

1 Magnetospheric chaos and dynamical complexity response during storm time disturbance

2 Irewola Aaron Oludehinwa¹, Olasunkanmi Isaac Olusola¹, Olawale Segun Bolaji^{1,2}, Olumide
3 Olayinka Odeyemi¹, Abdullahi Ndzi Njah¹

4 ¹Department of Physics, University of Lagos, Nigeria

5 ²Department of Physics, University of Tasmania, Australia

6 Abstract

7 In this study, we examine the magnetospheric chaos and dynamical complexity response to the
8 disturbance storm time (D_{st}) and solar wind electric field (VB_s) during different categories of
9 geomagnetic storm (minor, moderate and major geomagnetic storm). The time series data of the
10 D_{st} and VB_s are analyzed for the period of nine years using nonlinear dynamics tools (Maximal
11 Lyapunov Exponent, MLE, Approximate Entropy, ApEn and Delay Vector Variance, DVV). We
12 found a significant trend between each nonlinear parameter and the categories of geomagnetic
13 storm. The MLE and ApEn values of the D_{st} indicate that chaotic and dynamical complexity
14 responses are high during minor geomagnetic storms, reduce at moderate geomagnetic storms and
15 decline further during major geomagnetic storms. However, the MLE and ApEn values obtained
16 from VB_s indicate that chaotic and dynamical complexity responses are high with no significant
17 difference between the periods that are associated with minor, moderate and major geomagnetic
18 storms. The test for nonlinearity in the D_{st} time series during major geomagnetic storm reveals the
19 strongest nonlinearity features. Based on these findings, the dynamical features obtained in the
20 VB_s as input and D_{st} as output of the magnetospheric system suggest that the magnetospheric
21 dynamics is nonlinear and the solar wind dynamics is consistently stochastic in nature.

22 **Keywords:** D_{st} signals, Solar wind electric field (VB_s) signals, Geomagnetic storm, Chaotic
23 behaviour, Dynamical complexity, Nonlinearity.

24

25 1.0 Introduction

26 The response of chaos and dynamical complexity behaviour with respect to magnetospheric
27 dynamics varies (Tsurutani et al., 1990). This is due to changes in the interplanetary electric fields
28 imposed on the magnetopause and those penetrating the inner magnetosphere and sustaining
29 convection thereby initiating geomagnetic storm (Dungey, 1961; Pavlos et al., 1992). A prolonged
30 southward turning of interplanetary magnetic field (IMF, B_z), which indicates that solar wind-
31 magnetosphere coupling is in-progress was confirmed on many occasions for which such
32 geomagnetic storm was driven by Corotating Interaction Regions (CIRs), or by the sheath
33 preceding an interplanetary coronal mass ejection (ICME) or by a combination of the sheath and
34 an ICME magnetic cloud (Gonzalez and Tsurutani, 1987; Tsurutani and Gonzalez, 1987;
35 Tsurutani et al., 1988; Cowley, 1995; Tsutomu, 2002; Yurchyshyn et al., 2004; Kozyra et al., 2006;
36 Echer et al., 2008; Meng et al., 2019; Tsurutani et al., 2020). The sporadic magnetic reconnection
37 between the southward component of the Alfvén waves and the earth's magnetopause leads to
38 isolated substorms/convection events such as the high intensity long-duration continuous AE
39 activity (HILDCAA) which are shown to last from days to weeks (Akasofu, 1964; Tsurutani and
40 Gonzalez, 1987; Hajra et al., 2013; Liou et al., 2013; Mendes et al., 2017; Hajra and Tsurutani,
41 2018; Tsurutani and Hajra, 2021). Notably, the introduction of Disturbance Storm Time (D_{st})
42 index (Sugiura, 1964; Sugiura and Kamei, 1991) unveiled the quantitative measure of the total
43 energy of the ring current particles. Therefore, the D_{st} index remains one of the most popular
44 global indicators that can precisely reveal the severity of a geomagnetic storm (Dessler and Parker,
45 1959).

46 The D_{st} fluctuations exhibit different signatures for different categories of geomagnetic storm.
47 Ordinarily, one can easily anticipate that fluctuations in a D_{st} signal appear chaotic and complex.

48 These may arise from the changes in the interplanetary electric fields driven by the solar wind-
49 magnetospheric coupling processes. At different categories of geomagnetic storm, fluctuations in
50 the D_{st} signals differ (Oludehinwa et al., 2018). One obvious reason is that as the intensity of the
51 geomagnetic storm increases, the fluctuation behaviour in the D_{st} signal becomes more complex
52 and nonlinear in nature. It has been established that the electrodynamic response of the
53 magnetosphere to solar wind drivers are non-autonomous in nature (Price and Prichard, 1993;
54 Price et al., 1994; Johnson and Wings, 2005). Therefore, the chaotic analysis of the
55 magnetospheric time series must be related to the concept of input-output dynamical process
56 (Russell et al., 1974; Burton et al.,1975; Gonzalez et al., 1989; Gonzalez et al., 1994).
57 Consequently, it is necessary to examine the chaotic behaviour of the solar wind electric field
58 (VB_s) as input signals and the magnetospheric activity index (D_{st}) as output during different
59 categories of geomagnetic storms.

60 Several works have been presented on the chaotic and dynamical complexity behaviour of the
61 magnetospheric dynamics based on autonomous concept, i.e using the time series data of
62 magnetospheric activity alone such as auroral electrojet (AE), Amplitude Lower (AL) and D_{st}
63 index (Vassiliadis et al.,1990; Baker and Klimas, 1990; Vassiliadis et al.,1991; Shan et al., 1991;
64 Pavlos et al., 1994; Klimas et al., 1996; Valdivia et al., 2005; Mendes et al., 2017; Consolini,
65 2018). They found evidence of low-dimensional chaos in the magnetospheric dynamics. For
66 instance, the report by Vassiliadis et al. (1991) shows that the computation of Lyapunov exponent
67 for AL index time series gives a positive value of Lyapunov exponent indicating the presence of
68 chaos in the magnetospheric dynamics. Unnikrishnan, (2008) studied the deterministic chaotic
69 behaviour in the magnetospheric dynamics under various physical conditions using AE index time
70 series and found that the seasonal mean value of Lyapunov exponent in winter season during quiet

71 periods ($0.7 \pm 0.11 \text{ min}^{-1}$) is higher than that of the stormy periods ($0.36 \pm 0.09 \text{ min}^{-1}$).
72 Balasis et al. (2006) examined the magnetospheric dynamics in the D_{st} index time series from
73 pre-magnetic storm to magnetic storm period using fractal dynamics. They found that the transition
74 from anti-persistent to persistent behaviour indicates that the occurrence of an intense geomagnetic
75 storm is imminent. Balasis et al. (2009) further reveal the dynamical complexity behaviour in the
76 magnetospheric dynamics using various entropy measures. They reported a significant decrease in
77 dynamical complexity and an accession of persistency in the D_{st} time series as the magnetic storm
78 approaches. Recently, Oludehinwa et al. (2018) examined the nonlinearity effects in D_{st} signals
79 during minor, moderate and major geomagnetic storm using recurrence **plots** and recurrence
80 quantification **analyses**. They found that the dynamics of the D_{st} signal is stochastic during minor
81 geomagnetic storm periods and deterministic as the geomagnetic storm increases.

82 Also, studies describing the solar wind and magnetosphere as a non-autonomous system have been
83 extensively investigated. Price et al. (1994) examine the nonlinear input-output analysis of AL
84 index and different combinations of interplanetary magnetic field (IMF) with solar wind
85 parameters as input **functions**. They found that only a few of the input combinations show any
86 evidence whatsoever for nonlinear coupling between the input and output for the interval
87 investigated. Pavlos et al. (1999) presented further evidence of magnetospheric chaos. They
88 compared the observational behaviour of the magnetospheric system with the results obtained by
89 analyzing different types of stochastic and deterministic input-output systems and asserted that a
90 low dimensional chaos is evident in magnetospheric dynamics. Devi et al. (2013) **studied the**
91 **magnetospheric dynamics using AL index and the southward component of IMF(B_z)**. They
92 observed that the magnetosphere and turbulent solar wind have values corresponding to nonlinear
93 dynamical system with chaotic behaviour. The modeling and forecasting approach have been

94 applied to magnetospheric time series using nonlinear models (Valdivia et al., 1996; Vassiliadis et
95 al., 1999; Vassiliadis, 2006; Balikhin et al., 2010). These efforts have improved our understanding
96 **that the concept** of nonlinear dynamics can reveal some hidden dynamical information in the
97 observational time series. In addition to these nonlinear effects in D_{st} signals, a measure of the
98 exponential divergence and convergence within the trajectories of a phase space known as
99 Maximal Lyapunov Exponent (MLE), which has the potential to depict the chaotic behavior in the
100 D_{st} and VB_s time series during a minor, moderate and major geomagnetic storm have not been
101 investigated. In addition, to the best of our knowledge, computation of Approximate Entropy
102 (ApEn) that depicts the dynamical complexity behaviour during different categories of
103 geomagnetic storm has not been reported in the literature. The test for nonlinearity through delay
104 vector variance (DVV) analysis **that reveals the nonlinearity features** in D_{st} and VB_s time series
105 during minor, moderate and major geomagnetic storms is not well known. It is worth to note that
106 understanding the dynamical characteristics in the D_{st} and VB_s signals at different categories of
107 geomagnetic storms will provide useful diagnostic information to different conditions of space
108 weather phenomenon. Consequently, this study attempts to carry out comprehensive numerical
109 **analyses** to unfold the chaotic and dynamical complexity behaviour in the D_{st} and VB_s signals
110 during minor, moderate and major geomagnetic storm. In section 2, our methods of data
111 acquisition are described. Also, the nonlinear analysis that we employed in this investigation are
112 detailed. In section 3, we unveiled our results and engage the discussion of results in **section 4**.

