Response to Anonymous Referee #2 on "Identification of linear response functions from arbitrary perturbation experiments in the presence of noise

Part I. Method development and toy model demonstration"

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We would like to thank Anonymous Referee #2 for the positive review of our paper. We largely agree with the referee's suggestions and will address them in the revised manuscript. Below we give a point-by-point reply to the issues raised in the review.

***AR#2*: In the introduction the authors state that typically methods estimating the response rely on some "prior information". They state that their method does not need prior information. That seems not quite true (they assume monotonicity, Eqn (8), Picard condition etc).**

Authors: We appreciate the comment by the referee. Indeed the way these sentences were framed might give the impression that our method does not need prior information. Actually with these (malformed) sentences we tried to point out exactly what the referee requests, namely that additional prior information is needed, particularly in the form of additional data from an unperturbed experiment. But additionally to that, we agree with the referee that we make even more assumptions. We will change the text accordingly.

***AR#2*: Following up on my previous point, their method requires a few assumptions on the underlying dynamical system under consideration such as (8) and what they call the Picard condition. It would be nice to have these assumptions listed somewhere.**

Authors: We agree with the referee and will add such a list to the revised manuscript.

***AR#2*: It is well known that one can formulate regularization such as ridge regression in a Bayesian framework where the regularization corresponds to a prior. Could the authors comment on what the meaning of this prior is in their context?**

Authors: Although we are mostly acquainted with deterministic regularization methods (e.g., Bertero et al., 1995; Engl et al., 1996; Hansen, 2010), it seems to us that from the Bayesian perspective by assuming that the prior distribution for the quantity of interest $\boldsymbol{q}$ and the distribution for the additive noise $\boldsymbol{\eta}$ are Gaussian (of the type $\mathcal{N}(\mathbf{0}, \sigma^2 I)$) and independent, the maximum a posteriori estimator for $\boldsymbol{q}$ gives the same regularized solution that we arrive at (e.g. example 4.26 in Vogel, 2002). But since this is known from the literature and not needed to understand our method, we would prefer not to include such a discussion in the text.

***AR#2*: Introducing the noise η in (5) can be justified by the central limit theorem, I assume, which could be mentioned.**

Authors: We do not really understand what the referee is having in mind here. The noise term η arises in Eq. (5) simply as the remainder when shifting from the ensemble average to a particular realization. There is no statistical assumption behind this that could be made precise by invoking the central limit theorem. But maybe our formulation in line 134 that the noise term must show up "as a consequence of dropping the ensemble average" is not sufficiently clear. We will think about a better formulation.

***AR#2*: Although I appreciated that the authors went through some trouble in explaining the details necessary to understand their approach, the manuscript could gain by being more succinct. For example, the sentence right after Eqn (24) just reiterates what has been described before.**

Authors: We agree with the reviewer. We will go through the manuscript and see where we can be more succinct.

***AR#2*: In the introduction the authors give a nice account of the use of linear response theory in the climate sciences. Their exposition, however, might give the false impression that linear response should be expected. Whereas it is now proven that systems driven by noise satisfy linear response theory (Hairer and Majda 2010), the situation for deterministic systems as initiated by Ruelle is far more complicated. Viviane Baladi and co-workers in fact showed that very simple dynamical systems such as the logistic map do not obey linear response. Moreover, examples of dynamical systems in the climate sciences are known that exhibit a rough parameter dependency (Chekroun et al 2014). The question of how to reconcile the fact that generic high-dimensional dynamical systems satisfy linear response theory even when their individual microscopic constituents do not, was addressed by Wormell and Gottwald (2018, 2019). The following references are relevant for this discussion: (...)**

Authors: We agree with the reviewer that it is useful to extend the literature review as suggested.

***AR#2*: There are other recent methods dealing with response theory from a numerical point of view, either detecting it or calculating the response, which the authors may want to include: (...)**

Authors: We will refer to these additional numerical methods in the revised manuscript.

***AR#2*: Page 15, after Eqn (31): "Since almost every linear system can be diagonalised, we assume" —> "We assume .."**

***AR#2*: Page 17, l439, delete "extremely"**

Authors: The text will be changed accordingly.

With best regards,

Guilherme L. Torres Mendonça, Julia Pongratz and Christian H. Reick

References

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