Negative Differential Resistance, Instability, and Critical Transition in Lightning Leader

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Abstract. The phenomena of leader extinction and restrike during lightning events, such as multiple strokes in ground flashes or recoil leaders in cloud flashes, present significant challenges. A key aspect of this issue involves the discussion of the channel's negative differential resistance and its instability. From the perspective of bifurcation theory in nonlinear dynamics, this paper suggests an inherent consistency among the channel's negative differential resistance, channel instability, and the critical transition from insulation to conduction. This study examines the differential resistance characteristics of the leader-streamer system in lightning development. We correlate the differential resistance characteristics of the leader-streamer channel with the channel's state and instability transitions, investigating the critical current and potential difference conditions required for the stable transition of the leader-streamer channel.

Key words: negative differential resistance; instability; critical transition; lightning

1 Introduction

Natural lightning exhibits distinct intermittent characteristics that differentiate it from long-gap discharges observed in laboratory settings (Gou et al., 2010; Gou et al., 2018a; Iudin et al., 2022). This intermittency is intrinsically linked to the fractal structure and critical dynamics of the lightning process (Bulatov et al., 2020; Gou et al., 2018b,Sterpka et al., 2021; Iudin & Syssoev, 2022; Syssoev et al., 2022). Additionally, the inherent polarity asymmetry in bidirectional leader development introduces instability in the discharge process, leading to destabilization and re-excitation phenomena in various lightning events (Van der Velde & Montanya, 2013; Williams, 2006; Williams & Heckman, 2012; Iudin, 2021; da Silva et al., 2023; Scholten et al., 2023).

The intermittent nature of lightning is particularly evident in ground flashes, where negative ground flash discharges are characteristically separated by extended periods of dim luminosity. These periods are marked by a distinctive sequence: when the downward negative channel decays and eventually terminates, the still-active intracloud positive component intermittently creates conditions that facilitate the formation of dart or dart-stepped leaders (Van der Velde & Montanya, 2013; Stock et al., 2014; Stock et al., 2023; Lapierre et al., 2017; Jensen et al., 2023). A similar intermittent behavior is observed in cloud flashes, where the active positive leader exhibits marked temporal asymmetry compared to the negative leader, resulting in K-processes or recoil leaders within the cloud. These transient phenomena are generally attributed to channel instability, which manifests primarily through negative differential resistance characteristics of the channel (Williams & Heckman, 2012; Mazur, 2016b).

In gas discharge physics, negative differential resistance is fundamentally associated with bistability, hysteresis, and critical transitions (Bosch & Merlino, 1986; Lozneanu et al., 2002; Agop et al., 2012; Raizer & Mokrov, 2013). In lightning discharges, the leader and streamer form a strongly coupled system with complex interactions: streamers supply the energy and current essential for leader development, while the highly conductive leader channel maintains the electric field and potential required for continuous streamer propagation. Given this intricate coupling, this study expands the investigation of negative differential resistance properties to encompass the entire leader-streamer system, rather than focusing solely on the leader channel. Our analysis specifically examines the stability characteristics during lightning development and identifies the critical conditions—in terms of current and potential difference—that govern channel stability.

2 Method

2.1 Negative differential resistance in lightning

Lightning, as a natural large-scale arc discharge, exhibits negative differential resistance characteristics in its channel (Heckman, 1992; da Silva et al., 2019). This behavior manifests when channel temperature and conductivity increase with current, leading to a decrease in the internal electric field needed to maintain the current, thus reducing the voltage across the channel ($dV/dI < 0$). This characteristic was identified by Krehbiel et al. (1979) as a potential mechanism for channel attenuation.

Heckman (1992) and Williams & Heckman (2012) investigated how negative differential resistance relates to the multiplicity of negative ground flashes. They proposed that while a channel with negative differential resistance connected to an extended streamer source is inherently unstable, parallel channel resistance and capacitance could provide stabilization. Specifically, channel stability occurs when the electrical response time constant (RC) exceeds the thermal attenuation constant τ, with a critical stability current of approximately 100 A (Heckman, 1992; Williams, 2006; Williams & Heckman, 2012).

However, Mazur & Ruhnke (2014) and Mazur (2016a, b) challenged the appropriateness of modeling the leader channel as a parallel RC circuit. They argued that channel stability depends primarily on the minimum potential difference in the streamer zone at the channel tip, rather than negative differential resistance characteristics (Bazelyan & Raizer, 2000; Mazur, 2016a). Given that leader development is guided by numerous streamers at the front, we suggest that differential resistance analysis should consider both the leader channel and its associated streamers.

