“Brief communication: An innovation-based estimation method for model error covariance in Kalman filters”

Revision for NPG, received on November 15, 2021.

The article discusses a method for estimating the Q error covariance matrix of a dynamic model in ensemble data assimilation. This matrix is important because it plays the role of additive inflation in EnKF. The proposed methodology assumes that the error covariance matrix R of the observations is known, which is a strong assumption. Due to this simplification, Q is estimated using the second-order moment of the innovation. The proposed method is online and therefore dependent on two adjustment parameters, a forgetting factor and a first guess on Q. These parameters are important in practice. The article needs to take into account the estimation of these tuning parameters.

Major comments:

- The proposed methodology is an online estimation method, meaning that Q(t) is estimated synchronously with the state x(t). Authors suggest that Q is contact in time but they could have considered a time varying Q matrix.
- The proposed methodology is highly dependent on \rho, the forgetting factor, and Q(t_0), the initial model error covariance. The estimation of an adaptive \rho parameter should be addressed in this paper. Moreover, iterative procedures like the EM algorithm (Dreano et al. 2017 or Pulido et al. 2018) should be considered to estimate Q(t_0).

Minor comments:

- L. 52, please write P^p(t_{i+1}).
- L. 53-59, not sure the discussion is useful.
- L. 79, what is the meaning of "vec"?
- L. 87, not sure to understand the explanation of "bandwidth 20", can you clarify?
- L. 90, looking at the Lorenz-96 equations, x_1 and x_40 are neighbors and should be positively correlated. This is not what is shown in Fig.2 (a). I think you should consider such covariance between neighbors in Q.
- L. 98, please remind the reader that \rho is the forgetting factor.

References:

- Dreano, D., Tandeo, P., Pulido, M., Ait-El-Fquih, B., Chonavel, T., & Hoteit, I. (2017). Estimating model-error covariances in nonlinear state-space models using Kalman smoothing and the expectation–maximization algorithm. Quarterly Journal of the Royal Meteorological Society, 143(705), 1877-1885.
- Pulido, M., Tandeo, P., Bocquet, M., Carrassi, A., & Lucini, M. (2018). Stochastic parameterization identification using ensemble Kalman filtering combined with maximum likelihood methods. Tellus A: Dynamic Meteorology and Oceanography, 70(1), 1-17.