Response to Referee #1

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1 General Comments

The manuscript is well written, model equations are documented well. I found Section 3 a bit hard to follow since the majority of it describes surrogate models not DA. But it is fine since those are used only in the ensemble DA experiments. However, the DA system description is left limited; this may make the manuscript hard to follow for non-expert readers on DA while looking at the experiments in Section 5.

Furthermore, the validation of the experiments can be done in a more quantitative way. The skill of the posterior mean is documented in Tables 6-8 but they can be extended to prior mean to see how much the DA improves with respect to the forecast. Another way of doing it is using the non-assimilative experiments in Section 4 as a baseline to see how the sparse and plenty observation cases improve when observations are incorporated. Another interesting diagnostic can be to look at the spread of the prior and posterior ensemble to see the uncertainty before and after the assimilation.

Finally, conclusions can be extended considering my specific comments below. Overall, I would expect a more quantitative assessment of the results. I would be happy to see a revised version.

Response: We sincerely thank the reviewer for the detailed comments and very helpful suggestions. We have carefully addressed each comment and made the changes accordingly throughout the revised manuscript. The details are as follows.

2 Specific Comments

Introduction: The data assimilation literature related to the work should be extended to give the reader a view of which studies have been done in line with the investigation presented in Section 5.

We thank the reviewer's comment and we agree that we need to add the literature review of data assimilation in the related area. In the revised manuscript, we have added the following additional literature overview in the section of introduction:

> *" Recent advances in sea-ice data assimilation encompass a broad spectrum of approaches. For example, [Lisæter et al., 2003, Massonnet et al., 2014] assimilated passive-microwave concentration and altimetry-derived thickness into coupled ice–ocean models with an ensemble Kalman filter, substantially reducing drift and thickness errors.*

[Riedel and Anderson, 2024] accounted for the bounded, non-Gaussian statistics of sea-ice variables within the observation operator, which refines the posterior analyses of both ice and snow states. At the fully coupled level, [Penny et al., 2019] introduced a strongly coupled data-assimilation (SCDA) framework that puts sea-surface and ice increments directly into the atmospheric analysis, further improving the short-term forecasts in the marginal-ice zone. With the traditional Eulerian approaches, [Chen et al., 2022b, Deng et al., 2025] developed an efficient Lagrangian scheme that reconstructs mesoscale currents and vorticity from a limited set of tracked floes, even if only partial trajectories are observed due to clouds. Nevertheless, current data-assimilation frameworks for fully coupled atmosphere–ocean–ice models still lack a consistent treatment of cloud and precipitation effects. "

Section 2: Sections 2.2.4 and 2.2.4 can be subsections of 2.2.2. Similarly, 2.2.6 and 2.27 can be subsections of 2.2.5.

Response: We thank the reviewer for the helpful suggestion. We have combined Subsections 2.2.2–2.2.4 into one subsection titled "Oceanic forcing", and Subsections 2.2.5–2.2.7 into "Atmospheric forcing".

Section 3: how costly is the coupled system? Can you be a bit precise in terms of cpu time? Is the code parallelized? What is the gain with the reduced-order models in this case?

Response: We thank the reviewer for the question. We have revised Section 3 to clarify the computational cost of the coupled model and the role of the reduced-order model (ROM).

The full coupled atmosphere–ice–ocean model is implemented in a sequential MATLAB code and is *not* parallelized. A single forward run over one simulated day (i.e., 86,400 seconds) requires approximately 3.70 CPU-hours, corresponding to a wall-clock time of 0.82 hours on an Apple M1 Max CPU with 32 GB of RAM. This computational cost makes ensemble-based data assimilation prohibitively expensive for even with a small ensemble size. While a parallelized implementation could mitigate this computational burden, it would require substantial effort in restructuring the code and managing inter-process communication, which is beyond the scope of this study.

By contrast, the proposed reduced-order models (ROMs) for the coupled system are several orders of magnitude faster and enable efficient ensemble forecasting. In addition, we aim to demonstrate the effectiveness of data assimilation using standard ROMs that preserve the essential system dynamics under observation scenarios where ice floes are irregularly obscured by cloud cover.

In line 265 of the revised manuscript, we have added the following:

For instance, the full coupled atmosphere–ice–ocean model is implemented as a sequential MATLAB *code without parallelization. A single forward simulation over one model day (i.e., 86,400 seconds) requires approximately 3.70 CPU-hours, corresponding to a wall-clock time of 0.82 hours on an Apple M1 Max processor with 32 GB of RAM. It is therefore not feasible to use this full order model directly in data assimilation settings due to its high computational cost.*

Section 3.2 it is not clear to me how the total water content is linked to the ice floe area?

