Nonlin. Processes Geophys. Discuss.,
https://doi.org/10.5194/npg-2020-48-SC1, 2020

Nonlinear Processes in Geophysics
Discussions
Open Access

Interactive comment on "A Waveform Skewness Index for Measuring Time Series Nonlinearity and its Applications to the ENSO-Indian Monsoon Relationship" *by* Justin Schulte et al.

Paul Pukite

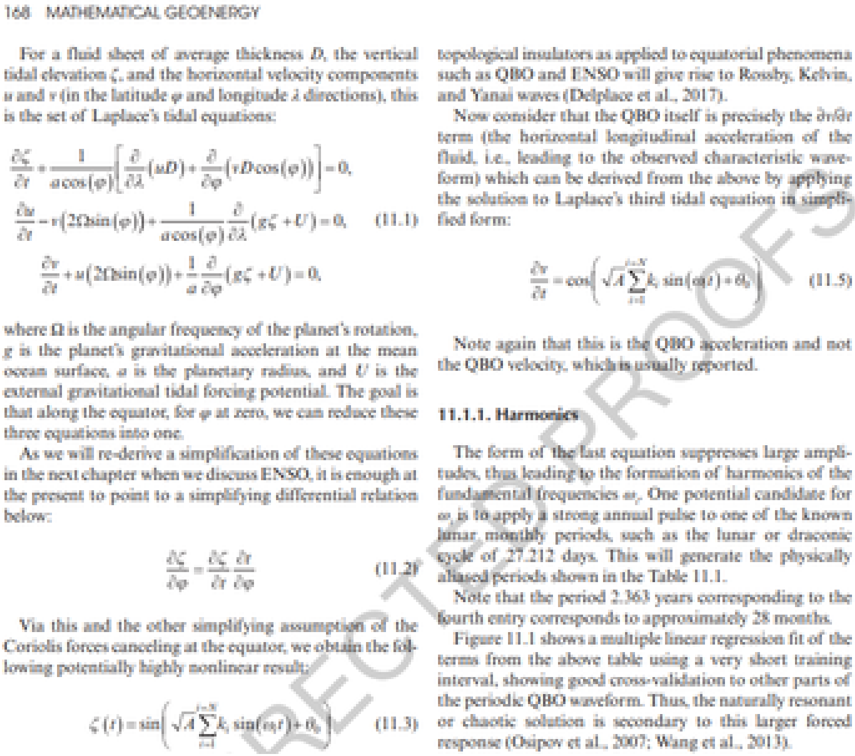
puk@umn.edu

Received and published: 19 December 2020

FYI, showed that skewness results when solving the wave equations along the equator, e.g. for ENSO. This generates a nonlinear amplification that can differ with respect to the sign of the amplitude excursion.

Pukite, P., Coyne, D., & Challou, D. (2019). Mathematical Geoenergy: Discovery, Depletion, and Renewal (Vol. 241). John Wiley & Sons. https://agupubs.onlinelibrary.wiley.com/doi/10.1002/9781119434351.ch12

Interactive comment on Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2020-48, 2020.



For a fluid sheet of average thickness D, the vertical tidal elevation ζ, and the horizontal velocity components u and v (in the latitude φ and longitude λ directions), this is the set of Laplace's tidal equations:

$$\frac{\partial \zeta}{\partial t} + \frac{1}{a\cos(\varphi)}\left[\frac{\partial}{\partial \lambda}(uD) + \frac{\partial}{\partial \varphi}(vD\cos(\varphi))\right] = 0,$$
$$\frac{\partial u}{\partial t} - v(2\Omega\sin(\varphi)) + \frac{1}{a\cos(\varphi)}\frac{\partial}{\partial \lambda}(g\zeta + U) = 0, \quad (11.1)$$
$$\frac{\partial v}{\partial t} + u(2\Omega\sin(\varphi)) + \frac{1}{a}\frac{\partial}{\partial \varphi}(g\zeta + U) = 0,$$

where Ω is the angular frequency of the planet's rotation, g is the planet's gravitational acceleration at the mean ocean surface, a is the planetary radius, and U is the external gravitational tidal forcing potential. The goal is that along the equator, for φ at zero, we can reduce these three equations into one.

As we will re-derive a simplification of these equations in the next chapter when we discuss ENSO, it is enough at the present to point to a simplifying differential relation below:

$$\frac{\partial \zeta}{\partial \varphi} = \frac{\partial \zeta}{\partial t}\frac{\partial t}{\partial \varphi} \quad (11.2)$$

Via this and the other simplifying assumption of the Coriolis forces canceling at the equator, we obtain the following potentially highly nonlinear result:

$$\zeta(t) = \sin\left(\sqrt{A}\sum_{i=1}^{i=N} k_i \sin(\omega_i t) + \theta_0\right) \quad (11.3)$$

topological insulators as applied to equatorial phenomena such as QBO and ENSO will give rise to Rossby, Kelvin, and Yanai waves (Delplace et al., 2017).

Now consider that the QBO itself is precisely the $\partial v/\partial t$ term (the horizontal longitudinal acceleration of the fluid, i.e., leading to the observed characteristic waveform) which can be derived from the above by applying the solution to Laplace's third tidal equation in simplified form:

$$\frac{\partial v}{\partial t} = \cos\left(\sqrt{A}\sum_{i=1}^{i=N} k_i \sin(\omega_i t) + \theta_0\right) \quad (11.5)$$

Note again that this is the QBO acceleration and not the QBO velocity, which is usually reported.

11.1.1. Harmonics

The form of the last equation suppresses large amplitudes, thus leading to the formation of harmonics of the fundamental frequencies ω_i. One potential candidate for ω_i is to apply a strong annual pulse to one of the known lunar monthly periods, such as the lunar or draconic cycle of 27.212 days. This will generate the physically aliased periods shown in the Table 11.1.

Note that the period 2.363 years corresponding to the fourth entry corresponds to approximately 28 months.

Figure 11.1 shows a multiple linear regression fit of the terms from the above table using a very short training interval, showing good cross-validation to other parts of the periodic QBO waveform. Thus, the naturally resonant or chaotic solution is secondary to this larger forced response (Osipov et al., 2007; Wang et al., 2013).

Fig. 1. Solution approach

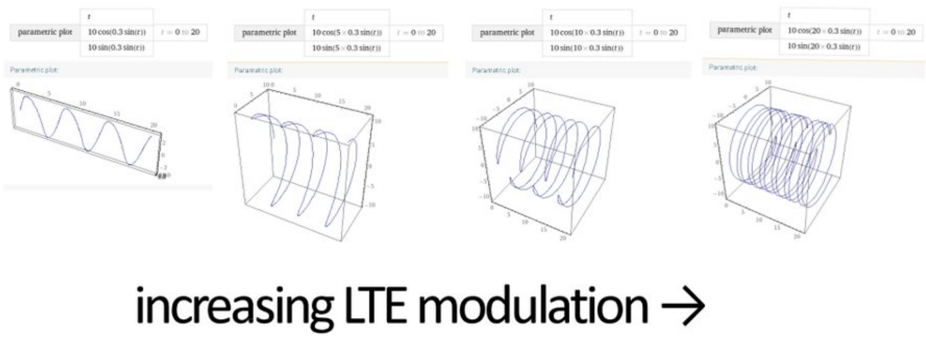


Fig. 2. Skewness results from asymmetric folding of the nonlinear mapping