

Dear Editor,

We have made major changes in our manuscript according to your comments:

- 1) In line 18, we added “electromagnetic” before Rossby-Khantadze;
- 2) In line 20, we removed “so called”;
- 3) In line 21 we changed the last word “waves” by “solitons”;
- 4) In line 19 at the end of the 1st sentence we added “zonal flow in the ionospheric E-layer is given”;

So, the revised abstract is as follows:

Abstract

The system of nonlinear equations for electromagnetic Rossby-Khantadze waves in a weakly ionized conductive ionospheric plasma with sheared zonal flow in the ionospheric E-layer is given. Use of multiple-scale analysis allows reduction of obtained set of equations to (1+1)D nonlinear modified KdV (MKdV) equation describing the propagation of solitary Rossby-Khantadze solitons.

- 5) In line 59 we changed “he has” by “it was pointed”;
- 6) We continued line 44 by line 45;
- 7) In line 67 instead the abbreviation we added “modified Korteweg-de Vries (mKdV) equation”;
- 8) In lines 69, 74 we changed MKdV by mKdV;
- 9) In Line 51, we changed “it was seen” by “it was revealed”, and inserted “electromagnetic” after “coupled”;
- 10) In Line 52, we added new reference [T.D. Kaladze, L.V. Tsamalashvili, L.Z. Kahlon, Rossby-Khantadze electromagnetic planetary vortical motions in the ionospheric E-layer, *J. Plasma Phys.*, (2011), v. 77, 813-828]
- 11) At the end of line 68 we added the sentence on experimental evidence of planetary Rossby waves in the ionospheric E-region: “Some premises of the possibility of existence of planetary Rossby waves in the dynamo E-area of weakly ionized ionosphere and corresponding experimental interpretation was discussed by Forbes [Forbes, J.M., Planetary waves in the thermosphere-ionosphere system // *J. Geomag. Geoelectr.*, v. 48, 91-98, (1996)]”
- 12) We removed the previous reference [19] and inserted new reference “19. Song Jian, Yang Lian-Gui, DA Chao-Jiu, and Zhang Hui-Qin, mKdV equation for the amplitude of solitary Rossby waves in stratified shear flows with a zonal shear flow, *Atmospheric Oceanic Science Letters*, 2009, #1, 18-23”
- 13) In the last paragraph of Introduction (lines 69-72) we made the following changes: “In previous publications [12, 16, 17, 18, new 19 by Song Jian], (1+1)D modified Korteweg-de Vries (mKdV) equations for the amplitude of solitary Rossby waves under the action of zonal shear flow derived in case of neutral fluids. In the given manuscript we generalized these results for the weakly ionized conducting ionospheric E-region plasma incorporating along with stream-function evolution of geomagnetic field for electromagnetic RK waves,

which to the best of our knowledge was not reported so far and thus provides novelty to this work.” **We think the motivation should be clear.** Then continue : “In Sec. 2, by using ...”

- 14) Answering your comment (V) we added: a) in line 193 the reference [16], b) instead lines 212-216 we changed: “Amid numerous exact solutions of mKdV equation (15) (see e.g. [Abdul-Majid Wazwaz, Partial Differential Equations and Solitary Waves Theory, Springer, 2009], we are interesting in soliton like traveling wave solution (**next line is Eq. (18)**), where c is the traveling wave velocity, and the coefficients D , and N are defined by Eqs. (16)-(17)”.
- 15) Concerning the origin of Eq. (1) we added new reference [T.D. Kaladze, L.Z. Kahlon, L.V. Tsamalashvili, Excitation of zonal flow and magnetic field by Rossby-Khantadze electromagnetic planetary waves in the ionospheric E-layer, Phys. plasmas, v.19, 022902 (2012); doi: 10.1063/1.3681370] and instead lines 80-104 we changed the following:

