Brief Communication: Residence Time of Energy in the Atmospheres of Venus, Earth, Mars and Titan

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Abstract.

The ~~concept of~~ residence time of energy in a planetary atmosphere~~τ_R~~, τ, recently introduced and computed for the Earth's atmosphere (Osácar et al., 2020), is here extended to the atmospheres of Venus, Mars and Titan. ~~After a global thermal perturbation, τ_R is the time scale the atmosphere needs to return to equilibrium.The residence times of energy in the atmospheres of Venus, Earth, Mars and Titan have been computed~~τ is the timescale for the energy transport across the atmosphere. In the cases of Venus, Mars and Titan, these computations are mere lower bounds due to a lack of some energy data. If the analogy between τ and the solar Kelvin-Helmholtz scale is assumed, then τ would also be the time the atmosphere needs to return to equilibrium after a global thermal perturbation.

1 Introduction

When the inflow, F_i, of any substance into a box is equal to the outflow, F_o, then the amount of that substance in the box, M, is constant. This constitutes an equilibrium or steady state. Then, the ratio of the stock in the box to the flow rate (in or out) is called the residence time and is a ~~time scale~~ timescale for the transport of the substance in the box.

$$t = \frac{M}{F}. \tag{1}$$

In equation (1) it is assumed that the substance is conserved. A good example of this type is the parameter defined in atmospheric chemistry (Hobbs, 2000) as the average residence time of each individual gas, defined as (Eq. 1). M is the total average mass of the gas in the atmosphere and F the total average influx or outflux, which in time average for the whole atmosphere are equal.

In this work we extend the substance that flows in the box from matter to energy and the residence time is

$$\tau = \frac{E}{F}, \tag{2}$$

where E is the total energy in the box (a planetary atmosphere) and F the energy flux that enters or leaves it.

~~By~~ Here, by using (2), we estimate the average residence time of energy in several planetary atmospheres. But first it is worth recalling that several authors have previously considered the energy-residence time relation in other type of problems

(Mcilveen, 1992, 2010; Harte, 1988). ~~Harte (1988) uses this concept to estimate the anomalous temperature in urban heat islands.~~

~~The residence time of energy can also be considered in the Sun. The Kelvin-Helmholtz time scale (Kippenhahn and Weigert, 1994) is the time the Sun would need to recover from a global thermal perturbation:~~

$$\tau_{KH} = \frac{G M_\odot^2}{R_\odot L_\odot} \sim 10^7\,\mathrm{yr}.$$

~~As $L_\odot$ is the luminosity of the Sun, equation (10) is also a form of expressing the residence time of energy in this star. This is because~~

$$\tau_{KH} = \frac{G M_\odot^2}{R_\odot L_\odot} \sim \frac{|\mathrm{gravitational\,energy}|}{L_\odot}.$$

~~And as in a star like the Sun, the Virial Theorem (Kippenhahn and Weigert, 1994) links~~

$$|\mathrm{gravitational\,energy}| = 2\,|\mathrm{total\,energy}|,$$

~~hence:~~

$$\tau_{KH} \sim 2\,\frac{|\mathrm{total\,energy}|}{L_\odot}.$$

~~References Stix (2003) and Spruit (2000) are worth reading in this respect.~~

The planetary atmospheres ~~and the Sun~~ constitute steady state problems because the storage of energy in their interior is not systematically increasing or decreasing. Section 2 addresses the numerator of Eq. (2) E, while Section 3 deals with the denominator F. In Section 4 the residence time of energy is considered for the Sun. In Section 5, the radiative constant is introduced and compared with the atmospheric residence timescale. In Section 6 we comment on some final points.

2 Forms of energy in a planetary atmosphere

~~In a planar atmosphere, the~~ The most important forms of energy in ~~a planetary~~ an atmosphere are: the thermodynamic internal energy, U,

$$U = \int_0^\infty c_v\, T(z)\, \rho(z)\, \mathrm{d}z,$$

the potential energy due to the planet's gravity, P,

$$P = \int_0^\infty g\, z\, \rho(z)\, \mathrm{d}z, = \int_0^\infty p(z)\, \mathrm{d}z$$

the kinetic energy, K, and the latent energy, L, related to the phase transitions.

