

1 **Brief Communication: Modified KdV equation for Rossby-Khantadze waves in a**
2 **sheared zonal flow of the ionospheric E-layer**

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15
16 **Abstract**

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18 The system of nonlinear equations for electromagnetic Rossby-Khantadze waves in a
19 weakly ionized conductive ionospheric plasma with sheared zonal flow in the ionospheric E-
20 layer is given. Use of multiple-scale analysis allows reduction of obtained set of equations to
21 (1+1)D nonlinear modified KdV (mKdV) equation describing the propagation of solitary
22 Rossby-Khantadze solitons.
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24 **Keywords:** Rossby-Khantadze waves; nonlinearity, sheared zonal flow
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26 **1. Introduction**
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28 Different satellite and ground-based investigations indicate presence of zonal flows in
29 various atmospheric regions around the Earth (Pedlosky, 1987). The presence of sheared flow
30 along the meridians with inhomogeneous velocity, is connected with the ultra-low-frequency
31 perturbations in E and F regions of the ionosphere (Sato, 2004; Champeaux et al. 2008; Shukla
32 et al., 2003; Onishchenko et al. 2004; Kaladze T.D., 2007; Kaladze et al., 2008). Effect of
33 sheared flow can be seen in the linear and nonlinear properties of the waves, and conditions
34 suitable for that are available in Earth's ionosphere. This gives rise to a variety of linear and
35 nonlinear phenomena like zonal flows. Rossby waves may exist and this may occur in the upper
36 atmosphere and in the oceans; and these play a significant role in the global atmospheric
37 circulation.

38 The reason for the existence of zonal flows is the non-uniform warming of the Earth's
39 atmospheric regions by the sun. In recent decades, several nonlinear phenomena related to the
40 excitation of sheared (zonal) flows were investigated. For instance, Benkadda et al., (2011)
41 investigated the excitation of zonal flows in fusion plasmas by taking drift waves into account.
42 Using Hasegawa Wakatani model, Champeaux et al. (2008) investigated the excitation of zonal
43 flows in drift wave turbulence. Lately, zonal flow generations for short wavelength
44 electromagnetic Rossby waves have also been studied by taking Reynolds stresses into account
45 (Shukla et al., 2003; Onishchenko et al. 2004). The production of zonal flow in E-ionospheric
46 layer by Rossby waves was investigated by Kaladze et al. (2007). The authors considered the
47 effects of the zonal flows on nonlinear structures in Rossby waves and it was shown that such
48 structures split into various segments based on the collection of zonal flow energy (Kaladze et
49 al., 2008). A different concept was considered by Benkadda et al. (2011) where they

50 emphasized on the interaction of high frequency drift waves with those having lower
51 frequencies.

52 More recently, it was revealed, that propagation of coupled electromagnetic Rossby-
53 Khantadze (RK) waves could be self-organized into dipolar (solitary) vortices (Kaladze, 2011;
54 Kaladze, 2014). The spatial inhomogeneity of Coriolis parameter and ambient magnetic field
55 along the meridians causes the propagation of such coupled RK waves. The numerical
56 simulation of the same problem was carried out by Kaladze, et al. (1999) where the possibility
57 of modulation mechanism in ionospheric plasmas was discussed, in turn leading to the
58 formation and transport of dust flows and particles respectively. Numerical solutions of RK
59 waves with sheared zonal flow in weakly ionized ionospheric E-plasma region was investigated
60 as well (Futatani et al., 2015). In his work it was pointed out the splitting of vortices, where the
61 energy is transported by sheared flow into multiple pieces. The equatorial Rossby waves were
62 discussed under similar sheared flows in the beginning of the millennia (Qiang et al., 2001)
63 and the existence of solitons was confirmed by the observations of *Freja and Viking satellites*
64 (Qiang et al., 2001; Bostrom, 1992; Lindqvist et al., 1994; Dovner et al. 1994, YunLong Shi et
65 al. 2018). The generation of shear flow by Rossby-Khantadze waves in E-ionospheric region
66 has also been discussed (Kaladze, et al. 2014). Earlier, Kaladze et al. (2009) had earlier
67 investigated the properties of magnetized solitary Rossby waves with the interaction of zonal
68 flows (sheared) and developed the modified Korteweg-de Vries (mKdV) equation. By
69 considering the β -plane approximation, similar work was done by Jian et al., (2009). Some
70 premises of the possibility of existence of planetary Rossby waves in the dynamo E-area of
71 weakly ionized ionosphere and corresponding experimental interpretation was discussed by
72 Forbes, 1996.