We consider weakly ionized E-ionospheric region comprising of small concentration of electrons, ions and bulk of neutral particles, where the ionospheric plasma is enclosed in a spatially inhomogeneous geomagnetic field $\mathbf{B}_0 = (0, B_{0y}, B_{0z})$ and the Earth’s angular velocity $\mathbf{\Omega} = (0, \Omega_{0y}, \Omega_{0z})$. In such E-layer therefore, two-dimensional consideration of the wave motion provides complete information about its propagation in terms of stream function $\psi(x, y, t)$, $\mathbf{v} = (u, v, 0)$, with $u = -\frac{\partial\psi}{\partial y}$ and $v = \frac{\partial\psi}{\partial x}$.

We introduce a local Cartesian system of coordinates with zonal x , latitudinal y , and z in local vertical direction. Then the nonlinear behavior of the sheared electromagnetic Rossby-Khantadze waves can be described by the following system of 2D equations,

$$\begin{cases} \frac{\partial\Delta\psi}{\partial t} + \beta \frac{\partial\psi}{\partial x} + J(\psi, \Delta\psi) - \frac{1}{\mu_0\rho} \beta_B \frac{\partial h}{\partial x} = 0, \\ \frac{\partial h}{\partial t} + J(\psi, h) + \beta_B \frac{\partial\psi}{\partial x} + c_B \frac{\partial h}{\partial x} = 0, \end{cases} \quad (1)$$

The first equation is the z -component of vorticity ($\zeta_z = \mathbf{e}_z \cdot \nabla \times \mathbf{v} = \Delta\psi$) of the single-fluid momentum equation under the action of the geomagnetic field, \mathbf{v} is the velocity of the neutral incompressible gas. The second equation is the z -component of the perturbed magnetic induction h obtained through Faraday’s law, and $\beta = \frac{\partial f}{\partial y} = \frac{2\partial\Omega_{0z}}{\partial y}$ describes the latitudinal inhomogeneity present in the vertical component of angular velocity. Also the parameter $c_B = \beta_B/en\mu_0$ with $\beta_B = \frac{\partial B_{0z}}{\partial y}$, describes the latitudinal inhomogeneity in the background magnetic field, n is the number density of the charged particles, μ_0 is the magnetic permeability and $J(a, b) = \frac{\partial a}{\partial x} \frac{\partial b}{\partial y} - \frac{\partial a}{\partial y} \frac{\partial b}{\partial x}$ is the Jacobian (responsible for the vector nonlinearity) and $\Delta = \partial_x^2 + \partial_y^2$. Note that the small concentration of charged particles (compared to the neutral particles) gives the contribution only in the inductive current (Kaladze, et al. 2013). It should also be noted that the ambient magnetic field and Coriolis parameter are spatially inhomogeneous, $f = 2\Omega_{0z}$ (Kaladze, et al., 2014). Details on the system (1) can be found in [T.D. Kaladze, L.Z. Kahlon, L.V. Tsamalashvili, Excitation of zonal flow and magnetic field by Rossby-Khantadze

electromagnetic planetary waves in the ionospheric E-layer, Phys. plasmas, v.19, 022902 (2012); doi: 10.1063/1.3681370]

- 16) As a private communication, we would like discuss your comment (IV). For the scientists engaged with the Rossby waves it will be clear that the first equation of the system (1) is written for the relatively small-scale structures $a \leq r_R$, where r_R is the

Rossby radius. With the opposite inequality the first term should be added by $-\frac{\partial}{\partial t} \frac{1}{r_R^2} \psi$.

Then obtained mKdV equation will contain quadratic nonlinear term (see Reference [18] by Shi et al.).

