Answers to both reviewers:

Dear Reviewers,

We are grateful to both reviewers for their corrections and comments as they have increased the quality of the paper. Please find our detailed answers and corrections to both reviewers comments (reproduced in black) below, written in blue. The corrections performed to the manuscript are given in red and an updated version of the manuscript with the corrections highlighted in red is provided.

The purpose of our work is to extend the theory of FTs which is at the heart of non-equilibrium statistical mechanics for the last 30 years to climate science with interacting components. We give examples of immediate applications but to our understanding, a major benefit resides in the development of a new theoretical framework to further our understanding of the fluctuating interaction between different components of the climate system and their predictability as well as its limits to it. In the previous work we have discussed the application of the concepts (Fluctuation dissipation relation, Fluctuation dissipation theorem and the Fluctuation Theorem) to air sea interaction on the basis of idealized bulk formulas. In the present work we discuss the applicability to data from satellite observations.

Both reviewers are concerned with what concrete benefits FTs provide to the understanding of air-sea interaction and climate science. To address this concern we did several changes in the manuscript (see detailed answer below) and also changed the last two paragraphs of the paper which now read:

Finally, we put the theory of FTs in the more general context of climate dynamics. A measurement, especially when coming from satellites always contains some averaging in space and time. A FT, when it applies, will help to relate averages over varying periods and is a powerful tool to guide the up and down-scaling of observational data in time and obtain the statistical information on shorter and longer time scales, which are not explicitly observed. More precisely, when the pdf of the power supply, and therefore also the symmetry function is known form observations for given averaging times the symmetry function can be calculated for shorter and larger averaging times and constrains “half” of the pdf. This is useful in down-scaling and the construction of statistical parameterizations of not directly observed dynamics over shorter time scales. On the other hand, the information can be useful for developing models for the persistence of events over large time-scales not yet observed. A FT can help to decide if the persistence in time of a phenomena is within the likeliness of the statistically stationary dynamics or due to external influences. Furthermore, when data from observations follow (or not) a FT, model data should do likewise. As such, the FT becomes a tool of investigating the fidelity of models.

We conclude by looking at our results from the stand point of dynamical systems. Statistical mechanics of systems in equilibrium are described by the Boltzmann distribution, which is completely determined by the temperature. In non-equilibrium statistical mechanics no such universal distribution is known (see i.e. Derrida (2007), Touchette (2009) and Frisch (1995)), but some quantities in some processes seem to follow a FT which constraints the pdf and might indicate some universality. The mechanical power-input to the ocean by air-sea interactions, as a forced and dissipative dynamical system, may thus belong to a class of particular non-equilibrium systems exhibiting a FT symmetry property and offer guidance for climate studies.

Furthermore we like to mention that the applications of FTs in climate science are just beginning and other applications will possibly arise. In the present work we base our investigation on previously published theoretical / numerical investigation which show the existence of a FT in power-supply to the ocean in idealized models. We used a 24 years time series at 6h resolution and have just enough data to start seeing FT like behavior. But to our understanding, there is no doubt that climate science is looking towards a rapid increase (in quantity and quality) of available data and the question about the presence of FT like symmetries in the data can be answered more decisively, also for different variables than the mechanical energy input into the ocean. This allows to analyze environmental data based on theories developed in non-equilibrium statistical mechanics for the last 30 years. We are therefore convinced that the reviewers concerns about the immediate benefit of FTs for ocean science concerning prediction and modeling of quantities will disappear automatically with time.

Sincerely,
Achim Wirth, Bertrand Chapron

Anonymous Referee #1

This study investigates empirically whether or not the time integrated input of mechanical power from the atmosphere to the ocean obeys a fluctuation theorem. If this were the case, observations of the very common case where momentum is transferred from the atmosphere to the ocean could be used to infer probabilities for the rare opposite case. The paper is overall well-written and easy to follow, even if the reader is not closely familiar with ocean dynamics or fluctuation theorems. The core idea is sufficiently interesting for publication in this journal and constitutes a natural next step after the first author's previous study of conceptual models (Wirth 2019). The results appear to be somewhat inconclusive but this fact alone should not exclude the paper from publication. I am mainly concerned with the data analysis in section 5 which is not very clearly presented, both in terms of the methodology and the actual discussion and plots.

