Responses to Reviewers

Responses to Reviewer #1' Comments

1. As I previously stated, the authors present a numerical study based on a code that they have developed. This is something important and difficult to do. However, they don't do any benchmark of their code, which makes the reader (very) doubtful of their numerical results.

   ...

   On the other hand, the lack of validation of the code renders their numerical experiments doubtful.

   Thanks for the reviewer's suggestions.

   We have added a Supplementary Material with benchmarking results, including comparisons with SPICE/SISMOWINE and SPECFEM2D:

   a. Comparison with the SPICE/SISMOWINE international standard program: We selected the standard test cases from this project and compared our simulation results with the provided reference solutions through waveform comparison and quantitative misfit analysis. The results show that our code's travel times, amplitude envelopes, and overall waveforms are in good agreement with the reference solutions.

   b. Validation with the SPECFEM2D spectral-element program: We also performed a cross-validation using the widely recognized open-source spectral-element package SPECFEM2D. We compared the waveforms from both codes under full-space and half-space models, respectively. The results again showed good consistency.

2. The authors present an analysis of seismograms (time histories) and they insist in plotting all components over the same line and present, in my opinion, a mathematical/statistical study of the waveforms. This, in my opinion, is not a proper way to understand seismic waveforms because: on the one hand we look to understand the physics of wave propagation when we analyze seismograms and, on the other hand, this kind of presentation obscures the information that

seismograms contain. In simpler words: when we look at differences in seismograms we look to analyze differences in different waveforms: P waves, S waves, Love and or Rayleigh waves, etc...

Thanks for the reviewer's suggestions.

We have recognized that the analysis of waveforms in the previous version was overly reliant on general statistical metrics and lacked an in-depth analysis of wave phases and physical mechanisms. Based on the reviewer's suggestion, we have revised the result analysis framework to focus on wave phases and physical mechanisms (please see Section 3 especially).

3. An easy way to invalidate the results presented in this paper is by looking at Figs. 11 and 12. If one makes a zoom we can see the similarity between trans.R and trans.T at the beginning of the seismogram and this simply cannot be happening in an isotropic medium. The first arrival that we are able to see in trans.R and trans.Z is the P wave, however, trans.T should not show any arrival whatsoever. The first arrival of the trans.T component is the S wave an before that one should see simply nothing, absolutely zeros. This is seismology 101.

   Thanks for the reviewer's suggestions.

   We reviewed our data processing procedure and discovered an error in calculating the azimuth angle when rotating the simulation results from the Cartesian coordinate system to the RTZ coordinate system. We have corrected this algorithm. In the revised manuscript, the seismograms shown in the RTZ coordinate system in the corresponding Figs. 8 and 9 have been processed with the correct coordinate rotation.

4. I thus suggest to simplify the study. The authors could simply run a 2D SH wave simulation to which the analytical solution is very easy to do for the code... and the authors can then focus on understanding the difference in their seismograms in different 2D SH heterogenous scenarios.

   Thanks for the reviewer's suggestions.

   Regarding the specific path the reviewer recommended, we would like to explain from two aspects:

a. Regarding code validation: During the revision period, we attempted to follow the 2D SH validation path. However, we encountered some technical difficulties in obtaining and implementing the relevant analytical solutions and could not complete it successfully. Therefore, we compared our code with the 3D standard model of the SPICE/SISMOWINE project and performed a cross-validation of the 2D P-SV wavefield with the widely used SPECFEM2D program. The relevant comparison results are presented in the supplementary material we submmited.

b. Regarding simplifying the research: After consideration, we decided to retain the current 3D model research framework. This is mainly because we want to investigate the spatial distribution characteristics of nonlinear effects excited by three-dimensional source mechanisms. The points we are interested in, such as how the efficiency of nonlinear mode conversion is controlled by the source radiation pattern, and the characteristics of the rotational motion itself, including all its components, exhibit anisotropic behavior in three-dimensional space. If the model were simplified to 2D, we would only be able to observe phenomena on a specific profile, which would inevitably lead to a incomplete understanding of the spatial characteristics. Therefore, to capture and analyze these phenomena with 3D properties, we retain the 3D simulation framework.

Responses to Reviewers #2' Comments

1. If the authors mention the simulation of a realistic event. Then I have to see data comparison. After the theoretical work done, which is credible, it makes no sense to talk about an important event and not compare to data. This will allow the reader to validate the methodology and importance of the work from the data point of view... The main idea is to analyze whether seismograms can be reproduced by a nonlinear model (or not).

   Thanks for the reviewer's suggestions.

   We agree with the reviewer's viewpoint that comparing simulation results with real observational data is the powerful means of validating a theory and method. However, at the current stage of this research, we plan to perform data comparison in future work to validate with real records, as suggested. Because the primary objective of this study is to isolate and understand the effects produced by the specific mechanism of geometric nonlinearity within a controlled numerical experiment environment. To achieve this goal, even when simulating the referenced earthquake events, we only used a simplified, horizontally layered medium model to exclude interferences from factors such as complex 3D structural heterogeneity, small-scale scattering, and site effects.

