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# Brief Communication: Residence Time of Energy in the Atmospheres of Venus, Earth, Mars and Titan

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**Abstract.**

The concept of residence time of energy in a planetary atmosphere $\tau_R$, recently introduced and computed for the Earth's atmosphere (Osácar et al., 2020), is here extended to the atmospheres of Venus, Mars and Titan. After a global thermal perturbation, $\tau_R$ is the time scale the atmosphere needs to return to equilibrium. The residence times of energy in the atmospheres of Venus, Earth, Mars and Titan have been computed. In the cases of Venus, Mars and Titan, these are mere lower bounds due to a lack of some energy data.

## 1 Introduction

When the inflow, $F_i$, of any substance into a box is equal to the outflow, $F_o$, then the amount of that substance in the box, $M$, is constant. This constitutes an equilibrium or steady state. Then, the ratio of the stock in the box to the flow rate (in or out) is called the residence time and is a time scale for the transport of the substance in the box.

$$t = \frac{M}{F}. \tag{1}$$

In equation (1) it is assumed that the substance is conserved. A good example of this type is the parameter defined in atmospheric chemistry (Hobbs, 2000) as the average residence time of each individual gas, defined as (Eq. 1). $M$ is the total average mass of the gas in the atmosphere and $F$ the total average influx or outflux, which in time average for the whole atmosphere are equal.

In this work we extend the substance that flows in the box from matter to energy and the residence time is

$$\tau = \frac{E}{F}, \tag{2}$$

where $E$ is the total energy in the box (a planetary atmosphere) and $F$ the energy flux that enters or leaves it.

By using (2), we estimate the average residence time of energy in several planetary atmospheres. But first it is worth recalling that several authors have previously considered the energy-residence time relation in other problems (Mcilveen, 1992, 2010; Harte, 1988). Harte (1988) uses this concept to estimate the anomalous temperature in urban heat islands.

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The residence time of energy can also be considered in the Sun. The Kelvin-Helmholtz time scale (Kippenhahn and Weigert, 1994) is the time the Sun would need to recover from a global thermal perturbation:

$$\tau_{KH} = \frac{GM_\odot^2}{R_\odot L_\odot} \sim 10^7\,\mathrm{yr}. \quad (3)$$

As $L_\odot$ is the luminosity of the Sun, equation (3) is also a form of expressing the residence time of energy in this star. This is because

$$\tau_{KH} = \frac{GM_\odot^2}{R_\odot L_\odot} \sim \frac{|\mathrm{gravitational\,energy}|}{L_\odot}. \quad (4)$$

And as in a star like the Sun, the Virial Theorem (Kippenhahn and Weigert, 1994) links

$$|\mathrm{gravitational\,energy}| = 2\,|\mathrm{total\,energy}|, \quad (5)$$

hence:

$$\tau_{KH} \sim 2\frac{|\mathrm{total\,energy}|}{L_\odot}. \quad (6)$$

References Stix (2003) and Spruit (2000) are worth reading in this respect.

The planetary atmospheres and the Sun constitute steady state problems because the storage of energy in their interior is not systematically increasing or decreasing. Section 2 addresses the numerator of Eq. (2) $E$, while Section 3 deals with the denominator $F$. In Section 4 we comment on some final points.

## 2 Forms of energy in a planetary atmosphere

In a planar atmosphere, the most important forms of energy in a planetary atmosphere are: the thermodynamic internal energy, $U$,

$$U = \int_0^\infty c_v\,T(z)\,\rho(z)\,\mathrm{d}z, \quad (7)$$

the potential energy due to the planet's gravity, $P$,

$$P = \int_0^\infty g\,z\,\rho(z)\,\mathrm{d}z, = \int_0^\infty p(z)\,\mathrm{d}z \quad (8)$$

the kinetic energy, $K$ and the latent energy, $L$, related to the phase transitions. In expressions (7) and (8), $c_v$ is the specific heat at constant volume, and $\rho(z)$ and $T(z)$ are the density and temperature of the mixture of gases of the atmosphere respectively. E stands for the total energy in the atmosphere:

$$E = U + P + K + L. \quad (9)$$

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**Table 1.** Forms of energy in planetary atmospheres

| | Venus | Earth | Mars | Titan |
|---|---|---|---|---|
| P $(\mathrm{J\,m^{-2}})$ | 1.24E+11 | 7.00E+08 | 6.05E+06 | 2.63E+09 |
| U $(\mathrm{J\,m^{-2}})$ | 4.31E+11 | 1.80E+09 | 2.10E+07 | 6.79E+09 |
| S $(\mathrm{J\,m^{-2}})$ | 5.55E+11 | 2.50E+09 | 2.71E+07 | 9.42E+09 |
| K $(\mathrm{J\,m^{-2}})$ | ... | 1.30E+06 | ... | ... |
| L $(\mathrm{J\,m^{-2}})$ | ... | 7.00E+07 | ... | ... |
| E $(\mathrm{J\,m^{-2}})$ | ... | 2.57E+09 | ... | ... |
| $C_p/R$ | 4.47 | 3.5 | 4.37 | 3.58 |