113 **2.0 Description of the Data and Nonlinear Dynamics**

114 The D_{st} index is derived by measurements from ground-based magnetic stations at low-latitudes
115 observatories around the world and depicts mainly the variation of the ring current, as well as the
116 Chapman-Ferraro Magnetopause currents, and tail currents to a lesser extent (Sugiura, 1964;

117 [Feldstein et al., 2005](#); [Feldstein et al., 2006](#); Love and Gannon, 2009). Due to its global nature, D_{st}
118 time series provides a measure of how intense a geomagnetic storm was (Dessel and Parker, 1959).
119 In this study, we considered D_{st} data for the period of nine years from January to December
120 between 2008 and 2016 which were downloaded from the World Data Centre for Geomagnetism,
121 Kyoto, Japan (<http://wdc.kugi-kyoto-u.ac.jp/Dstae/index.html>). We use the classification of
122 geomagnetic storms as proposed by Gonzalez et al. (1994) such that D_{st} index value in the ranges
123 $0 \leq Dst \leq -50nT$, $-50nT \leq Dst \leq -100nT$, $-100nT \leq Dst \leq -250nT$ are classified as
124 minor, moderate and major geomagnetic storms respectively and each **time series** is being
125 classified based on its minimum D_{st} value. The solar wind electric field (VB_s) data are archived
126 from the National Aeronautics and Space Administration, Space Physics Facility
127 (<http://omniweb.gsfc.nasa.gov>). **The sampling time of D_{st} and VB_s time series data was 1-hour.** It
128 is well known that the dynamics of the solar wind contribute to the driving of the magnetosphere
129 (Burton et al. 1975). Furthermore, we took the solar wind electric field (VB_s) as the input signal
130 (Price and Prichard, 1993; Price et al., 1994). The VB_s was categorized according to the periods
131 of minor, moderate and major geomagnetic storm. Then, the D_{st} and VB_s time series were
132 subjected to a variety of nonlinear analytical tools explained as follow:

133 **2.1 Phase Space Reconstruction and Observational time series**

134 An observational time series can be defined as a sequence of scalar measurements of some
135 quantity, which is a function of the current state of the system taken at multiples of a fixed sampling
136 time. In nonlinear dynamics, the first step in analyzing an observational time series data is to
137 reconstruct an appropriate state space of the system. [Takens \(1981\)](#) and [Mane \(1981\)](#) stated that
138 one time series or a few simultaneous time series are converted to a sequence of vectors. This

139 reconstructed phase space has all the dynamical characteristic of the real phase space provided the
140 time delay and embedding dimension are properly specified.

$$141 \quad X(t) = [x(t), x(t + \tau), x(t + 2\tau), \dots, x(t + (m - 1)\tau)]^T \quad (1)$$

142 Where $X(t)$ is the reconstructed phase space, $x(t)$ is the original time series data, τ is the time
143 delay and m is the embedding dimension. An appropriate choice of τ and m are needed for the
144 reconstruction of phase space which is determined by average mutual information and false nearest
145 neighbor, respectively.

146 **2.2 Average Mutual Information (AMI)**

147 The method of Average Mutual Information (AMI) is one of the nonlinear techniques used to
148 determine the optimal time delay (τ) required for phase space reconstruction in observational time
149 series. The time delay mutual information was proposed by Fraser and Swinney, (1986) instead of
150 an autocorrelation function. This method takes into account nonlinear correlations within the time
151 series data. It measures how much information can be predicted about one time series point, given
152 full information about the other. For instance, the mutual information between x_i and $x_{(i+\tau)}$
153 quantifies the information in state $x_{(i+\tau)}$ under the assumption that information at the state x_i is
154 known. The AMI for a time series, $x(t_i)$, $i = 1, 2, \dots, N$ is calculated as:

$$155 \quad I(T) = \sum_{x(t_i), x(t_i+T)} P(x(t_i), x(t_i + T)) \times \log_2 \left[\frac{P(x(t_i), x(t_i+T))}{P(x(t_i)) P(x(t_i+T))} \right] \quad (2)$$

156 where $x(t_i)$ is the i th element of the time series, $T = k\Delta t$ ($k = 1, 2, \dots, k_{max}$), $P(x(t_i))$ is the
157 probability density at $x(t_i)$, $P(x(t_i), x(t_i + T))$ is the joint probability density at the pair
158 $x(t_i), x(t_i + T)$. The time delay (τ) of the first minimum of AMI is chosen as optimal time delay
159 (Fraser and Swinney, 1986). Therefore, the AMI was applied to the D_{st} and VB_s time series and

160 the plot of AMI **versus** time delay is shown in Figure (3). We notice that the AMI showed the first
 161 local minimum at roughly $\tau = 15hr$. Furthermore, the values of τ near this value of $\sim 15hr$
 162 maintain constancy for both VB_S and D_{st} . In the analysis $\tau = 15hr$ was used as the optimal time
 163 delay for the computation of maximal Lyapunov exponent.

164 **2.3 False Nearest Neighbor (FNN)**

165 In determining the optimal choice of embedding dimension(m), the false nearest neighbor method
 166 was used in the study. The method was suggested by Kennel et al. (1992). The concept is based
 167 on how the number of neighbors of a point along a signal trajectory changes with increasing
 168 embedding dimension. With increasing embedding dimension, the false neighbor will no longer
 169 be neighbors, therefore by examining how the number of neighbors changes as a function of
 170 dimension, an appropriate embedding dimension can be determined. For instance, suppose we
 171 have a one-dimensional time series. We can construct a time series $y(t)$ of D -dimensional points
 172 from the original one-dimensional time series $x(t)$ as follows:

$$173 \quad y(t) = (x(t), x(t + \tau), \dots, x(t + (D - 1)\tau)) \quad (3)$$

174 **where** τ and D are time delay and embedding dimension. Using the formular from Kennel et al.
 175 (1992); Wallot and Monster, (2018), **if** we have a D -dimensional phase space and denote the r th
 176 nearest neighbor of a coordinate vector $y(t)$ by $y^{(r)}(t)$, then the square of the Euclidean distance
 177 between $y(t)$ and the r th nearest neighbor is:

$$178 \quad R_D^2(t, r) = \sum_{k=0}^{D-1} [x(t + k\tau) - x^{(r)}(t + k\tau)]^2 \quad (4)$$

179 Now applying the logic outlined above, we can go from a D -dimensional phase space to $(D + 1)$
180 dimensional phase space by time-delay embedding, adding a new coordinate to $y(t)$, and ask what
181 is the squared distance between $y(t)$ and the same r th nearest neighbor:

$$182 \quad R_{D+1}^2(t, r) = R_D^2(t, r) + [x(t + D\tau) - x^{(r)}(t + D\tau)]^2 \quad (5)$$

183 As explained above, if the one-dimensional time series is already properly embedded in D
184 dimensions, then the distance R between $y(t)$ and the r th nearest neighbor should not **change**
185 **appreciably** by some distance criterion R_{tol} (i.e. $R < R_{tol}$). Moreover, the distance of the nearest
186 neighbor when embedded into the next higher dimension relative to the size of the attractor should
187 be less than some criterion A_{tol} (i.e. $R_{D+1} < A_{tol}$). Doing this for the nearest neighbor of each
188 coordinate will result on many false nearest neighbors when embedding is insufficient or in few
189 (or no) false neighbors when embedding is sufficient. In the analysis, the FNN was applied to the
190 D_{st} and VB_s time series to detect the optimal value of embedding dimension (m). Figure (4) shows
191 a sample plot of the percentage of false nearest neighbor against embedding dimension in one of
192 the months under investigation (other months show similar results, thus for brevity we depict only
193 one of the results). We notice that the false nearest neighbor attains its minimum value at $m \geq 5$
194 indicating that embedding dimension (m) from $m \geq 5$ are optimal values. Therefore, $m = 5$ was
195 used for the computation of maximal Lyapunov exponent.