Specific comments:

p.4 l4-5 "fixed surface area" this is probably not very important but is the surface area actually fixed when the sea state can change over time? If you always consider fixed geographical regions, wouldn't calm conditions lead to a smaller surface area than rough seas?

The roughness of the surface is not considered here. I know changed to:

(the area which spans $10^o$ in the longitudinal and the latitudinal direction)

p.2 l22 "the focus" please make it clear whose focus you mean (the focus of most current research?)

I now changed to:

Furthermore, the research interest in many natural systems lies mostly in the fluctuations rather than in an average state, [...]

p.2 l33-34 "not only concerned with instantaneous values" if I understand correctly, eq.3 doesn't refer to instantaneous values at all, right? In that case you should cut "only" here.

Done.

p.3 l30 please make it unambiguous that the limit of large $\tau$ relates to both conditions and not just (ii). Also this is the first instance where $tau_0$ occurs, please explain what this refers to.

It is now changed to:

The Galavotti-Cohen fluctuation theorem (called FT in the sequel for brevity) holds for $\mathcal{P}$, if for averaging times larger than a characteristic time scale of the system ($\tau \gg \tau_0$), two conditions are satisfied: (i) the symmetry function depends linearly on the variable $z$, and (ii) on $\tau$:

p.5 l27f consider including a map of the world showing these four regions to give non-oceanographers at least some idea where they are located, how large they are and what factors might influence the different dynamics.

I did have a world map in a preliminary version of the paper, but the areas are rather small and not instantaneously visible. The solution is to put at least two maps, one for the North Atlantic and one for the North Pacific, but this takes too much space in my rather short paper and also I do already have many figures. Furthermore it is less the areas than their dynamic regimes which are important, which asks to include some current / wind information, which asks for individual zooms of the areas. Putting this might suggest that a FT can be eye-balled, which is of course not the case. When I give a talk on the subject I point towards the areas on a map, and show films of the current and the wind data considered, which resolves the problem. I therefore ask to keep as is. It is a personal preference and other choices are clearly possible.

p.5 l27f do you have some idea how sensitive your results are to the specific choice of your domains?

We show that the FT "works" in the re-circulation areas considered and that it does not work in the turbulent extensions of western boundary currents. It is written in the paper that: "During data analysis, we also found that a FT does not apply when islands or coastlines are present (not shown here). Departure from a FT for the power input to the ocean is found where horizontal dynamics dominates over the vertical ocean-atmosphere momentum exchanges."

Furthermore the analysis is very demanding in computer time which prohibits general investigation.

p.6 l1 what exactly do you mean by "an interval that spans twice the mean value [...] from the origin"? 0 +/- 2*mean( $E_tau$ ) ? In that case why is zero not at the center of the left parts of Fig.1-4 ?

We now replaced "mean" by variance. Not all the data obtained is shown in the graphs as the the averages over shorter time have a much lager variance. We adapted the range in the figs to have a good compromise showing the wide pdfs of short averaging and the narrow pdfs of the long averaging. Note that a convergence of the symmetry function is obtained for the limit in taking the long averaging times. Figures 1-4: Please add axis labels to both parts of the figures. Then the captions of Fig. 2-4 don't need to repeat that of Fig.1, "as Fig.1 but for case XY" would be sufficient. Please give the unit of the averaging time as well.

Axises are now labeled. And we added in the legend of the first figure:

The variable $\tau$ gives the length of the averaging interval in terms of observations done every 6 hours.

p.6 l11you state that you will verify Eq. 3 in two steps so the reader expects these two to be addressed in order. It is however unclear to me which of the following two paragraphs is supposed to refer to which aspect (see further comments below).

We now added:

That is, we first have to confirm that the lines in the right panels of figs. **??**, **??**, **??** and **??** converge towards straight lines for increasing averaging periods and second we see if the lines superpose when increasing averaging periods.

p.6 l12 you claim that you "determine the slope" but that that slope is never actually shown or discussed directly. Why not fit lines to your curves and show us the estimated slopes (see comment below)? In that way we could also compare whether or not the slope differs between the regions which is hardly possible by comparing curves indifferent plots with different y-axes.

