Review of the paper “Simulation characteristics of seismic translation and rotation under the assumption of nonlinear small deformation" by

Wei Li[1,2], Yun Wang[1,2], Chang Chen[1,2] and Lixia Sun[1,3]

[1] *“MWMC" group, School of Geophysics and Information Technology, China University of Geosciences, Beijing 100083, China*
[2] *State Key Laboratory of Geological Processes and Mineral Resources, China University of Geosciences, Beijing 100083, China*
[3] *Sinopec Research Institute of Petroleum Engineering Co., Ltd., Beijing 102206, China*

This is the second time that I review this paper, and just like before I have problems with the theoretical basis of this work. I do recognize that the authors present an interesting study of the *possible* non-linear elastic effects observed in seismological data, but the paper needs solid and clear theoretical basis as well as the main message of the contribution. In the next I elaborate the theoretical basis.

Theoretical basis The authors begin with a good explanation of the strain tensor and its meaning as a distance measure. They mention the non-linear term that is often avoided in research papers, which is usually done in the assumption that one is far away from the seismic source.

The expression for the strain tensor including high-order terms is given by

$$E_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i + \partial_i u_j \partial_j u_i\right) \quad \text{with} \quad i,j \in \{1,2,3\}. \tag{1}$$

This is equation (2) in the paper. Until this point everything is fine, however soon enough the authors star with a zoo of equations and terminology that is irrelevant for the study. For example, the authors re-write eq. (2) as eq. (5) (what for?) and then introduce eqs. (6)–(7) and (8) and start to talk about volumetric strains (what for?) to re-write the equation of motion in eq. (9). What is the need to talk about volumetric strains? write something like eq. (8)? what for? In addition to that, the authors write eq. (10) combining two notations (x, y, z) and $(1, 2, 3)$. This is a complete nonsense. Again, the authors write the equation of motion in eq. (9) using eq. (8), combining two notation, such a huge large equation as given in eq. (8). What is the need of such a huge bad notation and complication?

Please note that the first line of eq. (10) uses i, j so one assumes that $i, j \in \{1, 2, 3\}$ and in the next line of the same equation one reads i, j, k and x, y, z. One can again assume that $i, j, k \in \{1, 2, 3\}$ and some direct analogy to x, y, z. But that all. This is a mathematical error in notation and the huge confusion that this brings into the study just kills every possible contribution.

Having pointed out that there ar mathematical errors/confusions in the equations, I do not want to even look to the numerical implementation. In addition, I have said this, the numerical implementation has to be accompanied by a benchmark against another well tested numerical code. There many that can be used, for instance SPECFEM (Komatitsch and Vilotte, 1998), the codes from the group of Prof. Peter Moczo, etc. I do understand that these codes are discretizing the linear approximation. The idea should be that the code that the authors present should be benchmarked in the linear approximation with these well known codes (one of them) and later use the code to draw differences and similarities with the non-linear case.

I do not look forward to give a bad review one more time for this paper. For this reason I will write here the mathematical basis that this paper needs. One will see that eqs. (6)–(7) and (8) are completely unnecessary and confusing. Eqs. (11) should not be written that way... not even mentioning eq. (12) shouldn’t be either.

Theoretical basis (clarifications) If one looks to find the equations of motion related to any linear and symmetric strain measure, we can simply write the equations of motion as follows (for a good introduction to the topic see Slawinski (2010))

$$\rho \quad \underbrace{\partial_t^2 u_j}_{\text{acceleration term}} \quad = \quad \underbrace{\partial_i \sigma_{ij}}_{\text{Divergence of the stress}} \qquad \text{with} \quad i,j \in \{1,2,3\}, \tag{2}$$

where ρ is the material density, u the displacement vector and σ_{ij} is the (second-order) stress tensor defined as

$$\sigma_{ij} = \mathbb{C}_{ijkl} E_{kl}, \tag{3}$$

where $\mathbb{C}$ is a fourth-order tensor of elastic constants with the symmetries $\mathbb{C}_{ijkl} = \mathbb{C}_{jikl} = \mathbb{C}_{ijlk} = \mathbb{C}_{klij}$ (Slawinski, 2010) and E_{kl} the chosen strain measure. Note that we use the conventional notation $\{1,2,3\}$ for $\{x,y,z\}$.