In a planar atmosphere, in hydrostatic equilibrium and by using the state equation for an ideal gas, these magnitudes can be written as

$$U = \int_0^\infty c_v\, T(z)\, \rho(z)\, \mathrm{d}z = \frac{c_v}{R} \int_0^\infty p(z)\, \mathrm{d}z, \tag{3}$$

$$P = \int_0^\infty g\, z\, \rho(z)\, \mathrm{d}z = \int_0^\infty p(z)\, \mathrm{d}z, \tag{4}$$

In expressions (3) and (4), c_v is the specific heat at constant volume, R is the gas constant and $\rho(z)$ and $T(z)$ are the density and temperature of the mixture of gases of the atmosphere respectively. E stands for the total energy in the atmosphere:

$$E = U + P + K + L. \tag{5}$$

The sum $S = U + P$ will be called static energy, then

$$E = S + K + L. \tag{6}$$

It is important to remark that S is much bigger than the sum $K + L$. For example, for the Earth (Peixoto and Oort, 1992)

$$\frac{S}{K+L} = \frac{150}{6} = 25. \tag{7}$$

~~As the hydrostatic equilibrium holds, it is easy to show that~~ In ideal gases, S can be expressed as

$$S = (1 + \frac{c_v}{R})P = \frac{c_p}{R} P, \tag{8}$$

where c_p ~~and R are~~ is the specific heat at constant pressure~~and the constant of the mixture of gases that constitutes the atmosphere~~.

In the case of the Earth's atmosphere, the four terms U, P, K, and L are known (Peixoto and Oort, 1992), so we know E. However for the atmospheres of Venus, Mars and Titan we can only compute the terms U and P and estimate S but not E. We have carried out these computations by performing the numerical integration (4), using the vertical data $p(z)$ shown in (Sánchez-Lavega, 2011, page 212-227). The results of E or S for each planet are shown in Table 1.

In case S is not much bigger than $K + L$, our results will be a poor lower bound. Future observations will settle these numbers.

3 Energy fluxes absorbed and emitted by the planetary atmospheres. Residence times

The values of these fluxes for ~~Venus, Mars and Titan~~ all planets have been deduced from Read et al. (2016). For each planet, F_i and F_o represent the inflow or outflow of energy absorbed or emitted by the atmospheres. The so called 'Trenberth diagrams' are particularly suited to the identification of these fluxes. ~~The values adopted~~

Table 1. Forms of energy in planetary atmospheres

	Venus	Earth	Mars	Titan
P ($\mathrm{J\,m^{-2}}$)	1.24E+11	7.00E+08	6.05E+06	2.63E+09
U ($\mathrm{J\,m^{-2}}$)	4.31E+11	1.80E+09	2.10E+07	6.79E+09
S ($\mathrm{J\,m^{-2}}$)	5.55E+11	2.50E+09	2.71E+07	9.42E+09
K ($\mathrm{J\,m^{-2}}$)	...	1.30E+06	...	...
L ($\mathrm{J\,m^{-2}}$)	...	7.00E+07	...	...
E ($\mathrm{J\,m^{-2}}$)	...	2.57E+09	...	...
C_p/R	4.47	3.5	4.37	3.58

Table 2. Fluxes of energy and residence times in planetary atmospheres

	Venus	Earth	Mars	Titan
F_i ($\mathrm{W\,m^{-2}}$)	17292	509.6	49	6.88
F_o ($\mathrm{W\,m^{-2}}$)	17292	509.6	49	6.87
τ (days)	371	57	7	15916

As an example, in the case of Venus (see Read et al. (2016, Figure 3)), the fluxes absorbed by the atmosphere (F_i) are: $135\,\mathrm{W\,m^{-2}}$ from incoming solar radiation (shortwave) absorbed in the middle atmosphere, $3\,\mathrm{W\,m^{-2}}$ also from incoming solar radiation absorbed by the lower atmosphere, and $17154\,\mathrm{W\,m^{-2}}$ of longwave flux absorbed from surface. Thus, total influx is $17292\,\mathrm{W\,m^{-2}}$.