Response: We thank the reviewer for the careful reading. The link between ice-floe area and the total water content q_t operates in two ways. ***First***, q_t is a prognostic variable in the atmospheric model that does not directly relate to the sea-ice area. However, the ice floe area has a different evaporation rate, which contributes to different water transport from the ocean surface, and this can contribute to the different distribution of the total water over the ice floe and the ocean. Some numerical results are discussed in Section 4.4. ***Second***, the mean total water content $[q_t]$ over the lth ice floe is used as a quantitative indicator for cloud cover: larger $[q_t]$ indicates thicker or more extensive clouds, which obscure ice-floe edges and therefore is represented by larger observation uncertainties in floe position. Section 3.2 details how this uncertainty is incorporated.

In the end of Section 3.2, we have added the following to better illustrate the link:

> *In summary, the total water content q_t interacts with ice floes in two different ways. First, the spatial coverage of an ice floe over the ocean affects the local evaporation, which in turn modifies the distribution of q_t over ice and ocean. Second, the mean total water content $[q_t]$ above each floe is used to parameterize cloud-related observation uncertainty in floe localization, with higher $[q_t]$ indicating thicker clouds and thus higher uncertainty.*

Using "q" in the equations for both the PV and water content is confusing from time to time. Section 3.3.2 describes directly the data assimilation scheme (LETKF) not the localizations. This can be the introduction to Section 3.3. You don't detail the scheme, no KF equations for example, but you allocate a dedicated section to localization. What is the reason that you prefer to discuss localization explicitly for your application? How does your analysis benefit from it?

Response: We thank the reviewer for pointing out the potential confusion arising from using the symbol q. We have chosen to retain this notation in order to be consistent with prior work, particularly the formulation in [Qi and Majda, 2016, Chen et al., 2021, Chen et al., 2022a], which uses q for both quantities for potential vorticity. To clarify the meaning in our manuscript, we have added one sentence in the line 160 of section 2.3 where this notation first appears to remind the reader of the specific interpretation of q based on the context:

> *Here, $q_{(\cdot)}$ denotes the potential vorticity at the $(\cdot)$ layer, following the notation used in [Qi and Majda, 2016, Chen et al., 2021, Chen et al., 2022a]. Its precise definition may vary slightly depending on the component (e.g., atmosphere or ocean) and is clarified in the corresponding context.*

For the second point, we thank the reviewer for this thoughtful question. While the ensemble Kalman filter (EnKF) and its variants such as LETKF are well-established in the data assimilation (DA) literature such as [Evensen, 2009, Asch et al., 2016], we chose to put a dedicated section to localization due to its central importance in our application. Specifically, our coupled model features both Eulerian state variables (e.g., atmospheric and oceanic streamfunctions on a grid) and Lagrangian variables (i.e., individual ice floe trajectories), which evolve on different spatial frameworks and display different spreads of uncertainties. Therefore, localization plays a critical role in assimilating sparse observations in systems with mixed Eulerian–Lagrangian information.

By explicitly detailing our approach to localization, we aim to give the audience a clear presentation about how these different kinds of state variables are combined in the localization in data assimilation. Thus, while the LETKF algorithm is standard and well-known, the localization for different state variables in this work is both non-trivial and essential to the effectiveness of the assimilation procedure. For this reason, we believe it merits a focused discussion.

To make it clear, we have revised line 432 in the draft:

> *LETKF applies filtering exclusively in physical space for both the trajectories of ice floes which are in the* Lagrangian *frame of reference and the streamfunctions of the atmosphere and ocean that are in the* Eulerian *frame of reference.*

Section 4.1 Is the time step equal for all the components? If so, what are the implications for resolved scales for the supposedly fast evolving atmosphere and relatively slow ocean and sea ice? By the way, is there a specific reason that you choose the timestep as a decimal number in seconds? It is 58.2 seconds in line 399 while 1.2941 hours in Table 1. Not clear to me why they are different.

Response: We thank the reviewer for pointing this out. In our simulation, three distinct time steps are employed:

- Atmospheric model: $\Delta t = 58.2\,\mathrm{s}$
- Ocean model: $\Delta t = 1.2941\,\mathrm{hours}$
- Ice floe model: $\Delta t = 1.2941\,\mathrm{hours}$

These dimensional time steps are derived by rescaling from their non-dimensional counterparts based on characteristic scales appropriate for the Arctic regime used in our study. For example, the atmospheric model's dimensional time step of 58.2 s corresponds to the non-dimensional value $\Delta t = 5 \times 10^{-4}$, given the Arctic characteristic time scale T used in the QG model.

