

With this review response letter, we will submit the revisions to the manuscript (MS# npg-2020-41) entitled “Recurrence analysis of extreme event like data” for publication in *Nonlinear Processes in Geophysics*. We would like to thank the referee for careful review and constructive suggestions which, we believe, have refined our manuscript. The responses to all the comments and suggestions put forth by the reviewers are enlisted below.

Comment 1: In the shifting part of the cost function, replacing the linear function with a non linear function is very interesting. I would like to ask about the choice of the sigmoid? Have you tested other non linear functions and compared them (Tanh, RELU)?

Reply 1: Thanks for the suggestion. The implementation of the logistic function is easy and we can use the idea of the *delay*, which has physical relevance in climate phenomena. The logistic function satisfies the triangular inequality criteria, proposed by Victor and Purpura.

The tanh function is a rescaled logistic function and therefore, comparing their results would be trivial.

The ReLU provides a linear cost function shifting, very similar to the original one of ED. The difference is an additional shift of the onset of the linear increase of the costs and, by this, introducing a zero cost for small shifts.

Nevertheless, we implemented the ReLU as a cost function according to our problem as follows:

$$f(t) = \begin{cases} 0 & \Delta t \leq \tau \\ \Lambda_0 \Delta t & \Delta t > \tau \end{cases} \quad (1)$$

where Λ_0 is computed using Eq. (2b) in the original manuscript and Δt is the gap between two events. The cost will increase with the increment of the distance between events (Fig. 1). In our work, we assign the cost for deletion/insertion to be 1. So, the cost shifting will be optimized by ReLU function below the maximum cost 1.

Please note that strictly speaking, the ReLU function implemented in this way does not become a metric because two events with a small difference can be shifted without any cost.

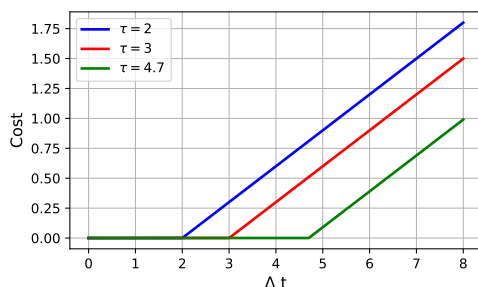


Figure 1: ReLU function for different τ value

We used the ReLU cost function for the recurrence analysis of the Poisson process, repeating Poisson process, and the periodical Poisson process. The recurrence plots for all the above cases are shown in the Fig. 2.

In case of ReLU cost function, the DET value is smaller (Fig. 2) than DET of the logistic function (Fig. 8a, 9a, and 10a in the original manuscript). We also compute the DET for different

value of τ (Fig. 3). From Fig. 3 we see that it is bit difficult to find a range of τ values where the variation in DET is minimum.

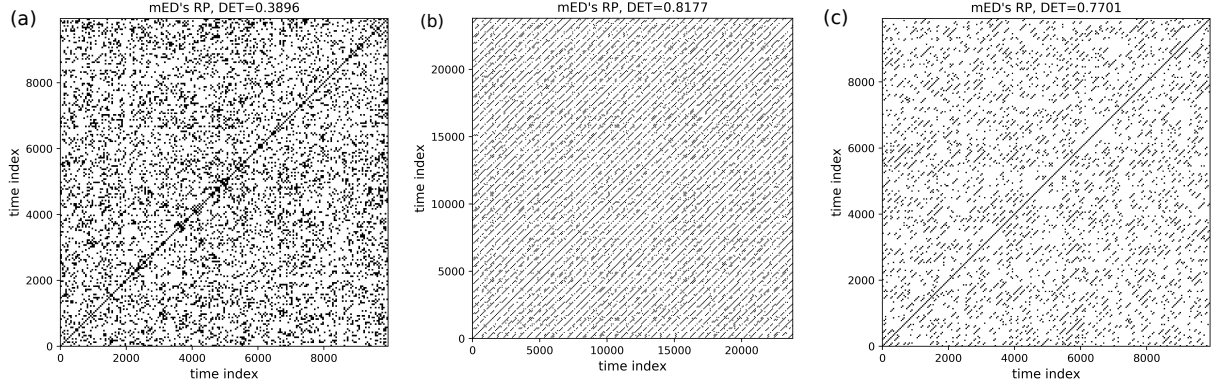


Figure 2: RP of (a) Poisson process, (b) repeating Poisson process, and (c) periodical Poisson process using ReLU function for optimizing the cost of shifting.

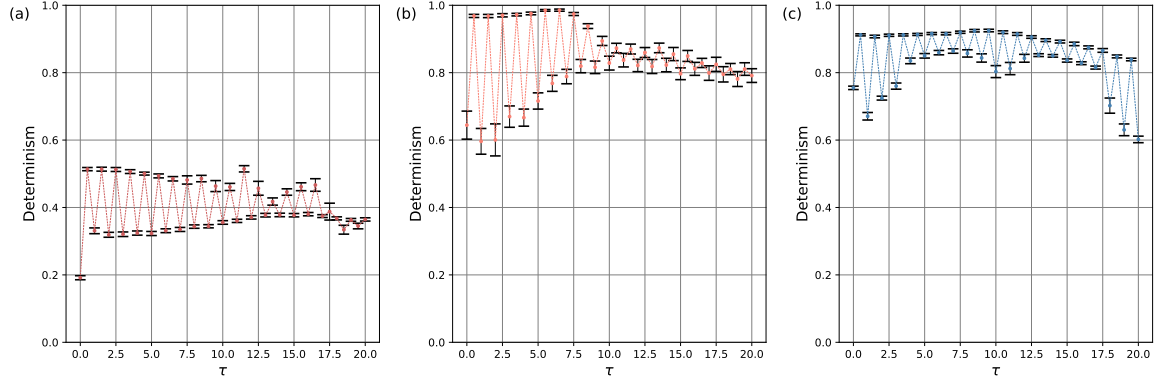


Figure 3: DET comparison for 300 realizations of (a) Poisson process, (b) repeating Poisson process, and (c) periodical Poisson process using ReLU function.