2.2 Negative differential resistance and bistability

To understand how negative differential resistance leads to bistability in lightning channels, we use a normalized relationship between current J and voltage φ that captures the essential nonlinear dynamics of the discharge process. This relationship emerges from the fundamental physics of plasma channel formation and maintenance (Agop et al., 2012, Manea et al., 2013):

$$\varphi = J\left(1+\frac{a}{1+J^2}\right) \tag{1}$$

where a is a dimensionless control parameter that governs the system's nonlinear characteristics.

As illustrated in Figure 1 (Manea et al., 2013), for parameter $a=18$, the $J-\varphi$ characteristic curve exhibits three distinct regions. There are two stable regions where $\varphi'(J) > 0$: a low-conductivity state (segment AB) and a high-conductivity state (segment CD), both characterized by a monotonically increasing current with voltage. These stable regions are separated by an unstable region where $\varphi'(J) < 0$, demonstrating negative differential resistance. The system displays bistability, with critical transitions occurring between the two stable states: at point B, the system abruptly transitions from the low-conductivity state to the high-conductivity state, while at point D, it reverts to the low-conductivity state. This results in hysteretic behavior, where the system follows different paths during voltage increase (A→B→C) and decrease (C→D→A).

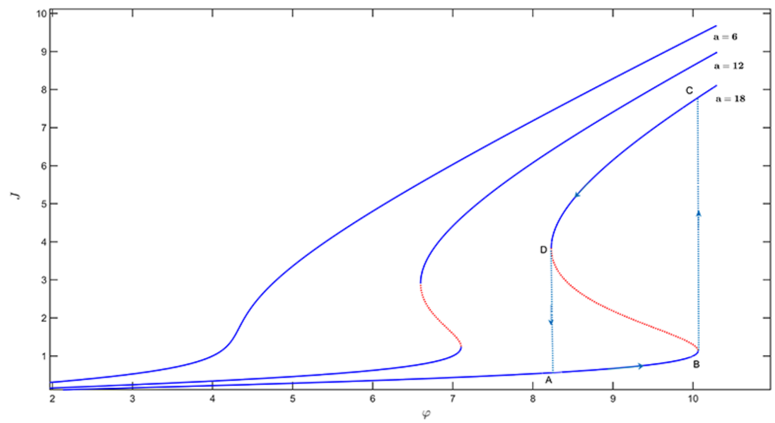

Fig. 1. Theoretical dependence of the normalized current on the normalized potential (adapted from Manea et al., 2013; original figure licensed under CC BY 4.0)

In nonlinear dynamics, negative differential resistance, bistability, and hysteresis are commonly observed. Considering the dynamic system $\frac{dJ}{dt} = f(\varphi, a, J)$, where J is the state variable and φ, a is a parameter. The equilibrium points are given by $f(\varphi, a, J) = 0$. At an equilibrium point, the system is unstable when $\partial f / \partial J > 0$ and stable when $\partial f / \partial J < 0$.

Let's define:

$$f(\varphi, a, J) = \varphi - J\left(1 + \frac{a}{1 + J^2}\right) \quad (2)$$

Then

$$\partial f / \partial J = -1 - a\frac{1 - J^2}{(1 + J^2)^2} \quad (3)$$

From equation (1), at equilibrium we have $\varphi = J(1+\frac{a}{1+J^2})$, and $\partial f/\partial J = -\varphi'(J)$. The stability condition $\partial f/\partial J > 0$ is equivalent to $\varphi'(J) < 0$, which corresponds to negative differential resistance. This mathematical analysis provides insight into how the sign of channel differential resistance determines the stability of lightning channel states and their transitions

Similar bistability, hysteresis, and critical transitions are widely observed in biological, atmospheric, ecological, and other systems, and they can be described by similar dynamical systems (Scheffer & Carpenter, 2003; Scheffer, 2009). The generation of instability and bistability can be illustrated by the rolling ball model shown in Figure 2, where the peaks and valleys represent unstable and stable points, respectively. Instability triggered by strong nonlinearities (positive feedback) is an important factor causing the bistability (polymorphism) of the system and the critical transition.