73 In previous publications [12, 16, 17, 18, 19], (1+1)D modified Korteweg-de Vries
74 (mKdV) equations for the amplitude of solitary Rossby waves under the action of zonal shear
75 flow derived in case of neutral fluids. In the given manuscript we generalized these results for
76 the weakly ionized conducting ionospheric E-region plasma incorporating along with stream-
77 function evolution of geomagnetic field for electromagnetic RK waves, which to the best of
78 our knowledge was not reported so far and thus provides novelty to this work. In Sec. 2, by
79 using the multiple scale analysis and perturbation approach from a system of nonlinear two-
80 dimensional equations we derive one-dimensional mKdV equation describing solitary Rossby-
81 Khantadze waves' dynamics along with zonal (shear) flows. In Sec. 3, includes the discussion
82 of the results.

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85 2. Mathematical Preliminaries

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88 We consider weakly ionized E-ionospheric region comprising of small concentration of
89 electrons, ions and bulk of neutral particles, where the ionospheric plasma is enclosed in a
90 spatially inhomogeneous geomagnetic field $\mathbf{B}_0 = (0, B_{0y}, B_{0z})$ and the Earth's angular
91 velocity $\boldsymbol{\Omega} = (0, \Omega_{0y}, \Omega_{0z})$. In such E-layer therefore, two-dimensional consideration of the
92 wave motion provides complete information about its propagation in terms of stream function
93 $\psi(x, y, t)$, $\mathbf{v} = (u, v, 0)$, with $u = -\frac{\partial\psi}{\partial y}$ and $v = \frac{\partial\psi}{\partial x}$.

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96 We introduce a local Cartesian system of coordinates with zonal x, latitudinal y, and z
97 in local vertical direction. Then the nonlinear behavior of the sheared electromagnetic Rossby-
98 Khantadze waves can be described by the following system of 2D equations,

$$97 \quad \begin{cases} \frac{\partial\Delta\psi}{\partial t} + \beta \frac{\partial\psi}{\partial x} + J(\psi, \Delta\psi) - \frac{1}{\mu_0\rho} \beta_B \frac{\partial h}{\partial x} = 0, \\ \frac{\partial h}{\partial t} + J(\psi, h) + \beta_B \frac{\partial\psi}{\partial x} + c_B \frac{\partial h}{\partial x} = 0, \end{cases} \quad (1)$$

98 The first equation is the z-component of vorticity ($\zeta_z = \mathbf{e}_z \cdot \nabla \times \mathbf{v} = \Delta\psi$) of the single-
99 fluid momentum equation under the action of the geomagnetic field, \mathbf{v} is the velocity of the
100 neutral incompressible gas. The second equation is the z-component of the perturbed magnetic
101 induction h obtained through Faraday's law, and $\beta = \frac{\partial f}{\partial y} = \frac{2\partial\Omega_{0z}}{\partial y}$ describes the latitudinal
102 inhomogeneity present in the vertical component of angular velocity. Also the parameter $c_B =$
103 $\beta_B/en\mu_0$ with $\beta_B = \frac{\partial B_{0z}}{\partial y}$, describes the latitudinal inhomogeneity in the background magnetic
104 field, n is the number density of the charged particles, μ_0 is the magnetic permeability and
105 $J(a, b) = \frac{\partial a}{\partial x} \frac{\partial b}{\partial y} - \frac{\partial a}{\partial y} \frac{\partial b}{\partial x}$ is the Jacobian (responsible for the vector nonlinearity) and $\Delta = \partial_x^2 +$
106 ∂_y^2 . Note that the small concentration of charged particles (compared to the neutral particles)
107 gives the contribution only in the inductive current (Kaladze, et al. 2013). It should also be
108 noted that the ambient magnetic field and Coriolis parameter are spatially inhomogeneous,
109 $f = 2\Omega_{0z}$ (Kaladze, et al., 2014). Details on the system (1) can be found in Kaladze, et al.
110 (2012).