196 **2.4 Maximal Lyapunov Exponent (MLE)**

197 The Maximal Lyapunov Exponent (MLE) is one of the most popular nonlinear dynamics tool used
198 for detecting chaotic behaviour in a time series data. It describes how small changes in the state of
199 a system grow at an exponential rate and eventually dominate the behaviour. An important
200 indication of chaotic behavior of a dissipative deterministic system is the existence of a positive

201 Lyapunov Exponent. A positive MLE signifies divergence of trajectories in one direction or
 202 expansion of an initial volume in this direction. On the other hand, a negative MLE exponent
 203 implies convergence of trajectories or contraction of volume along another direction. The
 204 algorithm proposed by Wolf et al. (1985) for estimating MLE is employed to compute the chaotic
 205 behavior of the D_{st} and VB_S time series at minor, moderate and major geomagnetic storm. Other
 206 methods of determining MLE includes Rosenstein's method, Kantz's method and so on. In this
 207 study, the MLE at minor, moderate and major geomagnetic storms periods was computed with
 208 $m = 5$ and $\tau = 15hr$ as shown in figures (5 & 6-bar plots) for D_{st} and VB_S . The calculation of
 209 MLE is explained as follows: given a sequence of vector $x(t)$, an m -dimensional phase space is
 210 formed from the observational time series through embedding theorem as

$$211 \quad \{x(t), x(t + \tau), \dots, x(t + (m - 1)\tau)\} \quad (6)$$

212 **where** m and τ are as defined earlier, after reconstructing the observational time series, the
 213 algorithm locates the nearest neighbor (in Euclidean sense) to the initial point $\{x(t_0), \dots, x(t_0 +$
 214 $(m - 1)\tau\}$ and denote the distance between these two points $L(t_0)$. At a later point t_1 , the initial
 215 length will have evolved to length $L'(t_1)$. Then the MLE is calculated as:

$$216 \quad \lambda = \frac{1}{t_M - t_0} \sum_{k=1}^M \log_2 \frac{L'(t_k)}{L(t_{k-1})} \quad (7)$$

217 M is the total number of replacement steps. We look for a new data point that satisfies two criteria
 218 reasonably well: its separation, $L(t_1)$, from the evolved fiducial point is small. If an adequate
 219 replacement point cannot be found, we retain the points that were being used. This procedure is
 220 repeated until the fiducial trajectory has traversed the entire data.

221

222 2.5 Approximate Entropy (ApEn)

223 Approximate Entropy (ApEn) is one of the nonlinear dynamics tools that measure the dynamical
224 complexity in observational time series. The concept was proposed by Pincus, (1991) which
225 provides a generalized measure of regularity, such that it accounts for the logarithm likelihood in
226 the observational time series. For instance, a dataset of length, N , that repeat itself for m points
227 within a boundary will again repeat itself for $m + 1$ points. Because of its computational
228 advantage, ApEn has been widely used in many areas of disciplines to study dynamical complexity
229 (Pincus and Kalman (2004); Pincus and Goldberger (1994); McKinley et al. (2011); Kannathan et
230 al. (2005); Balasis et al. (2009); Shujuan and Weidong, (2010); Moore and Marchant (2017)). The
231 ApEn is computed using the formula below:

$$232 \quad ApEn(m, r, N) = \frac{1}{N-m+1} \sum_{i=1}^{N-m+1} \log C_i^m(r) - \frac{1}{N-m} \sum_{i=1}^{N-m} \log C_i^m(r) \quad (8)$$

233 where $C_i^m(r) = \frac{1}{N-m+1} \sum_{j=1}^{N-m+1} \Theta(r - \|x_i - x_j\|)$ is the correlation integral, m is the embedding
234 dimension and r is the tolerance. To compute the ApEn for the D_{st} and VB_s time series classified
235 as minor, moderate and major geomagnetic storm from 2008 to 2016, we choose ($m = 3, \tau =$
236 $1hr$). We refer the works of Pincus, (1991); Kannathal et al. (2005); and Balasis et al. (2009) to
237 interested readers where all the computational steps regarding ApEn were explained in details.
238 Figures (5 & 6) depict the stem plot of ApEn for D_{st} and (VB_s) from 2008 to 2016.

239 2.6 Delay Vector Variance (DVV) analysis

240 The Delay Vector Variance (DVV) is a unified approach in analyzing and testing for nonlinearity
241 in a time series (Gautama et al., 2004; Mandic et al., 2007). The basic idea of the DVV is that, if
242 two delay vectors of a predictable signal are close to each other in terms of the Euclidean distance,

243 they should have similar target. For instance, when a time delay (τ) is embedded into a time series
 244 $x(k)$, $k = 1, 2, \dots, N$, then a reconstructed phase space vector is formed which represents a set of
 245 delay vectors (DVs) of a given dimension.

$$246 \quad X(k) = [X_{k-m\tau}, \dots, X_{k-\tau}]^T \quad (9)$$

247 Reconstructing the phase space, a set (λ_k) is generated by grouping those DVs that are with a
 248 certain Euclidean distance to DVs ($X(k)$). For a given embedding dimension (m), a measure of
 249 unpredictability σ^{*2} is computed over all pairwise Euclidean distance between delay vector as

$$250 \quad d(i, j) = \|x(i) - x(j)\| \quad (i \neq j) \quad (10)$$

251 Then, sets $\lambda_k(r_d)$ are generated as the sets which consist of all delay vectors that lie closer to $x(k)$
 252 than a certain distance r_d .

$$253 \quad \lambda_k(r_d) = \{x(i) \| x(k) - x(i)\| \leq r_d\} \quad (11)$$

254 For every set $\lambda_k(r_d)$, the variance of the corresponding target $\sigma^{*2}(r_d)$ is

$$255 \quad \sigma^{*2}(r_d) = \frac{\frac{1}{N} \sum_{k=1}^N \sigma_k^2(r_d)}{\sigma_k} \quad (12)$$

256 where $\sigma^{*2}(r_d)$ is target variance against the standardized distance indicating that Euclidean
 257 distance will be varied in a manner standardized with respect to the distribution of pairwise
 258 distance between DVs. Iterative Amplitude Adjusted Fourier Transform (IAAFT) method is used
 259 to generate the surrogate time series (Kugiumtzis, 1999). If the surrogate time series yields DV
 260 plots similar to the original time series and the scattered plot coincides with the bisector line, then
 261 the original time series can be regarded as linear (Theiler et al., 1992; Gautama et al., 2004; Imitaz,
 262 2010; Jaksic et al., 2016). On the other hand, if the surrogate time series yields DV plot that is not

263 similar to that of the original time series, then the deviation from the bisector lines indicates
264 nonlinearity. The deviation from the bisector lines grows as a result of the degree of nonlinearity
265 in the observational time series.

$$266 \quad t^{D_{VV}} = \sqrt{\langle (\sigma^{*2}(r_d) - \frac{\sum_{i=1}^N \sigma_{s,i}^{*2}}{N_s}) \rangle} \quad (13)$$

267 where $\sigma_{s,i}^{*2}(r_d)$ is the target variance at the span r_d for the i^{th} surrogate. To carry out the test for
268 nonlinearity in the D_{st} signals, $m = 3$ and $n_d = 3$, the number of reference DVs=200, and number
269 of surrogate, $N_s = 25$ was used in all the analysis. Then we examined the nonlinearity response at
270 minor, moderate and major geomagnetic storm.

271 3.0 Results

272 In this study, D_{st} and VB_s time series from January to December were analyzed for the period of
273 nine years (2008 to 2016) to examine the chaotic and dynamical complexity response in the
274 magnetospheric dynamics during minor, moderate and major geomagnetic storms. Figures (1) &
275 (2), display the samples of fluctuation signatures of D_{st} and VB_s signals classified as (a): minor,
276 (b): moderate and (c): major geomagnetic storms. The plot of Average Mutual information against
277 time delay (τ) shown in Figure (3) depicts that the first local minimum of the AMI function was
278 found to be roughly at $\tau = 15$ hr. Furthermore, we notice that the values of τ near this value of
279 (~ 15 hr) maintain constancy for both VB_s and D_{st} . Also, in Figure (4), we display the plot of the
280 percentage of false nearest neighbor against embedding dimension (m). It is obvious that a
281 decrease in false nearest neighbor when increasing the embedding dimension drop steeply to zero
282 at the optimal dimension ($m = 5$), thereafter the false neighbors stabilizes at that $m = 5$ for VB_s
283 and D_{st} . Therefore, $m = 5$ and $\tau = 15$ hr was used for the computation of MLE at different
284 categories of geomagnetic storm, while $m = 3$ and $\tau = 1$ hr are applied for the computation of
285 ApEn values.