   We position this study as a fundamental theoretical and numerical investigation, and the suggestion represents a crucial next step for this research. In our future work, we will attempt to apply our nonlinear model to real data and analyze whether the nonlinear model can better explain the observed waveforms. We have also pointed this out in the Discussion part, identifying it as a future research direction.

2. The authors need to be aware that the term nonlinearity is used in the literature to describe many different physical effects. For example, the term nonlinearity can be used to describe visco-elastic plastic effects... However, in this study, nonlinearities are geometrical, to describe pre-stressed medium... I would like to read a discussion in the paper clarifying these differences to the reader.

   Thanks for the reviewer's suggestions.

We have added a clear distinction and explanation of different nonlinear mechanisms in the Introduction. In the revised manuscript, we explicitly state that the scope of this study is limited to geometric nonlinearity and is conducted within a linear elastic constitutive framework, avoiding confusion with studies on material nonlinearity.

3. The benchmark tests should be more explicitly explained inside the paper. Even if it is written as a supplementary information or as an appendix. There must be a discussion of the benchmark done to the code.

   Thanks for the reviewer's suggestions.

   We completely agree with the reviewer's suggestion. We have prepared a Supplementary Material dedicated to presenting the benchmarking results of our code. In the supplementary material, we have compared our code with the SPICE/SISMOWINE international standard program and the SPECFEM2D spectral-element program, covering both full-space and half-space models in waveform comparison plots and quantitative error analyses. Please see the supplementary.

   In Section 2.2 (Staggered-grid ginite-difference method), we have explicitly informed the readers that we have conducted this validation work and directed them to the supplementary material for detailed information.

4. Finally, the presentation of the time histories (seismograms) is not the most appropriate one (in my opinion). I still lack of a clear comparison of the transverse component. Simply because the transverse component should not have any energy before the S wave and I cannot see that in the plots. It seems tricky to me how the authors are rotating the seismograms.

   Thanks for the reviewer's suggestions.

   It's correct that in an isotropic or horizontally layered medium, the P-wave first arrival should not appear on the transverse component. After careful inspection, we found that an error occurred when rotating the simulation results from the Cartesian coordinate system to the RTZ coordinate system in the previous version. We have since corrected the coordinate rotating algorithm. In the revised manuscript, all figures showing RTZ components (Figs. 8 and 9) have been correctly processed. The transverse component is at the numerical noise level before the S-wave arrival, which is consistent with basic wave physics principles.

List of relevant changes made in the manuscript

We have made substantial revisions to the manuscript based on the reviewers' comments. The main changes are listed below:

I. Major methodological and structural revisions

1. Addition of code validation

In response to the critical concern about the lack of code validation (Reviewer #1, Comment 1.1; Reviewer #2, Comment 2.3), we have prepared a Supplementary Material dedicated to the benchmarking of our numerical code. This includes detailed waveform and misfit comparisons with two international standard benchmarks: the SPICE/SISMOWINE project and the SPECFEM2D package. A statement directing the reader to this material has been added to Section 2.2.

2. Correction of a fundamental physical error in seismograms

Reviewer #1 (Comment 1.3) and Reviewer #2 (Comment 2.4) correctly pointed out a fatal error in our previous results where P-wave energy appeared on the transverse (T) component. We have identified and corrected the error in our coordinate rotation algorithm (from Cartesian to RTZ). All figures showing RTZ seismograms (Fig. 8 and Fig. 9) have been re-processed and replaced. The corrected figures show no energy on the T component before the S-wave arrival, which is consistent with basic wave physics.

3. Complete revision of the analysis framework

As suggested by Reviewer #1 (Comment 1.2), we have shifted the focus of our analysis from a general statistical study to an in-depth investigation of physical mechanisms. The entire Chapter 3 has been restructured to analyze nonlinear effects in terms of their spatial radiation patterns, spectral characteristics, and their impact on micro-waveform properties (e.g., rectilinearity and rotation-translation correlation), with an emphasis on the physical interpretation of the results.

II. Revisions in specific sections

1. Introduction: Following the advice of Reviewer #2 (Comment 2.2), we have added a clear distinction between constitutive nonlinearity and geometric nonlinearity

in the ssecond and third paragraphs of the Introduction to clarify the scope of our study.

2. Results and Discussion (Chapters 3, 4, 5): The entire results and discussion sections have been rewritten to reflect our deeper physical analysis.
3. Conclusions: The conclusions have been completely rewritten to be more precise and strictly aligned with the results. We now clearly distinguish between the universal radiation pattern and the source-dependent excitation efficiency of the nonlinear effects, and have specified that the high sensitivity of rotation is primarily for S-wave-driven nonlinearity.