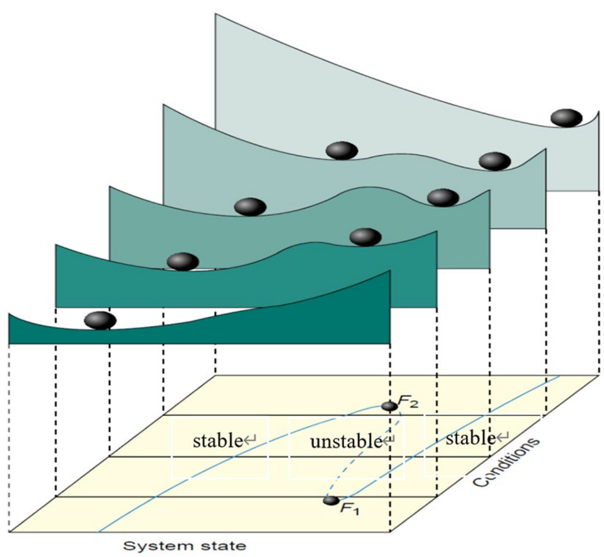

Fig. 2. Schematic representation of the locus of stability as a function of external conditions. Adapted from Scheffer & Carpenter (2003). Reprinted with permission from Elsevier.

2.3 The relationship between Lightning channel electric field and current.

The differential resistance characteristics of a gas discharge gap at a centimeter scale were measured first by King (1961). However, due to the effect of electrode vaporization, as pointed out by Mazur & Ruhnke (2014), King's results are only applicable to currents less than 10A with short gaps. In larger-scale lightning channels, the relationship between current and electric field is generally expressed in a power-law form. For example, Bazelyan et al. (2008) assumed that the leader channel current is inversely proportional to the electric field, , $E = 3400I^{-1}$, meanwhile, Larsson et al. (2005) proposed that the relationship between channel current and electric field can be described as:

$$E = 1600I^{-0.18} \tag{4}$$

This is consistent with the observations of Tanaka et al. (2003) and aligns with the suggestions of da Silva et al. (2019) , who suggested that the power law varies for different segments within the range of 10^2 to 10^4 A.

To develop a comprehensive description of the channel's electrical characteristics across different current regimes, we combined two complementary datasets:

1. King et al. (1961) data for I < 10 A, characterizing the initial breakdown phase where electrode effects dominate.

2. Larsson et al. (2005) measurements for I > 10 A, representing the fully developed leader channel. The combined dataset was then fitted using a double power-law model that captures both regimes

$$E = aI^{b} + cI^{d} \tag{5}$$

Where $a = 4278, b = -0.9788, c = 1799, d = -0.2006,$ The minimum current used for fitting was approximately 0.1 A.

Figure 3 shows the relationship between the electric field and current, with squares represent King's observations, circles representing Tanaka's (2003) experiments, and the solid green line representing the fitted curve.

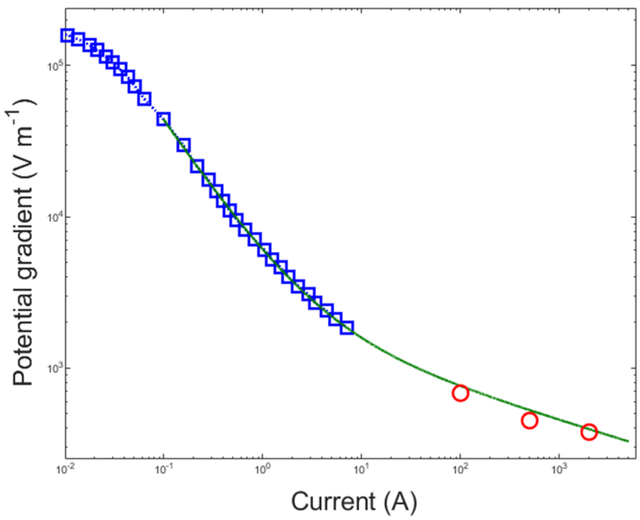

Fig. 3. Electric field versus current in arc channel

2.4 Differential resistance of the leader-streamer channel

The resistance of a streamer channel is determined by the potential difference ΔU_T across the streamer zone of the leader head and the channel current I, which can be expressed as (Bazelyan & Raizer, 2000):

$$I = q_c V_L = 2\pi\varepsilon_0 V_L \Delta U_T \quad (6)$$

where q_c represents the channel charge line density, and V_L is the channel development speed. The relationship between V_L and I follows a power-law form (Bazelyan & Raizer, 2000; Popov, 2009):

$$V_L = kI^{\alpha} \quad (7)$$

Since the power exponent α varies significantly in different studies, such as $\alpha \approx 1/3$ (Hutzler & Hutzler, 1982; Bazelyan et al., 2007) and $\alpha \approx 0.66$ (Kekez & Savic, 1983), this study adopts $k = 1.88 \times 10^4$, $\alpha = 0.67$ based on more recent works (Andreev et al., 2008; Popov, 2009; Bazelyan et al., 2009).