286 The results of MLE (bar plot) and ApEn (stem plot) for D_{st} at minor, moderate and major
287 geomagnetic storms are shown in Figure 5. During minor geomagnetic storms, we notice that the
288 value of MLE ranges between 0.07 and 0.14 for most of the months classified as minor
289 geomagnetic storm. Similarly, the ApEn (stem plot) ranges between 0.59 and 0.83. It is obvious
290 that strong chaotic behaviour with high dynamical complexity are associated with minor
291 geomagnetic storms. During moderate geomagnetic storms, (see b part of Figure 5), we observe a
292 reduction in MLE values (0.04 – 0.07) compared to minor geomagnetic storm periods. Within
293 the observed values of MLE during moderate geomagnetic storms, we found a slight rise of MLE

294 in the following months (Mar 2008), (Apr 2011), (Jan 2012, Feb 2012, Apr 2012), (Jul 2015, Aug
295 2015, Sept 2015, Oct 2015, Nov 2015) and (Nov 2016). Also, the ApEn revealed a reduction in
296 values between 0.44 and 0.57 during moderate geomagnetic storms. The lowest values of ApEn
297 were noticed in the following months: May 2010, Mar 2011, and Jan 2016. During major
298 geomagnetic storms as shown in Figure 5, the minimum and maximum value of MLE is
299 respectively 0.03 and 0.04 implying a very strong reduction of chaotic behaviour compared with
300 minor and moderate geomagnetic storms. The lowest values of MLE were found in the months of
301 Jul 2012, Jun 2013 and Mar 2015. Interestingly, further reduction in ApEn value (0.29 – 0.40)
302 was as well noticed during this period. Thus, during major geomagnetic storms, chaotic behaviour
303 and dynamical complexity **subside** significantly.

304 We display in Figure 6, the results of MLE and ApEn computation for the VB_s which has been
305 categorized according to the periods of minor, moderate and major geomagnetic storms. The
306 values of MLE (bar plot) were between 0.06 and 0.20 for VB_s . The result obtained indicate strong
307 chaotic behaviour with no significant difference in chaoticity during minor, moderate and major
308 geomagnetic storm. Similarly, the results obtained from computation of ApEn (stem plot) for VB_s
309 depict a minimum value of 0.60 and peak value of 0.87 as shown in Figure 6. The ApEn values of
310 VB_s indicates high dynamical complexity response with no significant difference during the
311 periods of the three categories of geomagnetic storm investigated.

312 The test for nonlinearity in the D_{st} signals during minor, moderate and major geomagnetic storms
313 was analyzed through the DVV analysis. Shown in Figure 9 is the DVV plot and DVV scatter plot
314 during minor geomagnetic storm for January 2009 and January 2014. We found that the DVV
315 plots during minor geomagnetic storms reveals a slight separation between the original and
316 surrogate data. Also, the DVV scatter plots shows a slight deviation from the bisector line between

317 the original and surrogate data which implies nonlinearity. Also, during moderate geomagnetic
318 storms, we notice that the DVV plot depicts a wide separation between the original and the
319 surrogate data. Also, a large deviation from the bisector line between the original and the surrogate
320 data was also noticed in the DVV scatter plot as shown in Figure (8) thus indicating nonlinearity.
321 In Figure (9), we display samples of DVV plot and DVV scatter plot during major geomagnetic
322 storm for Oct 2011 and Dec 2015. The original and the surrogate data showed a very large
323 separation in the DVV plot during major geomagnetic storm. While the DVV scatter plot depict
324 the greatest deviation from the bisector line between the original and the surrogate data which is
325 also an indication of nonlinearity. The DVV analysis of the VB_s time series during minor, moderate
326 and major geomagnetic storms shown in Figures (10-12) revealed a separation between the original
327 and surrogate data with no significant difference between the periods of minor, moderate and major
328 geomagnetic storm.

329 **4.0 Discussion of Results**

330 **4.1 The chaotic and dynamical complexity response in D_{st} at minor, moderate and major** 331 **geomagnetic storms**

332 Our result shows that the values of MLE for D_{st} during minor geomagnetic storm are higher,
333 indicating significant chaotic response during minor geomagnetic stormy periods (bar plot, Figure
334 5). This increase in chaotic behaviour for D_{st} signals during minor geomagnetic storms may be as
335 a result of asymmetry features in the longitudinal distribution of solar source region for the
336 Corotating Interaction Regions (CIR) signatures responsible for the development of geomagnetic
337 storms (Turner et al. 2006; Kozyra et al. 2006). CIR generated magnetic storms are generally
338 weaker than ICME/MC generated storms (Gonzalez et al., 1994; Tsurutani et al., 1995; Feldstein
339 et al., 2006; Richardson and Cane, 2011). Therefore, we suspect that the increase in chaotic

340 behaviour during minor geomagnetic storms is strongly associated with the asymmetry features in
341 the longitudinal distribution of solar source region for the Corotating Interaction Regions (CIR)
342 signatures. For most of these periods of moderate geomagnetic storms, the values of MLE
343 decreases compared to minor geomagnetic storms. This revealed that as geomagnetic stormy
344 events build up, the level of unpredictability and sensitive dependence on initial condition (chaos)
345 begin to decrease (Lorentz, 1963; Stogaz, 1994). The chaotic behaviour during major geomagnetic
346 storms decreases significantly compared with moderate geomagnetic storms. The reduction in
347 chaotic response during moderate and its further declines at major geomagnetic storms may be
348 attributed to the disturbance in the interplanetary medium driven by sheath preceding an
349 interplanetary coronal mass ejection (ICME) or combination of the sheath and an ICME magnetic
350 cloud (Echer et al., 2008; Tsurutani et al., 2003; Meng et al., 2019). Notably, the dynamics of the
351 solar wind-magnetospheric interaction are dissipative chaotic in nature (Pavlos, 2012); and, the
352 electrodynamics of the magnetosphere due to the flux of interplanetary electric fields had a
353 significant impact on the state of the chaotic signatures. For instance, the observation of strong
354 chaotic behaviour during minor geomagnetic storms suggests that the dynamics was characterized
355 by a weak magnetospheric disturbance. While the reduction in chaotic behaviour at moderate and
356 major geomagnetic storm period reveals the dynamical features with regards to when a strong
357 magnetospheric disturbance begins to emerge. Therefore, our observation of chaotic signatures at
358 different categories of geomagnetic storm has potential capacity to give useful diagnostic
359 information about monitoring space weather events. It is important to note that the features of D_{St}
360 chaotic behaviour at different categories of geomagnetic storm has not been reported in the
361 literature. For example, previous study of Balasis et al. (2009, 2011) investigate dynamical
362 complexity behaviour using different entropy measures and revealed the existence of low

363 dynamical complexity in the magnetospheric dynamics and attributed it to ongoing large
364 magnetospheric disturbance (major geomagnetic storm). The work of Balasis et al. (2009, 2011)
365 where certain dynamical characteristic evolved in the D_{st} signal was revealed was limited to one
366 year data (2001). It is worthy to note that the year 2001, according to sunspot variations is a period
367 of high solar activity during solar cycle 23. It is characterized by numerous and strong solar
368 eruptions that were followed by significant magnetic storm activities. This confirms that on most
369 of the days in year 2001, the geomagnetic activity is strongly associated with major geomagnetic
370 storms. The confirmation of low dynamical complexity response in the D_{st} signal during major
371 geomagnetic storms agree with our current study. However, the idea of comparing the dynamical
372 complexity behaviour at different categories of geomagnetic **storms** and reveal its chaotic features
373 was not reported. This is the major reason why our present investigation is crucial to the
374 understanding of the level of chaos and dynamical complexity involved during different categories
375 of geomagnetic storms. As an extension to the single-year investigation done by Balasis et al.
376 (2009, 2011) during a major geomagnetic storm, we further investigated nine years data of D_{st}
377 that covered minor, moderate and major geomagnetic **storms** (see Figure 5, stem plots) and
378 unveiled their dynamical complexity behaviour. During major geomagnetic stormy periods, we
379 found that the ApEn values decrease significantly, indicating reduction in the dynamical
380 complexity behaviour. This is in agreement with the low dynamical complexity reported by Balasis
381 et al. (2009, 2011) during a major geomagnetic period. Finally, based on the method of DVV
382 analysis, we found that test of nonlinearity in the D_{st} time series during major geomagnetic **storms**
383 reveals the strongest nonlinearity features.

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386 **4.2 The chaotic and dynamical complexity behaviour in the VB_s as input signals.**

387 The results of the MLE values for VB_s revealed a strong chaotic behaviour during the three
388 categories of geomagnetic storms. Comparing these MLE values during minor to those observed
389 during moderate and major geomagnetic storms, the result obtained did not indicate any significant
390 difference in chaoticity (bar plots, Figure 6). Also, the ApEn values of VB_s during the periods
391 associated with minor, moderate and major geomagnetic storms revealed high dynamical
392 complexity behaviour with no significant difference between the three categories of geomagnetic
393 storms investigated. These observation of high chaotic and dynamical complexity behaviour in the
394 dynamics of VB_s may be due to interplanetary discontinuities caused by the abrupt changes in the
395 interplanetary magnetic field direction and plasma parameters (Tsurutani et al., 2010). Also, the
396 indication of high chaotic and dynamical complexity behaviour in VB_s signifies that the solar wind
397 electric field is stochastic in nature. The DVV analysis for VB_s revealed nonlinearity features with
398 no significant difference between the minor, moderate and major geomagnetic storms. It is worth
399 mentioning that the dynamical complexity behaviour for VB_s is different from what was observed
400 for D_{st} time series data. For instance, our results for D_{st} times series revealed that the chaotic and
401 dynamical complexity behaviour of the magnetospheric dynamics are high during minor
402 geomagnetic storms, reduce at moderate geomagnetic storms and further decline during major
403 geomagnetic storms. While the VB_s signal revealed a high chaotic and dynamical complexity
404 behaviour at all the categories of geomagnetic storm period. Therefore, these dynamical features
405 obtained in the VB_s as input signal and the D_{st} as the output in describing the magnetosphere as a
406 non-autonomous system further support the finding of Donner et al. (2019) that found increased
407 or not changed in dynamical complexity behaviour for VB_s and low dynamical complexity

408 behaviour during storm using recurrence method. Thus, suggesting that the magnetospheric
409 dynamics is nonlinear and the solar wind dynamics is consistently stochastic in nature.