**Table 2.** Fluxes of energy in planetary atmospheres

| | Venus | Earth | Mars | Titan |
|---|---|---|---|---|
| $F_i\,(\mathrm{W\,m^{-2}})$ | 17292 | 509.6 | 49 | 6.88 |
| $F_o\,(\mathrm{W\,m^{-2}})$ | 17292 | 509.6 | 49 | 6.87 |
| $\tau$ (days) | 371 | 57 | 7 | 15916 |

The sum $S = U + P$ will be called static energy, then

$$E = S + K + L. \tag{10}$$

It is important to remark that $S$ is much bigger than the sum $K + L$. For example, for the Earth

$$\frac{S}{K+L} = \frac{150}{6} = 25. \tag{11}$$

As the hydrostatic equilibrium holds, it is easy to show that

$$S = \frac{c_p}{R} P, \tag{12}$$

where $c_p$ and $R$ are the specific heat at constant pressure and the constant of the mixture of gases that constitutes the atmosphere.

In the case of the Earth's atmosphere, the four terms $U$, $P$, $K$ and $L$ are known (Peixoto and Oort, 1992), so we know $E$. However for the atmospheres of Venus, Mars and Titan we can only compute the terms $U$ and $P$ and estimate $S$ but not $E$. We have carried out these computations by performing the numerical integration (8), using the vertical data $p(z)$ shown in (Sánchez-Lavega, 2011, page 212-227) . The results of $E$ or $S$ for each planet are shown in Table 1.

## 3 Energy fluxes absorbed and emitted by the planetary atmospheres

The values of these fluxes for Venus, Mars and Titan have been deduced from Read et al. (2016). For each planet, $F_i$ and $F_o$ represent the inflow or outflow of energy absorbed or emitted by the atmospheres. The so called 'Trenberth diagrams' are particularly suited to the identification of these fluxes. The values adopted for $F_i$ and $F_o$ are shown in Table 2.

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With the total energy values, $E$ or $S$ (in Table 1) and $F$ (Table 2), we estimate the value of residence time of energy in the atmosphere of each planet. However, as we stressed above, strictly speaking $E$ is only known in the Earth's case. In the other three cases, the ratio $(S/F)$ is a lower bound for the actual residence time.

$$\frac{S}{F} \leq \frac{E}{F} = \tau_R. \tag{13}$$

These results are shown in Table 2.

## 4 Final comments

The residence time of energy in the atmospheres of Venus, Earth, Mars and Titan have been computed. In the cases of Venus, Mars and Titan, they are mere lower bounds due to a lack of some energy data. In reference (Osácar et al., 2020) the value of $\tau_R$ for the Earth is computed by using the data provided by Hartmann (1994).

The response of an atmosphere to different perturbations, radiative, convective, etc, is not governed by a single time scale but by a range of time scales. The longest of these scales corresponds to the residence time as computed in Section 3. The residence time of energy in a planetary atmosphere characterizes the planet, and is computed in a model independent way.

Small perturbations, as for example small departures from radiative equilibrium temperatures, are damped by radiative transfer. The time scale for the dissipation of these small departures is identified and estimated in a perturbative way. See for example Wells (2012). The result is

$$\tau = \frac{c_p\,(p/g)}{\sigma T_{eff}^3}. \tag{14}$$

In this expression, $\sigma$ is the Stefan-Boltzmann constant, $p$ is pressure, $g$, the gravity acceleration and $T_{eff}$ the temperature corresponding to radiative equilibrium of the planet.

As about 80% of the radiative flux leaving an atmosphere comes from the cold top of the highest atmospheric opaque layer, we have estimated $\tau$ with the $T_{eff}$ at this height of maximum emission. In the case of the Earth's atmosphere, that level corresponds to a temperature $T_{eff} = 255\,\mathrm{K}$ and a pressure of $378\,\mathrm{mb}$. With these values, $\tau = 11.9$ days.

*Data availability.* The data of the energies used for the estimation of residence time in the Venus, Mars and Titan atmospheres were obtained from Sánchez-Lavega (2011, page 212-227) . Those for the Earth's atmosphere were extracted from Peixoto and Oort (1992).

The fluxes of energy for all the cases were deduced from Read et al. (2016).

*Author contributions.* Amalio Fernández-Pacheco conceived the idea; Carlos Osácar, Javier Pelegrina and Amalio Fernádez-Pacheco wrote the paper.

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