111 In Eq. (1), h represents the z-component of perturbed magnetic field and $\beta = \frac{\partial f}{\partial y} =$
112 $\frac{2\partial\Omega_{0z}}{\partial y}$ describes the latitudinal inhomogeneity present in the vertical component of angular
113 velocity. Also the parameter $c_B = \beta_B/en\mu_0$ with $\beta_B = \frac{\partial B_{0z}}{\partial y}$, describes the latitudinal
114 inhomogeneity in the background magnetic field, n is the number density of the charged
115 particles, μ_0 is the magnetic permeability and $J(a, b) = \frac{\partial a}{\partial x} \frac{\partial b}{\partial y} - \frac{\partial a}{\partial y} \frac{\partial b}{\partial x}$ is the Jacobian
116 (responsible for the vector nonlinearity) and $\Delta = \partial_x^2 + \partial_y^2$.

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118 The boundary conditions that are fulfilled in this system are given as,
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$$120 \quad \psi(0) = \psi(1) = 0, \quad (2)$$

121
122 which represents the flow's edges, specifically along the south and north direction [1, 2].
123

124 2.1 Perturbation and weakly nonlinear approach

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126 The background stream function is considered as:
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$$128 \quad \Psi(y) = - \int [U(y) - c_0] dy. \quad (3)$$

129 Here $U(y)$ describes the basic background flow with c_0 as a constant eigenvalue. The whole
130 stream function ψ is considered as the sum of background (zonal flow) stream function $\Psi(y)$
131 and a disturbed stream ψ' function, along with a normalized small parameter $\varepsilon \ll 1$. Which
132 forms a weakly nonlinear system, that is the subject of this study. While the perturbed magnetic
133 field is also characterized by a small a parameter ε . Therefore the stream function and the
134 magnetic perturbations takes the form,

$$135 \quad \begin{aligned} \psi &= \Psi(y) + \varepsilon\psi' = - \int [U(y) - c_0] dy + \varepsilon\psi', \\ h &= \varepsilon h' \end{aligned} \quad (4)$$

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137
138 Using Eq (4) into (1) gives
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$$\begin{cases} \frac{\partial \Delta \psi'}{\partial t} + (U(y) - c_0) \frac{\partial \Delta \psi'}{\partial x} + (\beta - U'') \frac{\partial \psi'}{\partial x} + \frac{\beta_B}{\mu_0 \rho} \frac{\partial h'}{\partial x} + \varepsilon J(\psi', \Delta \psi') = 0, \\ \frac{\partial h'}{\partial t} + \varepsilon J(\psi', h') + (U(y) - c_0) \frac{\partial h'}{\partial x} + \beta_B \frac{\partial \psi'}{\partial x} + c_B \frac{\partial h'}{\partial x} = 0. \end{cases} \quad (5)$$

where $U'' = \frac{d^2 U}{dy^2}$.

By using the multiple scale analysis, we obtain the asymptotic solution where we can take the spatial and temporal parameters as $X = \varepsilon x$ and time $T = \varepsilon^3 t$ respectively. Further by eliminating h' from 5(b) into 5(a) we get the single equation for ψ'

$$\mathcal{L}_0(\psi) + \varepsilon^2 \mathcal{L}_1(\psi) + \varepsilon J\left(\psi, \frac{\partial^2 \psi}{\partial y^2}\right) + \varepsilon^3 J\left(\psi, \frac{\partial^2 \psi}{\partial X^2}\right) + \varepsilon^4 \frac{\partial^3 \psi}{\partial T \partial X^2} = 0. \quad (6)$$

Throughout Eq. (6) the prime on the perturbed stream function is dropped.