410 **5.0 Conclusions**

411 This work has examined the magnetospheric chaos and dynamical complexity behaviour in the
412 disturbance storm time (D_{st}) and solar wind electric field (VB_s) as input during different categories
413 of geomagnetic storms. The chaotic and dynamical complexity behaviour at minor, moderate and
414 major geomagnetic storms for solar wind electric field (VB_s) as input and D_{st} as output of the
415 magnetospheric system were analyzed for the period of 9 years using nonlinear dynamics tools.
416 Our analysis has shown a noticeable trend of these nonlinear parameters (MLE and ApEn) and the
417 categories of geomagnetic storm (minor, moderate and major). The MLE and ApEn values of the
418 D_{st} have indicated that the chaotic and dynamical complexity behaviour are high during minor
419 geomagnetic storms, low during moderate geomagnetic storms and further reduced during major
420 geomagnetic storms. The values of MLE and ApEn obtained from VB_s indicate that chaotic and
421 dynamical complexity are high with no significant difference during the periods of minor,
422 moderate and major geomagnetic storms. Finally, the test for nonlinearity in the D_{st} time series
423 during major geomagnetic storms reveals the strongest nonlinearity features. Based on these
424 findings, the dynamical features obtained in the VB_s as input and D_{st} as output of the
425 magnetospheric system suggest that the magnetospheric dynamics is nonlinear and the solar wind
426 dynamics is consistently stochastic in nature.

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435 **Declaration of Interest statement**

436 The authors declare that there is no conflicts of interest.

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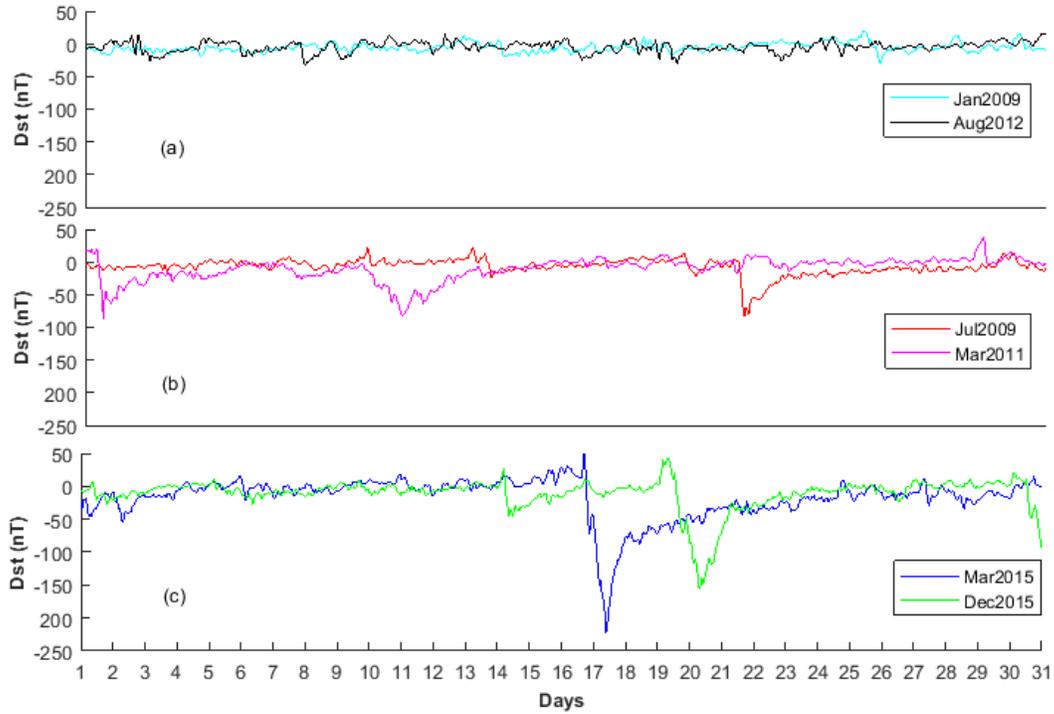
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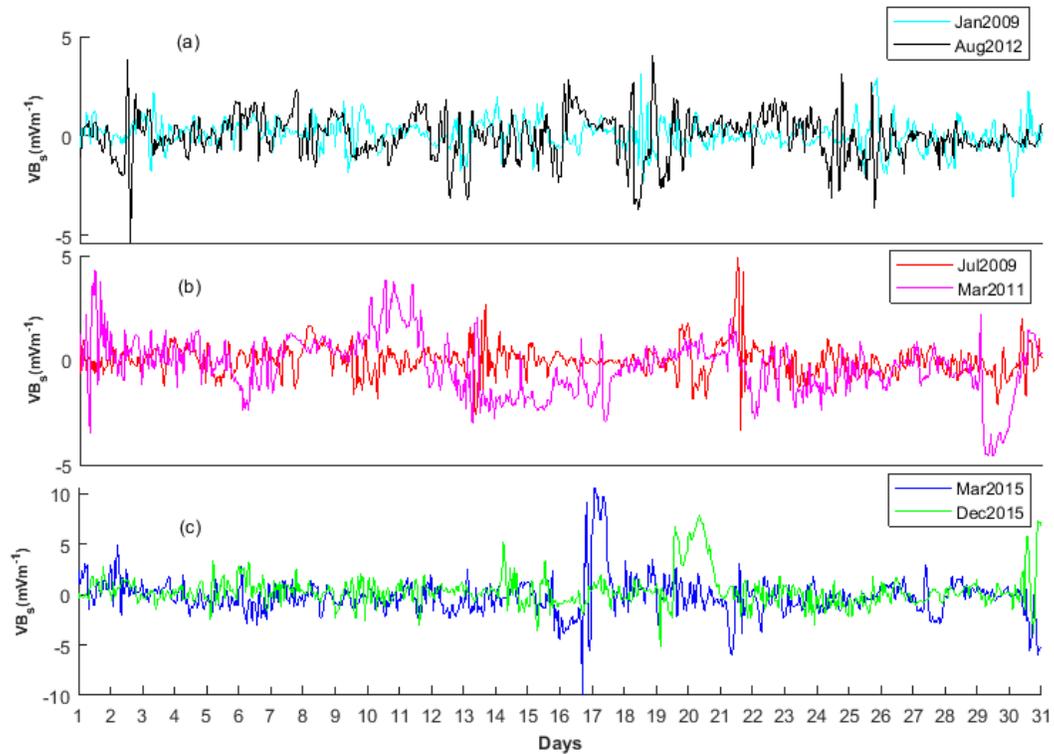
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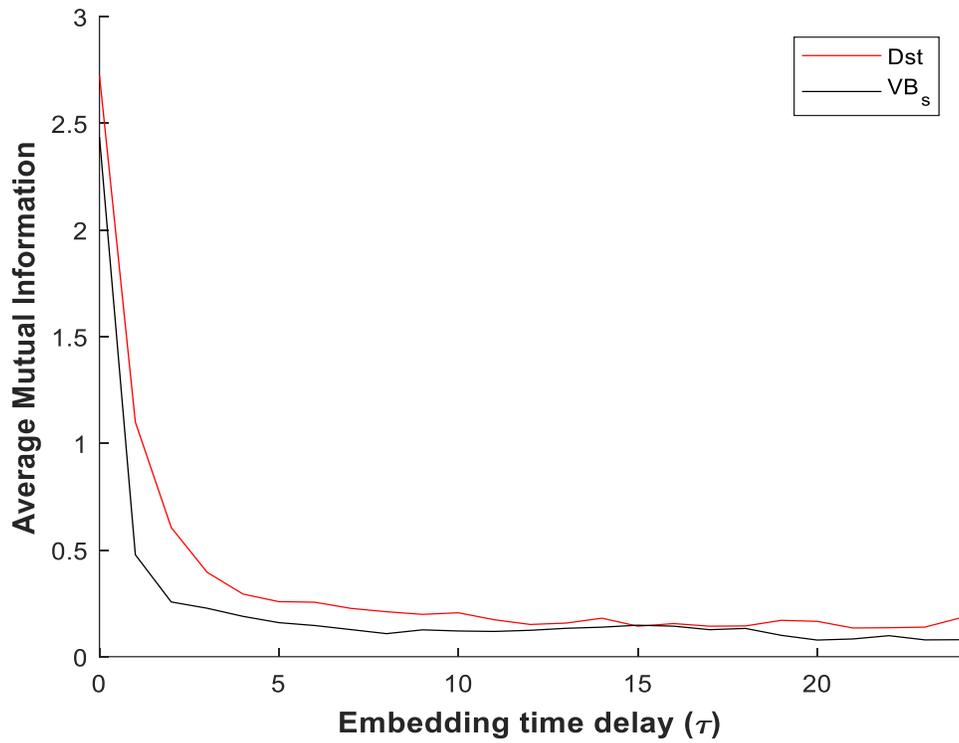
671 Figure 1: Samples of D_{st} signals classified as (a) Minor, (b) Moderate and (c) Major geomagnetic
 672 storm