Comment 2: The value of k is crucial for the behavior of the sigmoid and therefore for the cost function. Could you elaborate more on how you set the value of k . It would be interesting to see the curve of DET as a function of k (like with Figure in figure 8.C)

Reply 2: The logistic function has two parameters, among which k is the rate parameter. Although k determines the slope of the sigmoid, it does not have much impact on the outcome of the RQA, in particular of the DET measure. To demonstrate this, we consider a Poisson process and a periodical Poisson process and compute DET for different k values. For the Poisson process, we keep $\tau = 0$ as the DET value using mED is maximum at $\tau = 0$, and for the periodical Poisson process we choose

τ to be 0 (DET value using mED is lower than ED) and 10 (DET value using mED is higher than ED). We find that DET remains more or less constant for the entire range of values of k (Fig. 4). Whether the DET for the mED is higher or lower than ED is mostly determined by τ , which can be interpreted as a delay between events and, thus, has a well understandable physical relevance for the considered system (and can be selected accordingly).

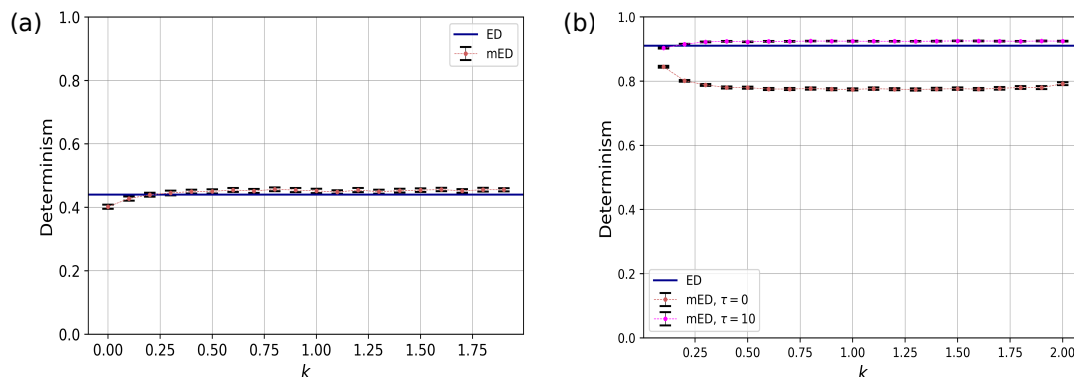


Figure 4: Comparison of DET for (a) Poisson process and (b) periodical Poisson process using mED for varying k and keep $\tau=0$, we use 300 realisations for each case.

Comment 3: Line 222, I could not understand the definition of p . What is the difference between p and L ?

Reply 3: We modify the manuscript as follows:

”Each window consists of n ($n \in \mathbb{N}, n > 1$) data points, so the window size is $w = n\Delta T$. For overlapping windows, a fraction of the data is shared between consecutive windows, denoted by $L \in \{1, 2, 3, \dots, n - 1\}$ as the number of data points within the overlapping range, where $L = \emptyset$ signifies the non-overlapping case. The overlapping range is $O_L = L\Delta T$ in terms of percentage $O_L(\%) = \frac{L}{n}$.”

Comment 4: Figure 12 and 13 have similar titles. Maybe, the overlap in figure 13 is 9 months.

Reply 4: We apologize for the typo. We will modify in the manuscript.

Comment 5: It would be interesting to plot the curve of DET as a function of figure for flood events in figures 12 and 13 (like in figures 8,9 and 10 for synthetic data using Poisson process)

Reply 5: Thank you for the comment. We will include the following discussion and figure in our manuscript:

”In order to compare *Edit distance*(ED) and the *modified Edit-distance*(mED) in case of flood events, we compute the RP and the *determinism* using both the methods (Fig. In case of mED, we vary the τ in the range 1 to 60 days. However, in case of ED we get the DET value for a particular

τ . For mED, we can vary the τ and compute the DET for different time scale. From Fig. 5, we find higher DET value for certain delay value.

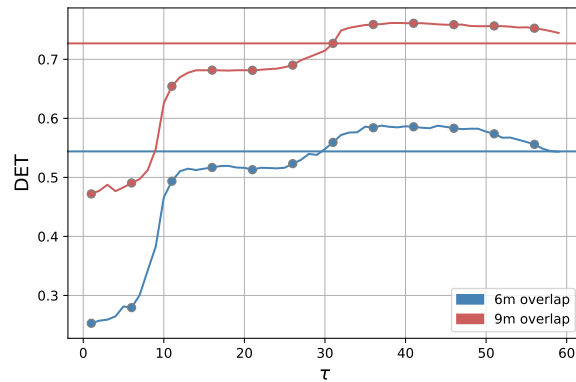


Figure 5: Comparison of DET for flood events using ED (horizontal line) and using mED (curved line) for 6 months overlapping (in blue color) and for 9 months overlapping (in red color)

Comment 6: The size of figures 11 and 13 could be reduced.

Reply 6: We will modify in the manuscript.

Comment 7: In figure 15, the mean of the surrogates is around 0.15. This value may depend on the shuffling process used in the generation of the surrogates from the original data. Could describe more how did you randomize the original set (uniform/Gaussian process)?

Reply 7: We exchange row (i) and column (j) simultaneously in the original recurrence plot with different pairs (i, j) as randomly chosen through a uniform process. In this way, we create 50 surrogate recurrence plots. We will include this information into the manuscript.