In addition, we have intentionally chosen different time steps for the three models to significantly reduce the computational cost. The primary computational burden in the coupled model arises from the ice floe component, which must resolve collisions as well as contact forces and torques among floes at every time step. If the ice floe model were computed using the atmospheric model's smaller time step, the computational expense would increase by approximately 80 times. Therefore, employing different time steps for each model is an effective strategy to balance computational efficiency and model accuracy.

To make this clearer, we have updated the time scale and time step information in Tables 2, 3, and 4, as well as added the following to the Section 4.1 of the manuscript:

> *It is worth noting that the dimensional time steps for the atmospheric and ocean models are obtained by rescaling their respective non-dimensional counterparts using their different characteristic scales appropriate to each component under the Arctic regime; for example, the atmospheric model's time step of* 58.2 s *corresponds to a non-dimensional value of* $\Delta t = 5 \times 10^{-4}$*, based on the characteristic time scale* T *employed in the PQG model.*

Section 4.3 The trajectory of the large floes seems to me more unpredictable including returns and changing directions. Is there a way to quantify this behavior?

Response: We thank the reviewer for highlighting this insightful observation. Indeed, large ice floes exhibit more complex trajectories characterized by frequent directional changes and occasional reversals. A quantitative assessment of this intricate behavior could involve analyzing metrics such as path curvature, directional persistence, or employing methods from stochastic trajectory analysis. However, a detailed quantification of these aspects lies beyond the scope of the current study and will be investigated systematically in our future work. In the revised manuscript, we have added one sentence:

> In addition, it would also be worthwhile to explore how the statistical properties of ice floe trajectories vary with floe size—while some aspects of this behavior, particularly for smaller floes, are illustrated through case studies in this work, a more systematic quantification is left for future investigation.

Section 5.1 Returning back to my comment on Section 3, did you try running an experiment with smaller ensemble size (instead of 300) using the full models (instead of surrogates)?

Response. We appreciate the reviewer's interest in a baseline test that would employ the full coupled model rather than the surrogate emulator. Unfortunately, even with a much smaller ensemble, such an experiment is prohibitively expensive:

- A single 66-day forecast with the full atmosphere–ocean–ice model costs approximately 244 CPU-hours. Running an ensemble of 50 members—the minimum size that still yields a well-conditioned sample covariance for this state dimension—would therefore require about 12200 CPU-hours per analysis cycle.

- The full coupled model is discretized on a much finer spatial grid than the surrogate (ROM), yielding a state vector whose dimension is orders of magnitude larger. Assimilating such high-dimensional state variables would take a substantial computational cost for each analysis cycle; even a minimum setup would be infeasible for a few data assimilation cycles in Section 5.1.

For these reasons we opted to use surrogate dynamics in combination with a larger ensemble (300 members) to preserve covariance accuracy at manageable cost. A systematic comparison between a ROM based ensemble and an FOM based ensemble in data assimilation is an important topic for future work once additional computational resources become available.

To make it clear about the computational cost for DA with the full order model, we have added the following statement at the beginning of Section 3 in the revised manuscript:

> *While it would be ideal to perform a baseline data assimilation experiment using the full coupled atmosphere–ocean–ice model, such a setup is computationally prohibitive. A single 66-day forecast with the full model requires approximately 244 CPU-hours, and an ensemble of 50 members, which is a relatively small ensemble size, would require over 12,000 CPU-hours per assimilation cycle. Furthermore, the full model operates on a much finer spatial grid than the reduced-order surrogate, resulting in a substantially higher-dimensional state variable and significantly more expensive computational costs.*

What are the state variables, all model variables? Section 5.2 It would be useful to provide the prior mean and spread of both the analysis and forecast to assess the improvements via DA. The posterior mean compared to truth is fine but doesn't show how much the state and the trajectory improved.

Response. We thank the reviewer for the helpful suggestion.

The *state variables* in our coupled atmosphere–ocean–sea ice model include:

- the streamfunctions for the upper and lower atmosphere layers, denoted by ψ_1^a and ψ_2^a,
- the streamfunction for the surface ocean, ψ_2^o, and
- the two-dimensional positions of each individual ice floe at time t, represented by $(x_l(t), y_l(t))$ for floe index l.

In response to the second point, we have revised Section 5.2, along with Figures 6–8 and Tables 6–8, to include both the *prior* (forecast) and *posterior* (analysis) means for the atmospheric and oceanic state variables, as well as the ice floe trajectories.

3 Technical corrections

Format of the citations are not adequate and should be corrected all over the text. e.g. Cámara-Mor et al. (2010); Kwok (2018) →(Cámara-Mor et al. 2010; Kwok 2018)

Response: We thank the reviewer for pointing this out. We have revised and corrected the citation format in the manuscript.

References

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