If one assumes isotopic symmetry for the elastic tensor $\mathbb{C}_{ijkl}$, we can write (Dahlen and Tromp, 1998)

$$\mathbb{C}_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \quad \text{with} \quad i,j,k \in \{1,2,3\}, \tag{4}$$

where λ, μ are Lamé parameters and δ the Dirac distribution. If we use the conventional linear strain tensor as given by the following expression

$$E_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right) \quad \text{with} \quad i,j \in \{1,2,3\}, \tag{5}$$

we arrive to the well known equation (doing step by step!)

$$\begin{aligned}
\partial_t^2 u_j &= \partial_i\left(\mathbb{C}_{ijkl}E_{kl}\right)\\
\partial_t^2 u_j &= \partial_i\left(\lambda\delta_{ij}\delta_{kl}\frac{1}{2}\left(\partial_k u_l + \partial_l u_k\right) + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\frac{1}{2}\left(\partial_k u_l + \partial_l u_k\right)\right)\\
\partial_t^2 u_j &= \partial_i\left(\lambda\delta_{ij}\partial_k u_k + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\frac{1}{2}\partial_k u_l + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\frac{1}{2}\partial_l u_k\right)\\
\partial_t^2 u_j &= \partial_i\left(\lambda\delta_{ij}\partial_k u_k + \mu\frac{1}{2}\partial_i u_j + \mu\frac{1}{2}\partial_j u_i + \mu\frac{1}{2}\partial_j u_i + \mu\frac{1}{2}\partial_i u_j\right)\\
\partial_t^2 u_j &= \partial_i\left(\lambda\delta_{ij}\partial_k u_k + \mu\left(\partial_i u_j + \partial_j u_i\right)\right) \quad \text{with} \quad i,j \in \{1,2,3\}.
\end{aligned} \tag{6}$$

If we consider the strain tensor defined as in eq. (1), just simply need to add the contribution of $\partial_i u_j \partial_j u_i$ to the previous equation, i.e.,

$$\begin{aligned}
\left[\lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\right]\partial_k u_l \partial_l u_k &= \lambda\delta_{ij}\delta_{kl}\partial_k u_l \partial_l u_k + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\partial_k u_l \partial_l u_k\\
&= \lambda\delta_{ij}\partial_k u_k \partial_k u_k + 2\mu\partial_i u_j \partial_j u_i
\end{aligned} \tag{7}$$

Thus, the equation of motion related to strain measure given in eq.(1) becomes

$$\partial_t^2 u_j = \partial_i\left(\lambda\delta_{ij}\left(\partial_k u_k + \partial_k u_k \partial_k u_k\right) + \mu\left(\partial_i u_j + \partial_j u_i + 2\partial_i u_j \partial_j u_i\right)\right) \quad \text{with} \quad i,j,k \in \{1,2,3\}. \tag{8}$$

No that the equation of motion is symmetric, we can simply interchange $j \to j$ to obtain a more familiar expression as follows

$$\partial_t^2 u_i = \partial_j\left(\lambda\delta_{ij}\left(\partial_k u_k + \partial_k u_k \partial_k u_k\right) + \mu\left(\partial_i u_j + \partial_j u_i + 2\partial_i u_j \partial_j u_i\right)\right) \quad \text{with} \quad i,j,k \in \{1,2,3\}. \tag{9}$$

Equation (9) uses a single notation for the coordinates $\{1,2,3\}$ and Einstein notation for repeated summation. For Finite-Difference calculation it is useful to write the equation of motion (9) in its displacement (or velocity)–stress notation as follows

$$\partial_t^2 u_i = \partial_j \sigma_{ji}, \tag{10}$$

with the stress tensor defined as

$$\sigma_{ji} = \left(\lambda\delta_{ij}\left(\partial_k u_k + \underbrace{\partial_k u_k \partial_k u_k}_{\text{additional term}}\right) + \mu\left(\partial_i u_j + \partial_j u_i + \underbrace{2\partial_i u_j \partial_j u_i}_{\text{additional term}}\right)\right). \tag{11}$$

Eq. (9) is the equation that should be analyzed in the paper and its velocity-stress formulation in the numerical implementation. The additional terms should be properly understood.

Conclusion I therefore cannot recommend this paper for publication as it is. It has confusing mathematical basis which do not allow to evaluate the correctness of the theory and the numerical predictions and in addition, the English is not well written. I understand that writing scientific English is not an easy task, but there are services offered on the web that one can use to write a sufficiently good English for a publication. One can read expressions like: *nonlinear effect unbearable in simulations...* (see line 496).

The authors show a potential interesting application but the paper needs substantial work to be done and once again it needs clear and solid mathematical basis with at least one benchmark of the code that are using.

The reviewer

References

Dahlen, F. and Tromp, J. (1998). *Theoretical global seismology*. Princeton university press.

Komatitsch, D. and Vilotte, J.-P. (1998). The spectral element method: an efficient tool to simulate the seismic response of 2D and 3D geological structures. *Bulletin of the Seismological Society of America*, 88(2):368–392.

Slawinski, M. (2010). *Waves and Rays in Elastic Continua*. World Scientific Publishing Company.