From Equations (6) and (7), we derive the voltage drop across the streamer zone at the leader head:

$$\Delta U_T = \frac{I}{2\pi\varepsilon_0 k I^{\alpha}} = \frac{I^{1-\alpha}}{2\pi\varepsilon_0 k} \qquad (8)$$

Considering the leader channel potential drop $U_C = LE$, where L is the leader channel length and E is the electric field of the channel as shown in Equation (4), and the streamer channel potential drop ΔU_T from Equation (8), the total potential drop U of the leader-streamer system is:

$$U = L(\mathrm{a}\mathrm{I}^{b} + c\mathrm{I}^{d}) + \frac{I^{1-\alpha}}{2\pi\varepsilon_0 k} \qquad (9)$$

Differentiating both sides with respect to I ,we obtain the total differential resistance:

$$\frac{dU}{dI} = L(\mathrm{ab}\mathrm{I}^{b-1} + cd\mathrm{I}^{d-1}) + (1-\alpha)\frac{I^{-\alpha}}{2\pi\varepsilon_0 k} \qquad (10)$$

3 Analysis results

Figure 4 illustrates how the differential resistance varies with channel current for different leader channel lengths, where the horizontal line represents zero differential resistance. The point at which the curve intersects the horizontal line marks a change in the sign of the differential resistance, with the corresponding current at this intersection representing the critical current.

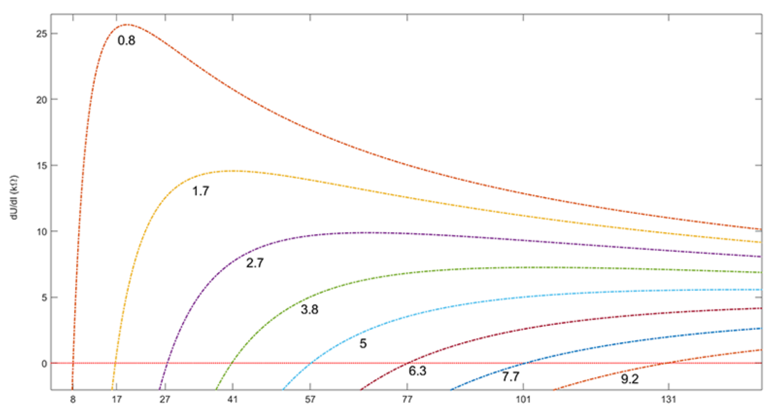

Fig. 4. Dependence of total differential resistance of channel on current with varying channel lengths (km). The horizontal line represents zero resistance

Figure 5 shows that both the critical currents and the critical potential differences in the streamer zone at the leader head increase with the channel length, which is consistent with theoretical predictions. This trend aligns with Heckman's 1992 study on critical currents, and supports the threshold condition for the critical potential difference proposed by Bazelyan & Raizer (2000) and Mazur (2016a).

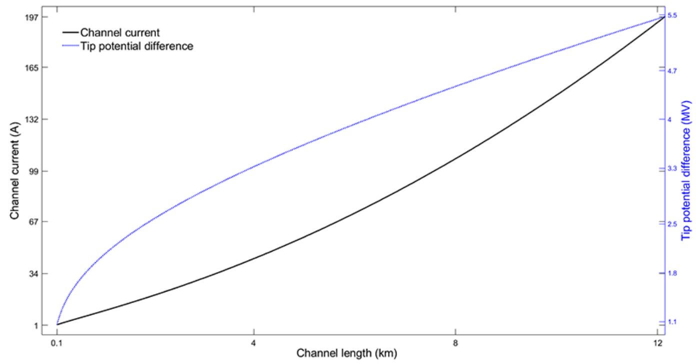

Fig. 5. Critical channel current and potential difference
of the streamer channel at the leader tip vary with channel length.

As the leader channel length increases, the ambient (stabilized) electric field decreases. Between 0.1 km and 12 km, the stabilized leader-streamer field drops from 15.5 kV/m to 1.1 kV/m. This trend is consistent with the findings of Lalande et al. (2002) and Becerra & Vernon (2006) who reported that the leader channel's ambient electric field decreases with channel height. Similarly, the internal electric field of the leader channel decreases (Figure 6). At a length of 0.1 km, the electric field is approximately 4.9 kV/m, while at 12 km, it drops to 0.65 kV/m. Syssoev & Shcherbakov (2001) found that stable thermal leader channels with long electric fields (30–50 m) had electric fields around 3–10 kV/m, which is consistent with our results.