There are already many lines the figures and adding lines makes the figures difficult to see. Furthermore in exps. GSE and KUE the behavior clearly fails to be linear, so lines can not be included. I choose to define the index gamma to investigate linearity. We do not give the value of the slope as we do not have a theory for the slope and how it is related to the dynamics. This is the case in all references on Fluctuation theorems obtained from experiments with turbulent fluids we know of (see e.g. Ciliberto et al. (2004)). We have of course tried to find a relation but did not succeed. Please note that it was and is written in the paper: "The contraction rate $\sigma > 0$ see Gallavotti and Cohen (1995a), Gallavotti and Cohen (1995b), Ciliberto et al. (2004) and Shang et al. (2005)) depends on the problem considered."

I our case it is influenced by the relation of the average wind to wind variability on different time-scales, the small scale turbulence in the boundary layer and the temperature stratification in the atmosphere.

We now added:

We did not manage to determine it from observed quantities.

p.6 l13 you again mention $tau_0$, can you at least give some rough estimate how long that time-scale might be, relative to the length of your time series? Could this be inferred from the power-spectrum of the time-series?

The reviewer is right, an estimate of $tau_0$ should be given, but we do not have enough data to provide such a solid estimate. To consider FTs huge amount of data is necessary, which is often not available yet in environmental sciences. In the present work we base our investigation on previously published theoretical / numerical investigation which show the existence of a FT and we have enough data to start seeing FT like behavior. Please note also that $tau_0$ strongly depends on the tail of the pdf, the rare negative events. Results indicate that in the cases where we observe a FT the symmetry function converges to a strait line in about 1 year. The power-spectrum gives information about the amplitude of a given frequency, but the phase is equally important to determine the occurrence of the high amplitude events (in the same manner as phase is important to determine coherent structures in turbulence). So the connection between the power-spectrum and $tau_0$ is subtle.

I now added:

For the extension of the domains within the recirculation area of the subtropical gyre a convergence towards a linear variation with $z$ is observed in less than $t_0 \approx 1$ year.

p.6 18f "This indicates the existence of a large deviation principle" isn't it more important that this convergence is predicted by the FT? What is the relationship between the existence of an LD principle and a FT? Also is this the first or the second part of the verification mentioned above?

The relation of FT and large deviation principal is often asked when I communicate about this work and I wanted to clarify the point here. If the LD exists for all z than the normalized symmetry function converges, but not necessarily to a straight line. So (ii) indicates the existence of a LD (but does not proof it), even if (i) does not hold. So the way it is said in the text is correct. I do not know how to say it correctly in a different and clearer way. If this sentence about LD leads to confusion it can be taken away. The rest of the paper is completely independent of it. I would prefer to keep it. We now changed :

For the domains within the recirculation area (ASG and PSG) of the subtropical gyre a convergence towards a linear variation with $z$ is observed in less than $t_0 \approx 1$ year. This points towards the existence of a FT, as both points put forward at the beginning of the previous paragraph are observed. For the extensions of the western boundary currents (GSE and KUE), the convergence does not achieve a linear behaviour of the normalised symmetry function. This shows that a FT does not hold, as the first point put forward at the beginning of the previous paragraph is not satisfied.

p.6 l19f "extension of the domains within ...", "extension of the western boundary current" please refer to the different regions by the acronyms you established before and also refer to the figures in which these results are shown.

Done

p.7 l1f I'm not sure why you chose to quantify the linearity of your curves by this specially designed index. If I understand correctly, the scaled symmetry functions corresponding to long averaging times should be linear across the whole range of z-values. Why not simply fit a line via least squares to calculate the overall slope? Use R^2 to get an idea of the goodness of fit and plot the slopes against tau to observe the convergence behavior. I understand that the statistical interpretation in terms of confidence intervals is questionable but I don't see why your index is more appropriate. Unless I misunderstood your definition, there are many non-linear curves for which gamma=1.

Yes, there are non-linear curves for which gamma=1. One known scenario when the FT fails is due to boundary conditions as briefly mentioned in the text. In this case there is a transition in the slope from high to low values, as we observe in our data. Based on this analytically explained scenario I choose gamma the way I did, other choices are clearly possible.

p.8 l7 "extreme events are often key" of course extreme events in general are interesting but your framework doesn't describe just any kind of weather extreme but specifically unusually small (negative) values of atmosphere-ocean momentum transfer. Can you explain a bit more specifically why a rare event wherein the wind in the atmosphere is sped up by the ocean is of interest?