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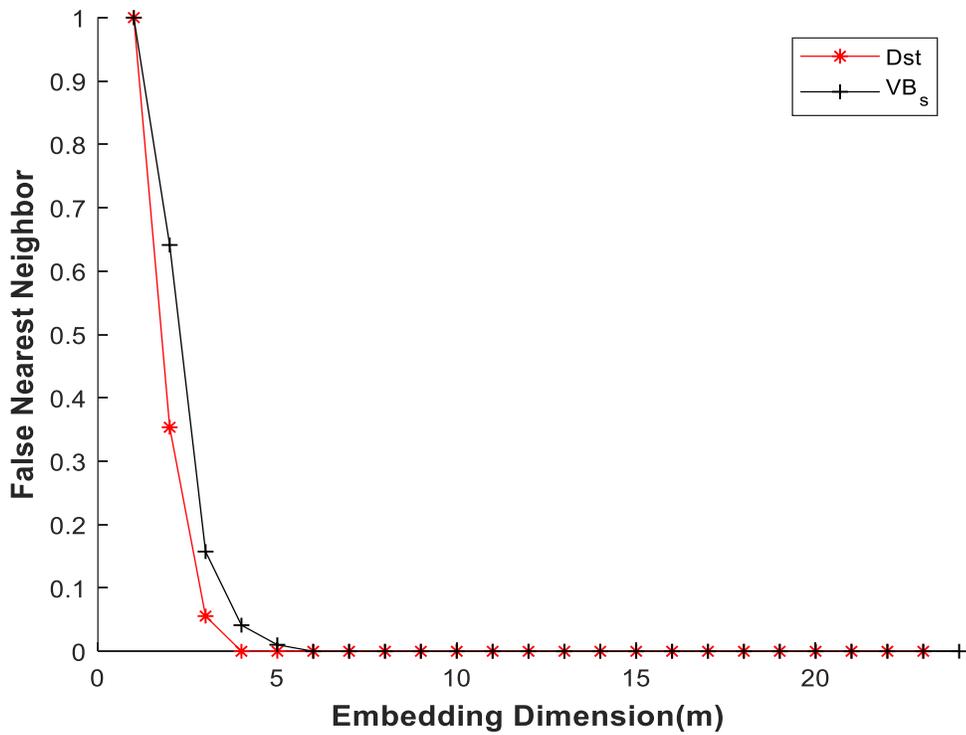
674 Figure 2: Samples of solar wind electric fields (VB_s) during (a) Minor, (b) Moderate and (c) Major
 675 geomagnetic storm.

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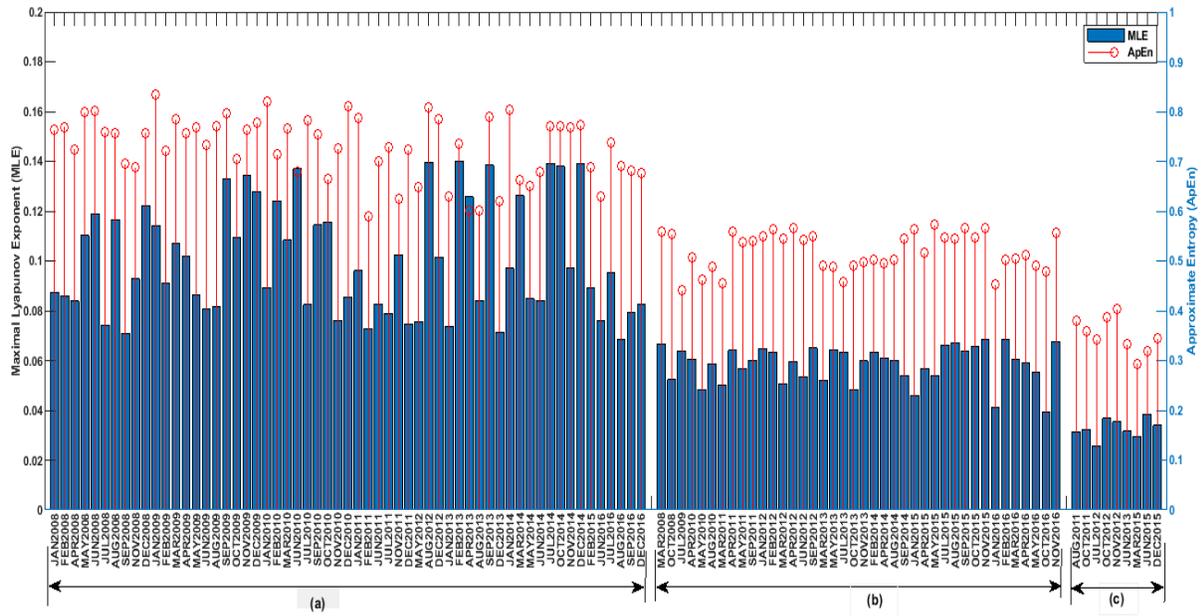
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678 Figure 3: The plot Average Mutual Information against embedding time delay (τ)



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680 Figure 4: The plot of Percentage of False Nearest Neighbors against embedding dimension (m)



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682 Figure 5: The MLE (bar plot) and ApEn (stem plot) of D_{st} at: (a) Minor, (b) Moderate and (c)
 683 Major geomagnetic storm.

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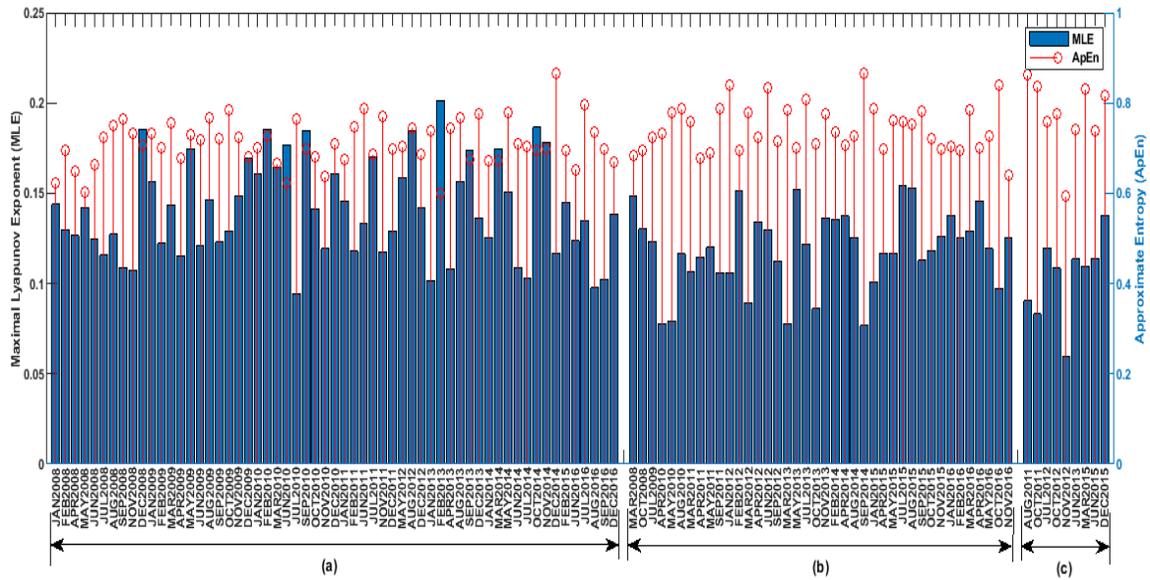
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692 Figure 6: The MLE (bar plot) and ApEn (stem plot) of solar wind electric field (VB_S) during: (a)
 693 Minor, (b) Moderate and (c) Major geomagnetic storm.