The emitted fluxes (F_o) are $17132\,\mathrm{W\,m^{-2}}$ of longwave radiation to surface and $160\,\mathrm{W\,m^{-2}}$, also longwave radiation emitted from atmosphere to space. The total outflux value is $17292\,\mathrm{W\,m^{-2}}$. Analogous calculations for the rest of planets give the values for F_i and F_o ~~are~~ shown in Table 2.

With the total energy values, E or S (in Table 1) and F (Table 2), we estimate the value of residence time of energy in the atmosphere of each planet. However, as we stressed above, strictly speaking E is only known in the Earth's case. In the other three cases, the ratio (S/F) is a lower bound for the actual residence time.

$$\frac{S}{F} \leq \frac{E}{F} = \tau_R. \tag{9}$$

These results are shown in Table 2.

4 ~~Final comments~~Energy residence time of energy in the Sun

~~The residence time of energyin the atmospheres of Venus, Earth, Mars and Titan have been computed. In the cases of Venus, Mars and Titan, they are mere lower bounds due to a lack of some energy data. In reference (Osácar et al., 2020) the value of τ_R for the Earth is computed by using the data provided by Hartmann (1994).~~

~~The response of an atmosphere to different perturbations, radiative, convective, etc, is not governed by a single time scale but by a range of time scales. The longest of these scales corresponds to~~ The Sun also constitutes a steady state for the energy. Thermonuclear energy is produced at the center and radiant energy is emitted across the surface as a blackbody at 6000 K. Temperatures in the interior are not systematically increasing or decreasing. In Stix (2003) it is shown that in solar physics Kelvin-Helmholtz (KH (Kippenhahn and Weigert, 1994)) timescale corresponds to both the time that takes a photon from the core until it goes out to the surface and the time necessary for the star to return to equilibrium after a global perturbation:

$$\tau_{KH} = \frac{GM_\odot^2}{R_\odot L_\odot} \sim 10^7\,\mathrm{yr}. \tag{10}$$

As $L_\odot$ is the luminosity of the Sun, equation (10) is also a form of expressing the residence time ~~as computed in Section 3. The residence time of energy in a planetary atmosphere characterizes the planet, and is computed in a model independent way.~~

~~Small perturbations, as for example small departures from radiative equilibriumtemperatures, are damped by radiative transfer . The time scale for the dissipation of these small departures is identified and estimated in a perturbative way. See for example Wells (2012b). The result is~~

$$\tau = \frac{c_p\,(p/g)}{\sigma T_{eff}^3}.$$

of energy in our star. This is because

$$\tau_{KH} = \frac{GM_\odot^2}{R_\odot L_\odot} \sim \frac{|\mathrm{gravitational\,energy}|}{L_\odot}. \tag{11}$$

And as in a star like the Sun, the Virial Theorem (Kippenhahn and Weigert, 1994) links

$$|\mathrm{gravitational\,energy}| = 2\,|\mathrm{total\,energy}|, \tag{12}$$

hence:

$$\tau_{KH} \sim 2\frac{|\mathrm{total\,energy}|}{L_\odot}. \tag{13}$$

Furthermore, Spruit (2000) shows that KH is the longest timescale for any solar perturbations.

