Response to Reviewer's Comments

Dear Reviewer

Thank you for your insightful comments and suggestions regarding our manuscript. We greatly appreciate your feedback, as it has helped us refine our presentation and ensure clarity. Below, we address your specific concerns regarding the interpretation of Equations (9) and (10) and the associated visual representation:

1. Response to Concern on Equations (9) and (10)

Reviewer's Comment:

You suggested that Equations (9) and (10) describe a non-autonomous dynamical system and could benefit from a stability analysis involving a two-dimensional dynamical system representation.

Our Response:

We appreciate your perspective and would like to clarify the purpose of Equations (9) and (10). These equations are not intended to represent a dynamical system, either autonomous or non-autonomous. Instead, they are static expressions for calculating the differential resistance $\frac{\mathrm{d}U}{dI}$, a key parameter used to analyze stability in lightning channels. Specifically:

$\frac{\mathrm{d}U}{\mathrm{d}I}$ captures the relationship between voltage and current in the channel. Its sign indicates stability: regions where $\frac{\mathrm{d}U}{dI}>0$ correspond to stable states, while regions where $\frac{\mathrm{d}U}{dI}<0$ correspond to unstable states.

These equations are used to identify critical transitions in stability, providing

a direct measure of the conditions under which negative differential resistance arises.

To avoid potential misunderstanding, we will revise the manuscript to better emphasize the role of Equations (9) and (10) as static analytical tools rather than dynamic models.

*2. Response to Concern on the Representation of Stability in Figures***

Reviewer's Comment:

You suggested adding figures to illustrate the stability profiles derived from Equations (9) and (10), as this would enhance clarity regarding the instability mechanisms in lightning channels.

Our Response:

We agree that visual representation is crucial for understanding the implications of the analysis. In fact, the variation of differential resistance $\frac{\mathrm{d}U}{dI}$ with current is already presented in Figure 4, where we illustrate how $\frac{\mathrm{d}U}{dI}$ changes under different conditions (e.g., channel lengths). This figure effectively visualizes the transitions between stable $\frac{\mathrm{d}U}{dI}>0$ and unstable $\frac{\mathrm{d}U}{dI}<0$ regions, offering a clear connection between theoretical predictions and practical dynamics of lightning channels.

However, based on your suggestion, we will enhance the manuscript as follows:

1. Provide additional context in the caption of Figure 4 to explicitly highlight how it relates to the stability analysis and the underlying mechanisms of negative

differential resistance.

2. Ensure the discussion in the text explicitly connects Figure 4 with the findings from Equations (9) and (10), reinforcing their practical relevance to lightning channel dynamics.

Conclusion

We are grateful for your thoughtful suggestions, which have helped us identify areas where additional clarity can be provided. By refining the explanations of Equations (9) and (10) and enhancing the context around Figure 4, we aim to address your concerns effectively. We hope these revisions meet your expectations, and we remain open to further suggestions for improvement.

Thank you again for your valuable feedback.

Best regards

On behalf of all authors