

## ***Interactive comment on “Behavior of the iterative ensemble-based variational method in nonlinear problems” by Shin’ya Nakano***

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I appreciate the referee #2 for taking time for reviewing the manuscript. The followings are the responses to the comments by the referee. Each response is numbered in accord with the referee’s comments.

### Responses to Major comments

1. Our group has applied an instance of the iterative method with the ensemble transform to a practical geodynamo model (Minami et al., 2020). However, I agree that the performance of a method with a randomly generated ensemble has not been demonstrated in any high-dimensional problems. According to the referee’s comment, I have

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applied the method to the Lorenz 96 model with 400 variables, of which the dimension is ten times higher. The iterative method with a randomly generated ensemble achieved a promising convergence even in high-dimensional problems. The attached figure shows the value of the objective function for each iteration in the experiment using the Lorenz 96 model with 400 variables. For the assimilation into the Lorenz 96 model with 400 variables, the convergence was attained in about 20 iterations with 200 ensemble members.

2. The iterative algorithm discussed in the present paper considers a highly uncertain problem, which corresponds to a situation before a spin-up process in typical data assimilation. This algorithm is thus applicable without spin-up. Rather, the iterations could be used for spin-up for another data assimilation system. In the experiments of Section 7, a spin-up procedure is not done. Instead, the initial ensemble was randomly drawn from a Gaussian distribution as described in L. 362.

3. The value of  $\delta$  in Figures 1–6 was chosen to contrast between a monotonic convergence case and a non-monotonic convergence case. At present, I can not provide any automatic way to determine the value of  $\delta$ . In practical applications, however, one can check whether the objective function  $J$  increases or not at each iteration just by running one forward simulation initialized at  $\bar{x}_{0,m}$ . In the iterative algorithm, most computational cost is spent for the ensemble simulation run. The pilot run, which is computationally much cheaper than the ensemble run, is a feasible way for tuning  $\delta$  in practical cases.

4. The iterative method discussed in the present paper is for variational problems. Therefore, it can not be fairly compared with the EnKFs. Comparing with the conventional 4DVar, the convergence rate of this iterative method would be a little poorer because it employs an ensemble approximation within a particular subspace at each iteration. Nonetheless, I believe that the iterative ensemble-based method is useful because it is much easier to implement. As mentioned in the introduction section, the conventional 4DVar requires an adjoint code which is usually time-consuming to de-

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velop. However, variational problems can be solved with the ensemble-based method which does not require an adjoint code. As discussed in the present paper, the iterative ensemble-based method, which does not require an adjoint code, can potentially achieve comparable accuracy to the conventional 4DVar. I thus believe it is a promising tool for data assimilation and various inverse problems in geosciences.

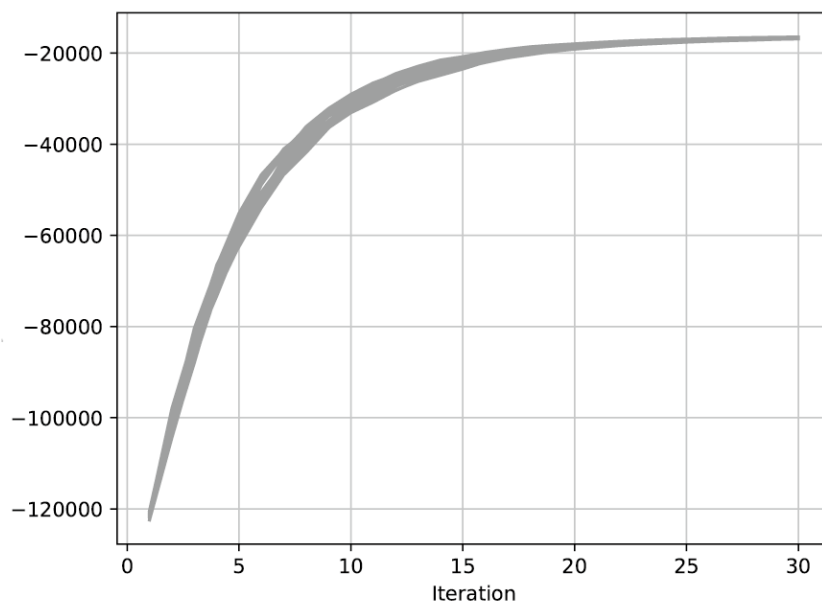
#### Responses to Minor comments

1. I appreciate the referee for the correction. This sentence should read “We can then obtain the local maximum in the full vector space if  $Q_m$  is taken to be full rank.”
2. I appreciate the referee for the correction.

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**Fig. 1.** The value of the objective function  $J$  for each iteration for 20 trials of the estimation in the experiment with 400 variables.

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