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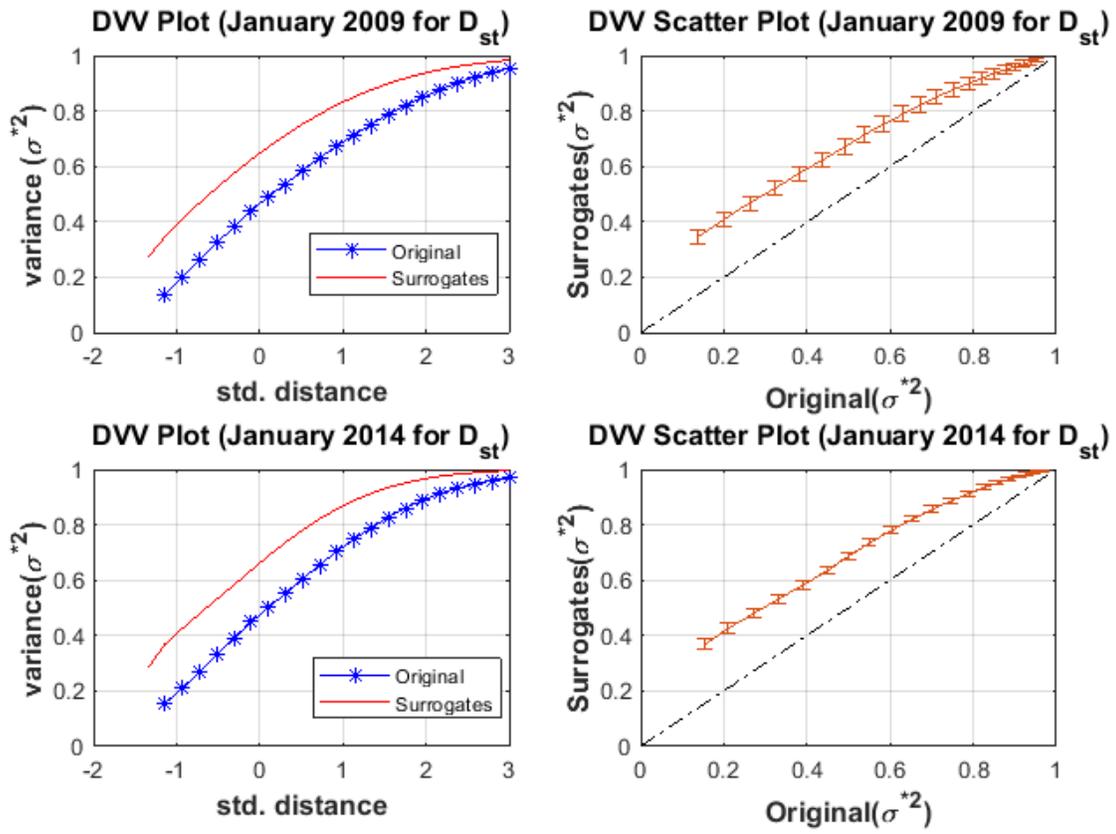
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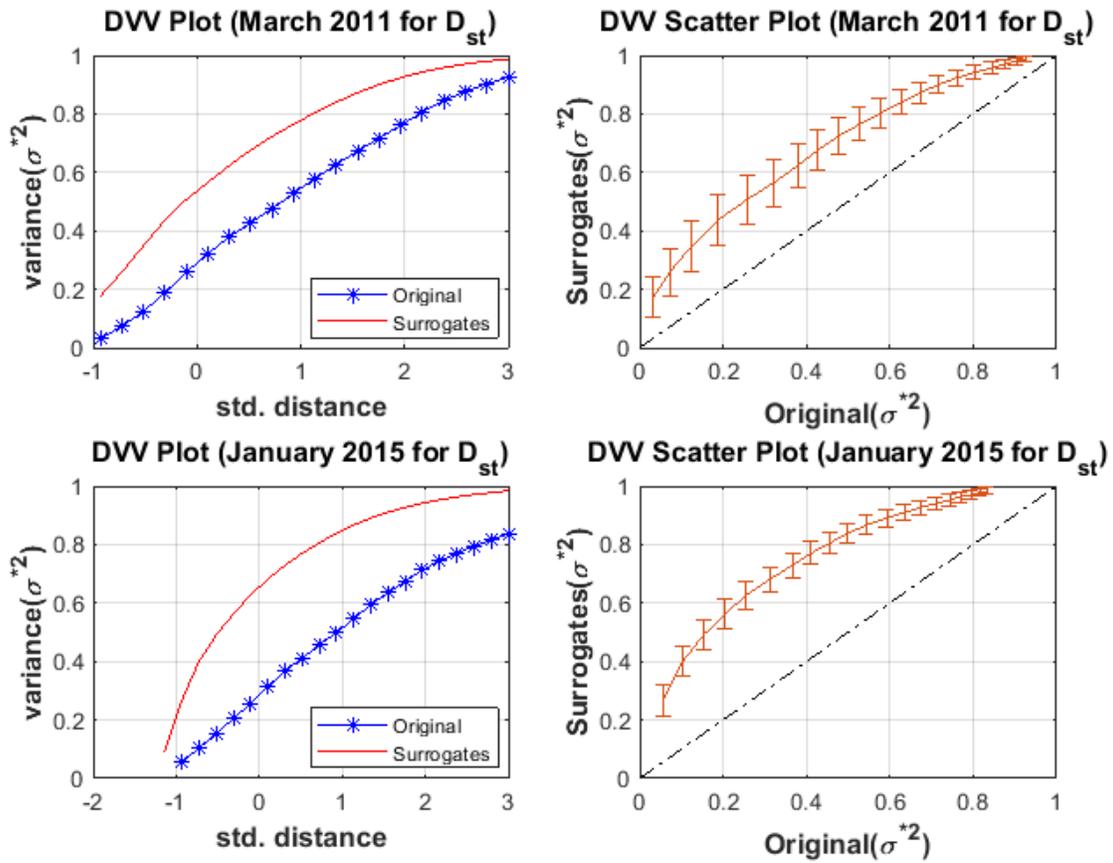
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708 Figure 7: The DVV plot and Scatter plot for D_{st} during minor geomagnetic storm for January
 709 2009 and January 2014.

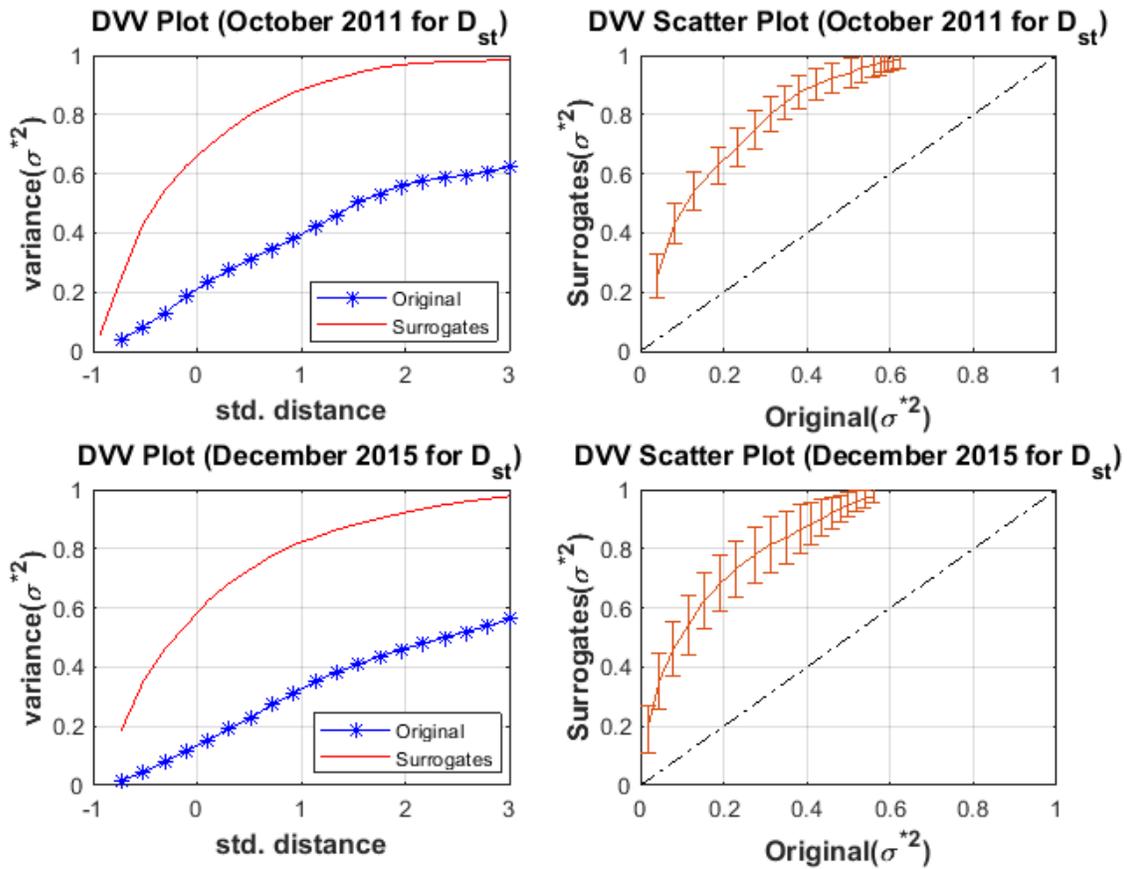


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712 Figure 8: The DVV plot and Scatter plot for D_{st} during moderate geomagnetic storm for March
 713 2011 and January 2015.