- 17) Remain comments on possible experimental observations are elucidated in Discussion, which should be read as follows:

3. Discussion

In the given paper, we studied the nonlinear dynamics of large-scale electromagnetic Rossby-Khantadze (RK) waves under the action of sheared zonal flow in the weakly ionized E-ionospheric plasma. Latitudinal inhomogeneity both of components of angular velocity of the earth's rotation and the geomagnetic field are taken into account. The inhomogeneity of the geomagnetic field with latitude is responsible for the existence of coupled RK waves. Such coupling results in an appearance of dispersion of Khantadze waves. To derive the nonlinear modified KdV equation we used the multiple scale analysis technique. From the lowest order of $O(\epsilon^0)$, we get an eigenvalue problem with constant eigenvalue c_0 along with the boundary conditions. The parameters $p(y)$ and $\alpha(y)$ have dependence on the variable y , making it not possible to solve this eigenvalue problem analytically. From the next order $O(\epsilon^1)$, by using separation of variables techniques and after doing some mathematical manipulations we arrive at the modified KdV equation (15) with cubic nonlinearity of (1+1) dimension. Traveling wave solitary solution of this equation is given by Eq. (18), where the parameter $\sqrt{\frac{6c}{N}}$ describes the amplitude of solitary RK structures. The obtained coefficients N and D depend on spatial inhomogeneous Coriolis force $\alpha(y)$ and background magnetic field $\beta(y)$, respectively.

Looking forward for the experimental observations of RK vortical motions in the weakly ionized ionospheric E-layer we give the following characteristics. Apart from the ordinary Rossby waves electromagnetic RK perturbations generated by the latitudinal gradient of the geomagnetic field and represent the variation of the vortical electric field $\mathbf{E}_v = \mathbf{v}_D \times \mathbf{B}_0$, where $\mathbf{v}_D = \mathbf{E} \times \mathbf{B}_0 / B_0^2$ is an electron drift velocity. They propagate along the latitude with the velocity $|c_B| \approx 2 - 20 \text{ km/s}$. Frequency ($\omega = k_x c_B$) and the phase velocity c_B depend on the charged particles' density and vary by one order of magnitude during the daytime and nighttime conditions. Such perturbations have relatively high frequency $(10^{-4} - 10^{-1}) \text{ s}^{-1}$ and the wavelength $\sim 10^3 \text{ km}$. Compared with the ordinary Rossby waves electromagnetic RK waves accompanied by the strong pulsations of the geomagnetic field 20-80 nT. Note that Khantadze waves in the middle and moderate latitudes

observed at the launching of spacecrafts [Burmaka, V.P., Lysenko, V.N., Chernogor, L.F., Chernyak, Yu.V. , Wave-like process in the ionospheric F region that accompanied rocket launches from the Baikonur Site, *Geomagn. Aeronomy*, **46**, 742-759, 2006] and by the world network of ionospheric and magnetic observations [Sharadze, Z.S., Japaridze, G.A., Kikvilashvili, G.B., Liadze, Z.L., Wave disturbances of non-acoustical nature in the middle-latitude ionosphere, *Geomagn. Aeronomy* **28**, 446-451, 1988 (in Russian); Sharadze, Z.S., Mosashvili, N.V., Pushkova, G.N., Yudovich, L.A., Long-period-wave disturbances in E-region of the ionosphere, *Geomag. Aeron.* **29**, 1032-1034, 1989 (in Russian); Sharadze, Z.S., Phenomena in the middle-latitude ionosphere, PhD Thesis, Moscow, 1991; Alperovich, L.S., Fedorov, E.N., *Hydromagnetic Waves in the Magnetosphere and the Ionosphere*. Springer, 2007]. Forbes (1996) provides data analyses for discussing the penetration of Rossby type planetary waves effects into ionospheric dynamo E-region (100-170 km) and the electrodynamic interactions which ensue there.

Discussed waves are mainly of zonal type and observed mainly during magnetic storms and substorms, earthquakes, artificial explosions, etc. They give valuable information on large-scale synoptic processes and about external sources and dynamical processes in the ionosphere. Thus theoretical investigations of electromagnetic Rossby type oscillations will only collect valuable information for further ionospheric experimental investigations.

Yours Sincerely,

Laila Zafar Kahlon