Response to Reviewer 1:

I would like to thank the reviewer for his/her careful reading of the manuscript and pointing out some interesting references to be added. I have taken almost all suggestions of the reviewer into account and hope that the manuscript is now acceptable. Here is a description to all the changes to improve the manuscript according to the suggestions of the reviewer.

As a general comment I would suggest to stress a bit more the question of "predictability" of tippings, especially in relation with the nature of tipping phenomena, since it is a central issue in many natural system and in the description with deterministic-stochastic approaches of natural phenomena.

I would like to thank the reviewer for the suggestion to additionally address the question of predictability in the review part of the manuscript. I have included a whole paragraph devoted to this question at the end of Sec. . This paragraph includes a short discussion of developments of methodology to identify early warning signals when approaching critical transitions. In addition, I shortly recall some approaches of how to predict critical transition including ideas of how to prevent them. This paragraph reads:

Prediction of tipping points and early warning signals In the course of climate change it becomes more and more important to find appropriate methods to predict tipping points and to identify early warning signals. One method that has been developed in the physics and chemistry literature is critical slowing down (CSD) of the restoring forces when a transition is approached (**????**). Resulting from these smaller restoring forces that bring the system back to its stable state after a perturbation, the response to inevitable noise is amplified, leading to a rising standard deviation (**??**) and a lag-1 autocorrelation when approaching the bifurcation (**??**). These methods—critical slowing down and noise amplification—have become extremely popular over the last decade as possible early-warning signals. Besides those methods other statistical approaches have been developed to estimate how close we are to tipping points in the climate system and in ecology (**????**) or how probable noise- and rate-induced transitions are (**?**). They have been used to estimate the proximity of several tipping points in climate such as e.g. the melting of the Greenland ice sheet (**?**), the collapse of the Atlantic Meridional Overturning (**??**) or the loss of the Amazon rainforest (**?**). Despite these various applications of early warning signals they have also been critically discussed from various perspectives (**???**).

Furthermore, another important issue to be highlighted could be the role of processes operating at different scales in changing the topology and the geometry of fixed point and attractors, as well as, the role of symmetries and scale-invariance (turbulence is one of the possible examples). Some possible suggested (optional) references are highlighted below.

I have added some suitable references suggested by the reviewer to emphasize more the impact of different scales on the topological structure in state space. This is now included in the introduction as follows: These timescale separations lead to partly unexpected behaviors. Such scale dependence has been studied in the literature in different contexts such as e.g. climate sensitivity (**?**), tipping in excitable systems (**??**), overshooting and reversing tippings (**??**) and the topological structure of invariant sets in complex systems including their characteristics like fractal dimensions (**??**).

I would like to thank the reviewer for the numerous corrections of English in the manuscript All the suggestions concerning notation and language have been taken into account. But there have been specific questions, which I answer again one-by-one:

Lines 37-49: I would suggest to add some general references to the different outlined points.

Since these are very general statements, I find it strange to assign them to some authors.

Eq. (4): the role of a rate-dependent b is like the albedo feedback in energy-balance models. It would be interesting to mention, if this is the case, as a possible example in the context of the present paper.

I would like to thank the reviewer for this interesting thought. The energy-balance model looks indeed quite similar. However, to see that we would have the same behavior here, would need a thorough investigation and this cannot be explained in a half-sentence. Therefore, I omitted it here, but will undertake this investigation later.

Lines 273-275: this corresponds to a quadratic map in Eqs. 4. Does this mean that an attractor and/or a closed basin is missing/forbidden?

After the saddle-node bifurcation only one attractor is indeed left and this is the attractor corresponding to extinction. To avoid here a bifurcation-induced tipping, the ramping interval has been restricted to a value of b below the saddle-node bifurcation.

Lines 389-390: is this an effect of the variations of the nature of the basins corresponding to $X_1 = 3, X_2 = 3$

I thank the reviewer for noticing that the explanation is not clear here. I have added it to the text now as follows: The reason for this behavior is twofold. One reason is the separation between the intrinsic dissipative timescale and the timescale of environmental change. The non-autonomous basins of attraction are computed starting from a grid of initial conditions, but all of them have already converged to the neighborhood of the attractor at $X_1 \simeq 3$, $X_2 = 3$, when the attractor $(X_1 \simeq 1, X_2 = 3)$ appears in the saddle-node bifurcation due to a faster intrinsic timescale. On the other hand, the example studied describes either chemical concentrations or abundances of species, which have to be positive or equal to zero. This restriction of possible valid initial conditions are another reason, why none of the used initial conditions can reach the new attractor. They can only do so in the frozen-in case, but not beyond a critical rate of environmental change.

Section 3.3: it is clear that "drastic" effects are more evident for changing rate of environmental than timescale of the process, this is crucial for predictability and particularly true for climate change. A few explanatory/additional lines on this would be desirable.

This explanation has been added as follows: Overall, we can conclude, that bifurcations which change the topological structure of the state space have a tremendous impact on the evolution of trajectories. This impact depends crucially on the relation between the intrinsic dissipative timescale and the timescale of environmental change. It turned out, that following the trajectories, which can be considered as observables do not necessarily detect those transitions. There is a detection limit beyond which bifurcations which happen in state space are not noticed by the observables. As a consequence transitions to those undetected states can happen without warning.

Line 551: Code availability the link seems not to work.'

Now the link should work, I corrected the mistake.

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