Here we introduce the following linear differential operators:

$$\mathcal{L}_0 = \left[(U - c_0) \frac{\partial^2}{\partial y^2} + p(y) + \frac{\alpha(y)}{U - c_0 + c_B} \right] \frac{\partial}{\partial x}, \quad \mathcal{L}_1 = \frac{\partial}{\partial T} \frac{\partial^2}{\partial y^2} + (U - c_0) \frac{\partial^3}{\partial X^3}, \quad (7)$$

where $\alpha(y) = \frac{\beta_B^2}{\mu_0 \rho}$ and $p(y) = \beta - U''$. Here the parameter α involves the spatial inhomogeneous background magnetic field which was not considered before in Kaladze et al. [16].

Furthermore, we denote the disturbed stream function ψ (the asymptotic expansion) as:

$$\psi = \psi_0 + \varepsilon \psi_1 + \varepsilon^2 \psi_2 + \dots \quad (8)$$

By using Eq. (8) into Eq. (6), from the lowest order $O(\varepsilon^0)$, we get the following equation with conditions of boundary:

$$\mathcal{L}_0[\psi_0] = 0, \quad \text{with} \quad \psi_0 = 0 \text{ for } y = 0, 1. \quad (9)$$

The above equation (9) is a linear differential equation. By performing a separation of variables method for $\psi_0 = A(X, T) \Phi_0(y)$ into this form and substitute it into Eq. (7) we get the following equation with conditions of boundary:

$$\left(\frac{d^2}{dy^2} + \frac{p(y)}{U - c_0} + \frac{\alpha(y)}{(U - c_0)(U - c_0 + c_B)} \right) \Phi_0 = 0, \quad \text{with} \quad \Phi_0(0) = \Phi_0(1) = 0. \quad (10)$$

Here we consider $U - c_0 \neq 0$ and $U - c_0 + c_B \neq 0$. This is an eigenvalue problem for eigen value c_0 . By specifying $p(y)$ and $\alpha(y)$, $\Phi_0(y)$ can be found. Since $p(y)$ and $\alpha(y)$ have dependence on the variable y , it is not easy to solve this eigen value problem analytically. From the lowest order $O(\varepsilon^0)$, we see that the problem is time independent, but cannot be analytically solved as we have not substituted any definite dependence on y for the parameters $p(y)$ and $\alpha(y)$. Thus, in order to gain more information about the amplitude of these waves, we go to the next order i.e. $O(\varepsilon^1)$ from Eqs. (7) and (8), we obtain

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$$185 \quad \mathcal{L}_0[\psi_1] = -J\left(\psi_0, \frac{\partial^2 \psi_0}{\partial y^2}\right) \equiv F_1 = A \frac{\partial A}{\partial X} \left(\frac{p(y)}{U-c_0} + \frac{\alpha}{(U-c_0)(U-c_0+c_B)} \right)_y \Phi_0^2, \quad (11)$$

186 Furthermore, we carry out a separation of variables in the following manner $\psi_1 =$ 187 $\frac{1}{2} A^2(X, T) \Phi_1(y)$ for non-singular neutral solutions into (11)

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$$189 \quad \left(\frac{d^2}{dy^2} + \frac{p(y)}{U-c_0} + \frac{\alpha}{(U-c_0)(U-c_0+c_B)} \right) \Phi_1 = \left(\frac{p(y)}{U-c_0} + \frac{\alpha}{(U-c_0)(U-c_0+c_B)} \right)_y \frac{\Phi_0^2}{(U-c_0)}, \quad (12)$$

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191 For the given boundary conditions $\Phi_1(0) = \Phi_1(1) = 0$. To get amplitude we solve Eqs.(7)192 and (8) in the next order i.e. $O(\varepsilon^2)$ which gives

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$$194 \quad \mathcal{L}_0[\psi_2] = -\mathcal{L}_1[\psi_0] - J\left(\psi_0, \frac{\partial^2 \psi_1}{\partial y^2}\right) - J\left(\psi_1, \frac{\partial^2 \psi_0}{\partial y^2}\right) \equiv F_2, \quad (13)$$

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196 with $\psi_2(0) = \psi_2(1) = 0$.

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198 Here it is pointed out that the dispersion effect competes with weakly nonlinear effect.