The reviewer is right. We now added:

Extreme negative events lead to strong transfer of energy to small-scale turbulence in the atmospheric and oceanic boundary layers, potentially causing strong mixing in the atmosphere and ocean.

p.8 l9f I like this example, perhaps it would be even more illustrative if you put in actual numbers for tau? Say one month or one year? This, however raises the question how large tau has to be for the FT to hold...

We now added:

The variable $\tau$ gives the length of the averaging interval in terms of observations done every 6 hours, that is $\tau = 400$ corresponds to a period of 100 days. A FT represents a tool to obtain the rare negative events from frequent positive events for all averaging times $\tau > \tau_0 \approx 1$ year

p.8 l12 "all averaging times" if I understand correctly, your FT only makes statements about long averaging times, right?

We now added:

$t > t_0 \approx 1$ year

p.9 l3 "exp2 & 4" please refer either to the figures or the abbreviations of the different, regions in a consistent manner, the terms "expN" were never explicitly introduced.

Oups, yes, now corrected.

p.9 l18 "guide the up and down-scaling" can you either give a reference for this claim or explain a little more how the FT could help with that?

We now added:

More precisely, when the pdf of the power supply, and therefore also the symmetry function is known form observations for given averaging times the symmetry function can be calculated for shorter and larger averaging times and therefore constrains "half" of the pdf. This is useful in down-scaling and the construction of statistical parameterizations of not directly observed dynamics over shorter time scales. On the other hand the information can be useful for developing models for the persistence of events over large time-scales not yet observed.

Technical corrections:

p.2 l14: case mismatch between "the importance [...] is, [...] their imprint", please re-formulate

Done.

p.2 l17-18: the sentence with "can not be understood or modelled" is repeated verba-tim, please cut or re-formulate.

Done.

p.2 l32: replace "i.e." by "e.g."

A negative event is when the ocean loses energy, so I would like to keep "i.e." meaning: "that is".

p.4 l7: replace "is" by "should be"

Done.

p.5 l6f "the production has been performed of ..." confusing sentence, do you mean "a near real-time data set, as well as a 24 year reanalysis, [...], have been produced" ?

Done.

p.5 l15 25 or 24 years ?

Now corrected. The ocean data is 25 years but the overlap with the atmospheric data is only 24 years.

p.5 l20 "6h in time and 1/4$^o$ in space" this is repeated from the previous sentence.

Yes, but the first time it considers the atmospheric data and the second time it is the atmospheric and oceanic data. We put it to emphasize that both are available at the same resolution in time and space. We now write:

[. . . ] at the same resolution in space and time.

p.5 l24 ", For" either change to lower case or start a new sentence

Done.

p.5 l30 "from" instead of "form"

Done.

p.7 l5 "these cases" or "this case"

Done.

p.8 l1 "is a currently a hot topic" cut one of the "a"s

Done.

p.8 l9 "slope" instead of "slops"

Done.

p.9 l5-6 replace "to which" by "in which"

Done.

p.9 l14 "growth" instead of "grows" or write "its surface grows quadratically"

Done.

Anonymous Referee #2

This paper aims to provide observational support in favour of the idea that the windpower input satisfies a fluctuation theorem (FT) in some regions of the ocean. Fts have only appeared recently in the literature and have been useful to justify the physical character of (rare) violations of the second law of thermodynamics. In this paper, it is the wind power input that is treated as the dominantly positive quantity and the analogue of the positive entropy production predicted by the second law, while the negative power input events are seen as the analogue of the rare events seemingly violating the second law. Review of the literature on the subject is pedagogical enough that it can be read and understood with little background on the part of the reader. Overall, the paper is relatively clear and easy to follow, while the analysis appears to be competently done although short on practical details. The main weakness of the paper, however, is that it appears to devote much time explaining why FTs are useful or important in general, without ever really explaining why they are useful or important in the particular case considered by the paper, namely ocean energetics. The negative power input events are presented as 'extreme' events, but it is unclear to what extent this is justified. Are these events related to the passing by of low- pressure systems that result in occasional reversal of the winds relative to prevailing conditions? The authors emphasise that extreme events are often 'key' for the systems considered (by others), but do not explain why these are key for the system they consider. The paper needs to improve on those aspects as well as on the specific points outlined below before it can be accepted for publication.

