edwards et al., 2020a, Edwards et al., 2020b], to maintain consistency and clarity.*

Is the replacement of contact forces with white noise intended to increase ensemble spread in DA? Please clarify.

Response. Thank you for raising this point. The stochastic replacement of deterministic contact forces by a zero-mean white-noise term has two purposes, and only the second is related to ensemble spread:

- *Sub-grid representation of unresolved collisions.* In reality, floes experience numerous brief contacts whose individual forces are not resolved at the model grid spacing and time step used here. A white-noise approximation ($\sigma_f\,\eta(t)$ with $\langle \eta(t)\eta(t')\rangle = \delta(t-t')$) is a standard surrogate for the net impulse of many small, rapidly varying collisions. The amplitude σ_f is tuned to match the observed mean-square horizontal velocity of floes in high-resolution simulations, ensuring that the noise has a physically interpretable magnitude.
- *Implicit model-error perturbation in the DA ensemble.* Because the same stochastic term is applied independently to each ensemble member, it also acts as a time-correlated model-error perturbation, naturally widening the spread of the prior without the need for explicit inflation. Thus the noise does increase ensemble spread, but that benefit is secondary; the primary motivation is to capture unresolved contact dynamics in a statistically consistent manner.

We have revised line 298 in Section 3.1.1:

> *"The stochastic forcing is used to effectively approximate the instantaneous floe-floe interactions in the forecast system, significantly reducing the computational cost while at the same time, providing a physically motivated source of model-error variance that maintains adequate ensemble spread. "*

Line 309: what does $\widetilde{\phi}_{bt,k}$ refer to?

Response. $\widetilde{\phi}_{bt,k}$ denotes the k-th Fourier coefficient of the barotropic component of the streamfunction ψ; the subscript "bt" stands for "barotropic".

To make it clear, in the revised manuscript, we added the following in Section 3.1.2:

> *"The subscript "bt" therefore labels the barotropic mode, while "bc" labels the baroclinic component."*

Line 367: Equation (50) suggests larger floes have higher uncertainty, which contradicts earlier claims. Please clarify or correct the formula.

Response. Thank you for highlighting this potential confusion. Equation (50) prescribes the *standard deviation of observation errors* σ_l^{obs} for each floe according to the cloud–water criterion:

- **Observed floes** [$q_t < \widetilde{q}_t$]— The floe is under mostly clear sky and is therefore detected unambiguously; its positional uncertainty is set equal to the measurement uncertainty: 5×10^2 m. This error is *independent* of floe size.
- **Unobserved floes** [$q_t \geq \widetilde{q}_t$] — Thick cloud cover prevents a trustworthy observation of most of the area of the floe. Instead of discarding a cloud-obscured floe, we still include its location as an observation, but we recognize that this guess may be wrong by an amount comparable to the floe's own diameter.

  $$\sigma_l^{\mathrm{obs}} = 2r_l,$$

  This radius-scaled uncertainty represents the realistic possibility that the true floe could lie anywhere close the floe outline, providing a physically upper bound on positional error. Because the corresponding Kalman gain is nearly zero, this surrogate observation has minimal influence on the analysis; the r_l scaling therefore does *not* imply that larger floes are inherently measured less accurately.

Thus, dependence on radius appears *only* for floes that are *not* observed; for any floe that is actually observed the uncertainty remains the same measurement uncertainty.

Line 385: There is again confusion between total water content and total water mixing ratio.

Response. We appreciate the reviewer's diligence in flagging this inconsistency. To avoid confusion, we have added one sentence in line 435 in the revised manuscript to state explicitly that total water and total water mixing ratio are used interchangeably in our presentation:

> *In addition, in this work, the terms "total water" and "total water mixing ratio" are used interchangeably, following the convention adopted in [Edwards et al., 2020a, Edwards et al., 2020b], to maintain consistency and clarity.*

Line 447: What is the difference between Equations (49) and (55)? They appear to be similar—clarification would help.

Response. We agree that Equations (49) and (55) are the same. Equation (55) was an inadvertent repetition introduced during a previous edit; it adds no new information beyond the definition already given in Eq. (49). To eliminate the redundancy and avoid confusion, we have deleted Eq. (55) and renumbered the subsequent equations accordingly.

Line 462: In Equation (50) the uncertainty below the threshold was set at 0.5 km.

Response. We thank the reviewer for identifying this typo. Line 462 has been corrected to read:

> *The observational uncertainty for the ice floe locations, when there is no cloud cover, is set to 0.5 km.*

Line 493: in Equation (50) is the threshold.

Response. We thank the reviewer for identifying this typo. Line 462 has been corrected to read:

> *The observational uncertainty, introduced via (50), shows that when cloud cover,* $[q_t]$, *remains below a certain threshold* $\widetilde{q}_t$, *the uncertainty stays small (around 500 m), allowing for accurate floe location recovery.*

Add a list of symbols with definitions and units of measurements.

We thank the reviewer for this suggestion. The symbols, definitions, and units for the variables used in the ice floe, atmospheric, and oceanic models are already provided in Tables 2–4 of the manuscript. These tables collectively serve as a reference for the key quantities associated with each model component. To improve clarity and ensure consistency in notation, we have added the following note at the beginning of Section 4.1:

> *The symbols with definitions and units for the ice floe model are listed in Table 2, for the ocean model in Table 3, and for the atmospheric model in Table 4.*

References

[Boisvert et al., 2023] Boisvert, L. N., Webster, M. A., Parker, C. L., and Forbes, R. M. (2023). Rainy days in the arctic. *Journal of Climate*, 36(19):6855–6878.

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