199

200 Furthermore, we take $\psi_2 = B(X, T)\Phi_2(y)$ and multiply Eq. (13) by ψ_0 and integrate over y ,

201 which yields

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$$203 \quad \int_0^1 dy \frac{F_2}{U-c_0} \Phi_0 = 0. \quad (14)$$

204 By substituting F_2 and using $\psi_1 = \frac{1}{2} A^2(X, T) \Phi_1(y)$ into above Eq. (14) results in the

205 modified KdV (mKdV) equation [17]

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$$207 \quad \frac{\partial A}{\partial T} + N A^2 \frac{\partial A}{\partial X} + D \frac{\partial^3 A}{\partial X^3} = 0. \quad (15)$$

208

209 Here

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$$211 \quad N = \frac{I_2}{I_0}, \quad D = -\frac{I_1}{I_0}, \quad (16)$$

212

$$213 \quad \left\{ \begin{array}{l} I_0 = \int_0^1 dy \Phi_0^2(y) \left[\frac{p(y)}{(U-c_0)^2} + \frac{\alpha}{(U-c_0)^2(U-c_0+c_B)} \right], \\ I_1 = \int_0^1 dy \Phi_0^2(y) \\ I_2 = \int_0^1 dy \frac{\Phi_0^2(y)}{U-c_0} \left\{ \begin{array}{l} \frac{3}{2} \left(\frac{p(y)}{U-c_0} + \frac{\alpha}{(U-c_0)(U-c_0+c_B)} \right)_y \Phi_1(y) \\ -\frac{1}{2} \Phi_0^2(y) \left[\left(\frac{p(y)}{U-c_0} + \frac{\alpha}{(U-c_0)(U-c_0+c_B)} \right)_y \frac{1}{U-c_0} \right]_y \end{array} \right\} \end{array} \right. \quad (17)$$

214

215 Kaladze et al. (2009) and Jian et al. (2009) also obtained the same **mKdV** equation for
 216 Rossby waves as equation (15) and pointed out that the background flow shear is a necessary
 217 condition for the existence of solitary waves, whereas in this work, we get the **mKdV** for the
 218 Rossby-Khantadze waves where the coefficients have been modified by inclusion of
 219 inhomogeneity in geomagnetic field. Moreover, the effect of shear basic flow on the spatial
 220 structure, propagation velocity and wave width of solitary Rossby waves have been studied.

221

222 Here the function $\beta(y)$, $\alpha(y)$ and $U(y)$ are related to the coefficients N and D .

223

224 Amid numerous exact solutions of mKdV equation (15) (see e.g. Wazwaz A-M., 2009),
225 we are interesting in soliton like traveling wave solution

$$226 \quad A(X, T) = \pm \sqrt{\frac{6c}{N}} \operatorname{sech} \left(\sqrt{\frac{c}{D}} (X - cT) \right). \quad (18)$$

227 where c is the traveling wave velocity, and the coefficients D , and N are defined by Eqs.
228 (16)-(17).

229

230

231 Discussion

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233 In the given paper, we studied the nonlinear dynamics of large-scale electromagnetic
234 Rossby-Khantadze waves with zonal flows in E-ionospheric plasma. Both the latitudinal
235 inhomogeneities in angular velocity of the earth's rotation and the geomagnetic field are taken
236 into account. The inhomogeneity of the magnetic field with latitude is responsible for coupled
237 Rossby-Khantadze waves. Such coupling results in an appearance of dispersion of Khantadze
238 waves. To derive the nonlinear modified KdV we used the multiple scale analysis technique.
239 From the lowest order of $O(\varepsilon^0)$, we get an eigen value problem with constant eigen value c_0
240 along with the boundary conditions. The parameters $p(y)$ and $\alpha(y)$ have dependence on the
241 variable y , making it not possible to solve this eigen value problem analytically. From the next
242 order $O(\varepsilon^1)$, by using separation of variables techniques and after doing some mathematical
243 manipulations we arrive at the modified KdV equation (15) with cubic nonlinearity of (1+1)
244 dimension. Traveling wave solitary solution of this equation is given by Eq. (18), where the

245 parameter $\sqrt{\frac{6c}{N}}$ describes the amplitude of solitary RK structures. The obtained coefficients N
246 and D depend on spatial inhomogeneous Coriolis force $\alpha(y)$ and background magnetic field
247 $\beta(y)$, respectively.