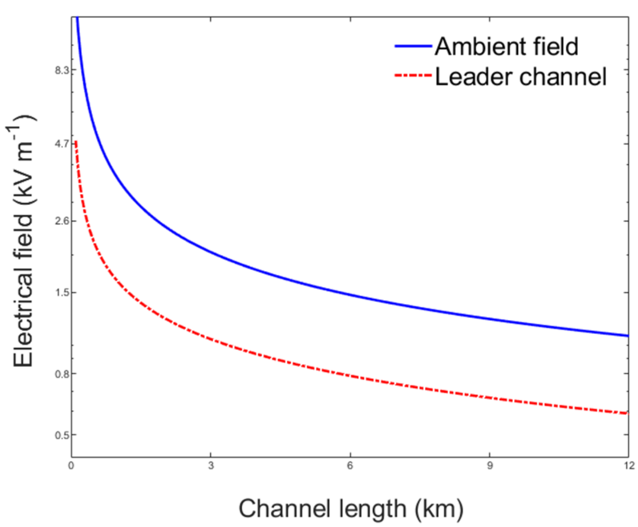

Fig. 6. variations of ambient electric field of the leader-streamer system and electric field of the leader channels with length

4 Discussion and conclusion

This paper extends the discussion on lightning discharge channel stability and differential resistance from the leader channel to the leader-streamer system. Using bifurcation theory and critical transition theory from nonlinear dynamics, we studied the extinction, re-excitation, and critical transition of intermittent events (such as recoil leaders) in the lightning process. By analyzing the sign changes in the differential resistance of the leader-streamer system, we determined the critical current and the critical potential difference at the channel end. Our results show that as the channel length increases, both the critical current and the critical potential difference at the channel end increase. Meanwhile, the average ambient electric field and the electric field required for stable transmission gradually decrease after an initial sharp drop. These findings are consistent with existing research.

The exact mechanism behind the sudden change in channel conductivity remains unclear, but it is likely related to instability caused by positive feedback within the channel. The re-excitation of a decayed leader channel is typically due to the uneven distribution of current and electric field, which becomes more pronounced with increasing channel length. This asymmetry is particularly evident in the channel structure: the leader head maintains higher charge concentration and conductivity, remaining active and often merging with adjacent channels, while the rear part exhibits lower conductivity and greater susceptibility to disconnection and splitting

In the case of negative ground flashes, the electric field in the upper channel becomes non-uniform due to the low current in the positive leader section, which is insufficient to maintain conductivity in the lower channel. Recent observations show that the low current in the positive leader and poor conductivity in the rear section lead to negative charge deposition in the center of the rear channel. This creates negatively polarized needle structures that trigger nonlinear instability (Williams & Montanya, 2019; Hare et al., 2019; Pu & Cummer, 2019; Hare et al., 2021). The current in the rear positive leader decreases, leading to disconnection from the negative leader. The increased potential difference at the paused negative leader causes re-breakdown and reconnection, resulting in multiple strokes. In contrast, positive ground flashes feature stronger current at the negative leader head, making the channel less prone to splitting, which typically results in a single stroke.

The transition from a semiconductor to a conductive state in the leader channel may be driven by ionization-thermal instability caused by positive feedback. As shown in previous studies (Bazelyan & Raizer, 2000; Popov, 2009; da Silva et al., 2019), the pulsed mechanism of the stepped leader is related to electric field inhomogeneity among the streamers at the leader head (Syssoev & Iudin, 2023). This instability may exacerbate the electrical inhomogeneity in the streamer zone, which is thought to be triggered by attachment instability (Douglas-Hamilton & Mani, 1974; Sigmond, 1984; Luque et al., 2016; Malagón-Romero & Luque, 2019). If the mechanism in positive stepped leaders is similar to that of negative stepped leaders, the excitation of the leader should occur in the streamer zone at the leader head (Tran & Rakov, 2016; Kostinskiy et al., 2018; Huang et al., 2020; Wang et al., 2020).

Furthermore, the inhomogeneities, instabilities, and critical transitions observed in the leader channel and streamer zone, whether during initiation or transmission, as well as the emergence of pulse events of various scales and the interactions between leader channels, streamers, and other streamers, all exhibit collective, fractal, and critical properties. These phenomena may require more unified explanations based on fractal analysis and critical dynamics.

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Figure 2: Adapted from "Catastrophic regime shifts in ecosystems: Linking theory to observation," by Marten Scheffer and Stephen R. Carpenter, published in Trends in Ecology & Evolution. Used with permission from Elsevier.

Code/Data Availability

The dataset used in this study is openly available at https://doi.org/10.5281/zenodo.14917985 (Gou et al., 2025).

Author Contributions

XG: Conceptualization, Methodology, Software, Validation, Formal Analysis, Investigation, Writing – Original Draft, Review & Editing, Visualization, Supervision, Project Administration, LX, CX: Software, Formal Analysis, PY, YZ, MC: Review & Editing.

Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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