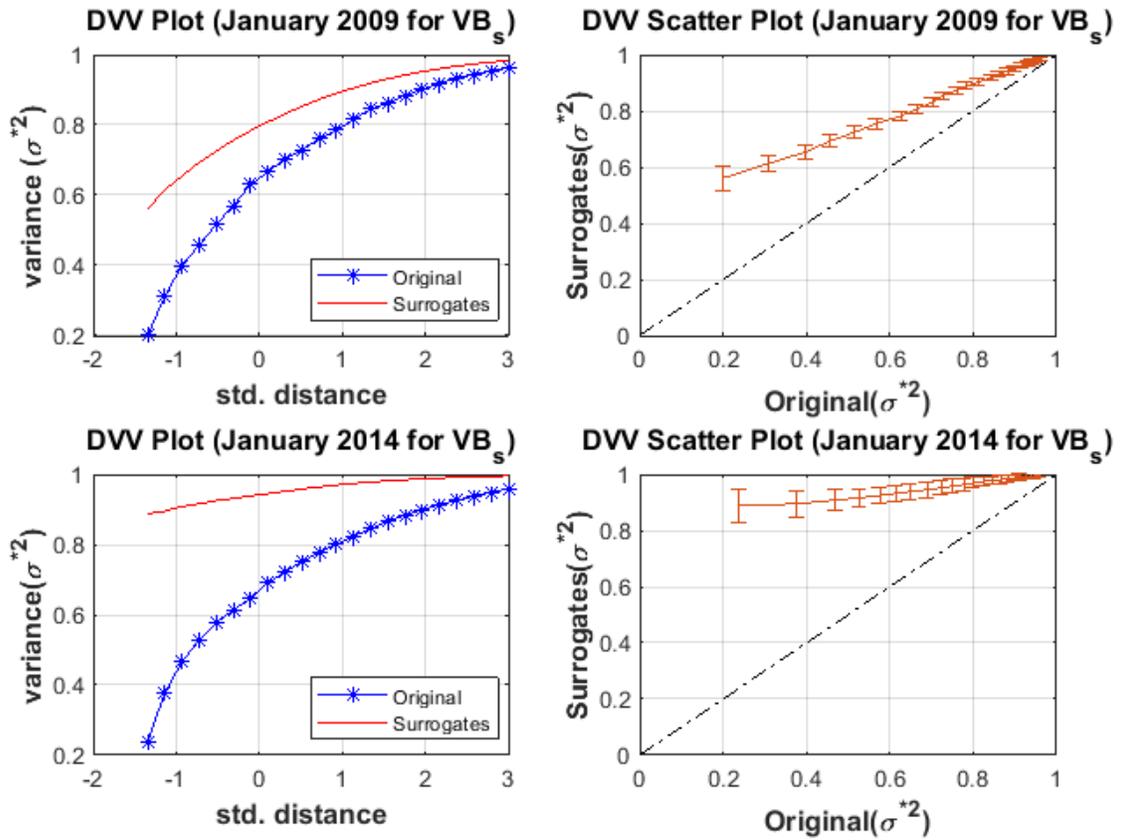
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716 Figure 9: The DVV plot and Scatter plot for D_{st} during major geomagnetic storm for October
 717 2011 and December 2015.

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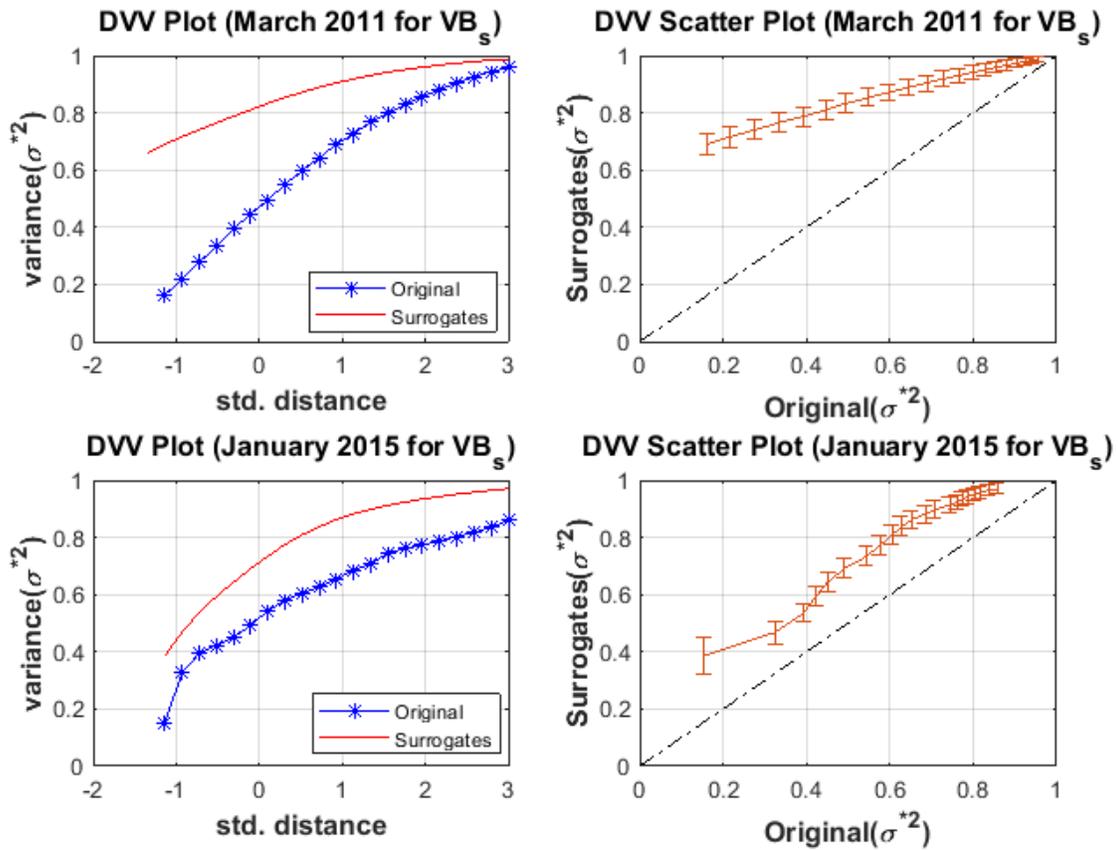


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720 Figure 10: The DVV plot and Scatter plot for VB_s during minor geomagnetic storm for January
 721 2009 and January 2014.

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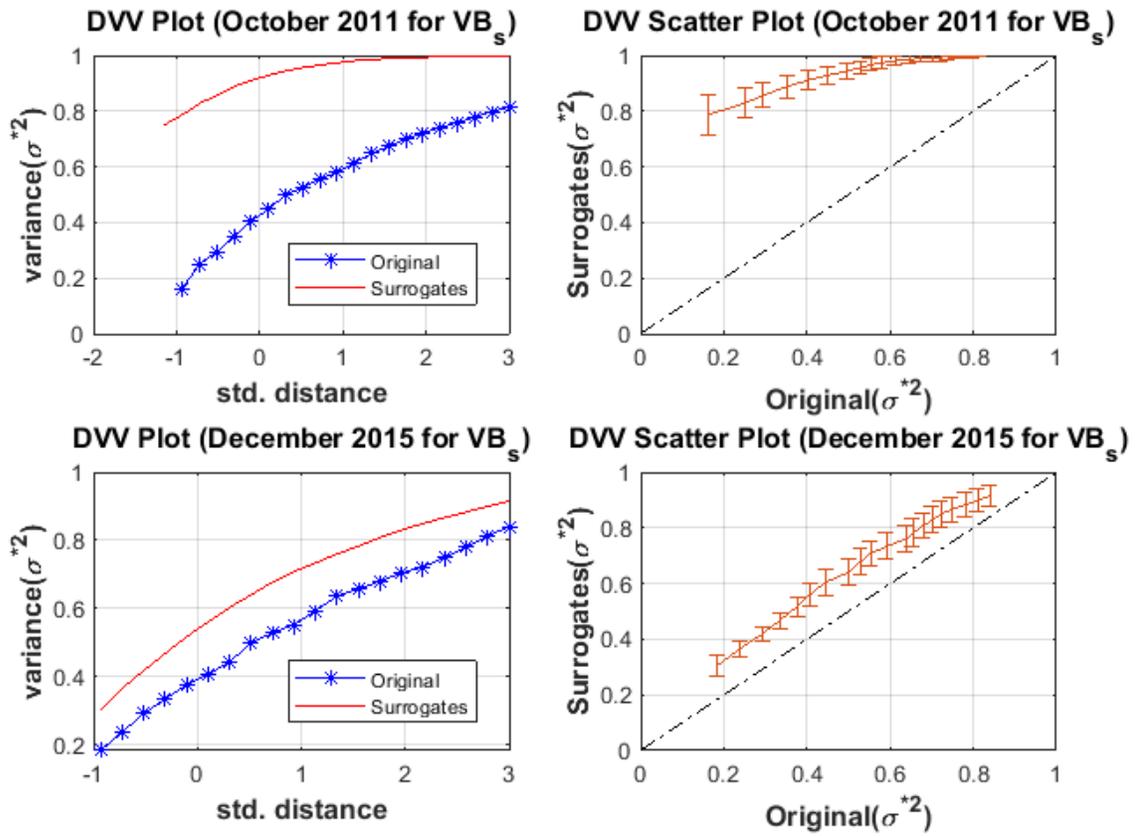


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725 Figure 11: The DVV plot and Scatter plot for VB_s during moderate geomagnetic storm for March
 726 2011 and January 2015.

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730 Figure 12: The DVV plot and Scatter plot for VB_s during major geomagnetic storm for October
 731 2011 and December 2015.

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