248 Looking forward for the experimental observations of RK vortical motions in the
249 weakly ionized ionospheric E-layer we give the following characteristics. Apart from the
250 ordinary Rossby waves electromagnetic RK perturbations generated by the latitudinal gradient
251 of the geomagnetic field and represent the variation of the vortical electric field $\mathbf{E}_v = \mathbf{v}_D \times \mathbf{B}_0$,
252 where $\mathbf{v}_D = \mathbf{E} \times \mathbf{B}_0 / B_0^2$ is an electron drift velocity. They propagate along the latitude with
253 the velocity $|c_B| \approx 2 - 20 \text{ km/s}$. Frequency ($\omega = k_x c_B$) and the phase velocity c_B depend on
254 the charged particles' density and vary by one order of magnitude during the daytime and
255 nighttime conditions. Such perturbations have relatively high frequency ($10^4 - 10^{-1} \text{ s}^{-1}$) and
256 the wavelength $\sim 10^3 \text{ km}$. Compared with the ordinary Rossby waves electromagnetic RK
257 waves accompanied by the strong pulsations of the geomagnetic field 20-80 nT. Note that
258 Khantadze waves in the middle and moderate latitudes observed at the launching of spacecrafts
259 Burmaka, et al. (2006) and by the world network of ionospheric and magnetic observations
260 Sharadze, et al. (1988); Sharadze, et al. (1989); Sharadze, (1991); Alperovich, et al. (2007).
261 Forbes (1996) provides data analyses for discussing the penetration of Rossby type planetary
262 waves effects into ionospheric dynamo E-region (100-170 km) and the electrodynamic
263 interactions which ensue there.

264

265 Discussed waves are mainly of zonal type and observed mainly during magnetic storms
266 and sub-storms, earthquakes, artificial explosions, etc. They give valuable information on
267 large-scale synoptic processes and about external sources and dynamical processes in the
268 ionosphere. Thus, theoretical investigations of electromagnetic Rossby type oscillations will
269 only collect valuable information for further ionospheric experimental investigations.

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AUTHOR DECLARATIONS:

Conflict of Interest

The authors have no conflicts to disclose.

Data Availability

The data that support the findings of this study are available within the article.

References

- [1] Pedlosky J., Geophysical Fluid Dynamics, Springer-Verlag, New York, 1987.
- [2] Satoh M., Atmospheric Circulation Dynamics and General Circulation Models, Springer, New York, 2004
- [3] Champeaux S. & Diamond P.H., Streamer and zonal flow generation from envelope modulations in drift wave turbulence, Phys. Lett. A **288**, 214–219, 2001.
- [4] Shukla P.K., & Stenflo L., Generation of zonal flows by Rossby waves, Phys. Lett. A, **307**, 154–157, 2003.
- [5] Onishchenko O.G., Pokhotelov O.A., Sagdeev R.Z., Shukla P.K., & Stenflo L., Generation of zonal flows by Rossby waves in the atmosphere, Nonlinear Process. Geophys. **11**, 241–244, 2004.
- [6] Kaladze T.D., Wu D.J., Pokhotelov O.A., Sagdeev R.Z., Stenflo L., & Shukla P.K., Rossby-wave driven zonal flows in the ionospheric E-layer, J. Plasma Phys. **73**, 131–140, 2007.
- [7] Kaladze T.D., Pokhotelov O.A., Stenflo L., Rogava J., Tsamalashvili L.V., & Tsik-Lauri M., Zonal flow interaction with Rossby waves in the Earth’s atmosphere: a numerical simulation, Phys. Lett. A **372**, 5177–5180, 2008.
- [8] Benkadda S., Klochkov D.N., Popel S.I., & Izvekova Yu.N., Nonlinear excitation of zonal flows and streamers in plasmas, Phys. Plasmas **18**, 052306, 2011.
- [9] Kaladze T.D., Tsamalashvili L.V., & Kahlon L.Z., Rossby-Khantadze electromagnetic planetary vortical motions in the ionospheric E-layer, J. Plasma Phys., v. **77**, 813-828 2011.
- [10] Kaladze T., Kahlon L., Horton W., Pokhotelov O., & Onishchenko O., Shear flow driven Rossby-Khantadze electromagnetic planetary vortices in the ionospheric E-layer, Europhys. Lett. **106**, 29001, 2014.
- [11] Kaladze T., Magnetized Rossby waves in the Earth’s ionosphere, Plasma Phys. Rep. **25** (4), 284–287, 1999.
- [12] Futatani S., Horton W., Kahlon L.Z., & Kaladze T.D., Rossby-Khantadze electromagnetic planetary waves driven by sheared zonal winds in the E-layer ionosphere, Phys. Plasmas **22**, 012906, 2015.
- [13] Qiang Z., Zuntao F., & Shikuo L., Equatorial envelope Rossby solitons in a shear flow, Adv. Atmos. Sci. **18**, 418–428, 2001.
- [14] Bostrom R., Observations of weak double layers on auroral field lines, IEEE Trans. Plasma Sci. **20**, 756–763, 1992.
- [15] Lindqvist P.A., Marklud G.T., & Blomberg L.G., Plasma characteristics determined by the Freja electric field instrument, Space Sci. Rev. **70**, 593–602, 1994.
- [16] Dovner P.O., Eriksson A.I., Bostrom R., Holback B., Freja multiprobe observations of electrostatic solitary structures, Geophys. Res. Lett. **21**, 1827–1830, 1994.
- [17] Kaladze T., Tsamalashvili L., Kaladze D., Ozcan O., Yesil A., & Inc M., Modified KdV equation for magnetized Rossby waves in a zonal flow of the ionospheric E-layer, Physics Letter A, **383**, 125888, 2009.

