

Response to the referees regarding the revised manuscript.

First, now that we have gone through and revised the manuscript in detail in accordance with the referee comments, we believe the paper has been significantly improved and strengthened by addressing all of the excellent remarks. We thank both referees for their suggestions, and in a nutshell, we have taken all their suggestions and made all the requested changes in the revised manuscript. We have also added new, unpublished versions of several of the figures, and added many new references to that are relevant to the topics we consider.

Blue font is used in the manuscript to denote changes.

Response to Referee #1.

Thanks again for all your suggestions. We have included the requested references, added citations in the requested locations, and briefly described this type of spectral method. We have also now included intuitive descriptions of Theorems 1 and 2, and the function $F(s)$ in Equation (5).

Response to Referee #2.

Thanks again for all your suggestions. Concerning points 1 and 2, the figure captions have been re-written to make these excellent issues more clear, particularly the caption in Figure 3. Also, concerning the excellent question as to why connectedness is signaled by the appearance of point measures near a spectral endpoint, from an intuitive standpoint it's helpful to keep in mind that $F(s)=p/s$ is the exact expression for a geometry with layers parallel to the applied field, where p is the volume fraction of the conducting phase. This is the simplest composite geometry where the conducting phase connects across the sample, parallel to the field, and whose spectral measure is a single Dirac point measure of mass p at the spectral endpoint $\lambda=0$. Perturbing this situation, while maintaining the connection across the sample, keeps similar spectral behavior near the endpoint. This singular behavior there goes away, and is replaced by a spectral gap, as the connectivity of the conducting phase across the sample is destroyed. For point 3, we have substantially re-written the advection diffusion section to address the points raised, and for point 4 we have added new material in the Introduction to address the issues brought up there, which we believe substantially improves the readability and applicability of the paper. All the technical comments were taken care of as requested. Thank you very much!