5 Radiative relaxation timescale

The most simple models that can be devised for the structure of the atmosphere are the static radiative ones. But if energy transfer with the surface is taken into account, the structure produced under radiative equilibrium cannot be maintained. Convection develops spontaneously and neutralizes the stratification introduced by radiative transfer. The new radiative–convective equilibrium produces two layers. Below a certain height, the thermal structure is controlled by convective overturning and constitutes the troposphere. In this layer, the vertical profile of temperature is adiabatic. In the layer above troposphere, which constitutes the stratosphere, the thermal structure remains close to radiative equilibrium, because radiative transfer stabilizes the stratification.

In this context, the concept of radiative relaxation timescale is introduced. It is defined as:

$$\tau_R = \frac{c_p p/g}{4\sigma T_{\text{eff}}^3}. \tag{14}$$

See, for example Wells (2012a). In this expression, ~~σ is the Stefan-Boltzmann constant,~~ p is pressure, c_p is the specific heat at constant pressure, g ~~, the gravity acceleration and T_{eff} the temperature corresponding to radiative equilibrium~~ gravity, σ is the Stefan-Boltzmann constant and T_{eff} is the blackbody effective temperature of the planet.

~~As about 80% of the radiative flux leaving an atmosphere comes from the cold top of the highest atmospheric opaque layer, we have estimated τ with the T_{eff} at this height of maximum emission~~In general, if a state of equilibrium is perturbed, the atmosphere uses the most efficient procedure at hand to neutralize it. The procedure can be convective, advective or radiative. τ_R is the time it would take to relax the perturbation by radiating the energy excess in the infrared.

Defining T_r and p_r as the temperature and pressure of the level from which infrared photons are emitted to the space, and T_s and p_s as the temperature and pressure at surface, if we assume that in the troposphere the temperature profile is given by a dry adiabat, then we have

$$p_r = p_s \left(\frac{T_r}{T_s}\right)^{\frac{C_p}{R}}. \tag{15}$$

Assuming this hypothesis (Pierrehumbert, 2010), for the Earth, we obtain $p_r = 670\,\text{mb}$. Earth's actual p_r is somewhat lower than this estimate because tropospheric temperature decays less strongly with height than the dry adiabat. The result for this value of p_r is about 22 days.

Due to the factor p in the numerator of Eq. 14, the value of τ_R decreases rapidly with height. So, radiation is not an efficient procedure to neutralize perturbations in the low troposphere. There τ_R is very long.

A clear example is in Venus, where τ_R varies from 116 days at 40 Km (lower cloud deck) to 0.5 hr at 100 Km (Sánchez-Lavega et al., 2017) .

If the analogy between the atmospheric τ and the solar scale KH is assumed, τ is the time necessary to return to equilibrium after a global perturbation, whilst τ_R is the timescale corresponding to small perturbations.

6 Final comments

In our opinion, the concept of "Residence time of energy in a planetary atmosphere" is completely original. This residence time has been computed for the atmospheres of Venus, Earth, Mars and Titan. In the ~~case of the Earth's atmosphere, that level corresponds to a temperature $T_{eff} = 255\,\text{K}$ and a pressure of $378\,\text{mb}$. With these values , $\tau = 11.9$ days~~cases of Venus, Mars and Titan, they are mere lower bounds due to a lack of data about kinetic and latent energies. In reference (Osácar et al., 2020) the value of τ for the Earth was computed by using the data provided by Hartmann (1994).

The analogy between τ and the KH solar timescale seems likely, although this does not constitute a proof.

The usual radiative timescales presented by other authors (e.g. Houghton (2002), Wells (2012a), Sánchez-Lavega (2011)) are calculated assuming that a small perturbation is produced in the temperature, i.e. it is a perturbative calculus. Furthermore, it depends on the values of pressure and temperature where it is computed. On the contrary, in the computation of the residence time of energy τ in planetary atmospheres only global averaged planetary parameters are used.

Data availability. The data of the energies used for the estimation of residence time in the Venus, Mars and Titan atmospheres were computed with p and T from Sánchez-Lavega (2011, page 212-227). Those for the Earth's atmosphere were extracted from Peixoto and Oort (1992). The fluxes of energy for all the cases were deduced from Read et al. (2016).

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