322 [18] Jian S., Lian-Gui Y., Chao-Jiu DA., & Hui-Qin Z., mKdV equation for the amplitude of
323 solitary Rossby waves in stratified shear flows with a zonal shear flow, Atmospheric Oceanic
324 Science Letters **1**, 18-23, 2009.

325 [19] Forbes, J.M., Planetary waves in the thermosphere-ionosphere system, J. Geomag.
326 Geoelectr. v. **48**, 91-98, 1996.

327 [20] Shi1 Y., Yang D., Feng X., Qi J., Yang H. and Yin B., One possible mechanism for
328 eddy distribution in zonal current with meridional shear, Scientific Reports, **8**, 10106, 2018.

329 [21] Kaladze T.D., Horton W., Kahlon L.Z., Pokhotelov O., and Onishchenko O., Zonal
330 flows and magnetic fields driven by large-amplitude Rossby-Alfvén-Khantadze waves in the
331 E-layer ionosphere, J. Geophys. Res.: Space Physics **118**, 1-12, 2013

332 [22] Kaladze T.D., Kahlon L.Z., & Tsamalashvili L.V., Excitation of zonal flow and
333 magnetic field by Rossby-Khantadze electromagnetic planetary waves in the ionospheric E-
334 layer, Phys. plasmas v.**19**, 022902, 2012

335 [23] Wazwaz A-M., Partial differential equations and solitary waves theory, Springer, 2009.

336 [24] Burmaka, V.P., Lysenko, V.N., Chernogor, L.F., & Chernyak, Yu..V. , Wave-like process
337 in the ionospheric F region that accompanied rocket launches from the Baikonur Site,
338 Geomagn. Aeronomy **46**, 742-759, 2006.

339 [25] Sharadze, Z.S., Japaridze, G.A., Kikvilashvili, G.B., & Liadze, Z.L., Wave disturbances
340 of non-acoustical nature in the middle-latitude ionosphere, Geomagn. Aeronomy **28**, 446-451,
341 1988 (in Russian)

342 [26] Sharadze, Z.S., Mosashvili, N.V., Pushkova, G.N., & Yudovich, L.A., Long-period-wave
343 disturbances in E-region of the ionosphere, Geomag. Aeron. **29**, 1032-1034, 1989 (in Russian)

344 [27] Sharadze, Z.S., Phenomena in the middle-latitude ionosphere, PhD Thesis, Moscow, 1991.

345 [28] Alperovich, L.S., & Fedorov, E.N., Hydromagnetic Waves in the Magnetosphere and the
346 Ionosphere, Springer, 2007.

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