Concerning the lack of concrete applications of FTs in air-sea interaction please see my answer to both reviewers in the beginning of this reply

General comments

Title: A more concise title would be: Empirical evidence of a fluctuation theorem for the wind mechanical power input in the ocean. I suggest using empirical because the estimation of the power input does not just involve satellite data. The authors need to explicitly state that the mechanical power input is due to the wind, as surface buoyancy fluxes also contributes to powering the ocean.

We agree and changed the title to:

Empirical evidence of a fluctuation theorem for the wind mechanical power input into the ocean

Aim: Could the authors clarify the precise aims of the paper? Is it intended to contribute to the literature about ocean energetics? If so, the authors should provide some review of the literature about ocean energetics. Is it intended to provide a constraint and metric by which to constrain ocean models? If so, the authors should expand on this some more and explain how one should go about it. Even better would be to repeat the calculations using model outputs where the authors find evidence for a FT to establish whether this would be a useful metric to assess models. As written, it is difficult to understand what issues of interest to the oceanographic community the present results are useful for.

Please see my answer to both reviewers in the beginning of this reply. Performing the same analysis on model data is planned, but this is another paper. Here we want to discuss the existence of FTs in observations. We added in the introduction:

For a general discussion on air-sea interaction we refer to Csanady (2001), for ocean energetics to Ferrari and Wunsch (2009) and for wind work to Wunsch (1998).

More specific comments

1. Abstract, line 3: ‘global satellite observations’ may be more specific . Scatterometer wind observations and surface current derived altimeter data.

Yes, but then there is also drifter data and in-situ measurements. We are afraid being at the same time to specific and not specific enough in the abstract. We prefer writing that the basis are ‘global satellite observations’ and being more specific in the Data section and most importantly referring to the work were this rather involved products are described in all detail. Other choices are clearly possible.

2. Page 1, lines 15-17: The wind stress also includes a form stress component due to the wind blowing creating negative and positive pressure anomalies on the surface waves

By shear we mean the difference of the wind and the currents near the surface. In the present paper we are not concerned with the details of the air-sea interaction at small scales but suppose that these are parameterized by bulk formulas. That is why we write : [. . . ] due to the difference between the atmospheric winds and the ocean currents near the surface in the corresponding planetary boundary layers.“ and not “at the surface”. We now added:

In the present work we do not discuss the various physical processes occurring at the air-sea interface which are important for the momentum transfer.

We now replace “shear“ by “shear-stress” in the text.

3. Page 1, lines 20-21: The energy exchange is not conservative and most of the mechanical energy is dissipated. I don’t understand what that means. Clearly, momentum is conserved and energy is transferred from the atmosphere to the ocean. Part of it goes into avaialbel potential energy to push down isopycnals or suck up isopycnals. Does it go into heat rapidly? Ultimately, sure. What are you trying to say here?

In air-sea interaction momentum is conserved but not energy (it resembles an inelastic collision of two objects, that stick together after collision). Most of the energy goes into 3D turbulence in the atmospheric and oceanic boundary layers with a direct energy cascade to dissipation into heat a large part goes into wave generation.

4. Page 2, line 5. ‘measure’ -> ‘estimate’ or ‘evaluate’. The power input is clearly not measured.

Done.

5. Page 2, line 12: ‘spacial’ -> ‘spatial’

Done. (The dictionary says that spacial is ok too)

6. Page 2, lines 16-17: and conversely, turbulent motion depend also on the mean. Does it matter for the arguments developed here?

It does, but here we want to emphasize the closure problem, that is the large scales we are usually interested in, in climate sciences, can not be modeled without some knowledge of the small scales. We now added:

, and vice versa.

7. Page 3, line 7: ‘existence of a FT was shown empirically’. ‘Shown’ sounds like a strong word. Suggested sounds more accurate

Done.

8. Page 3, line 13. ‘Satellite measurements’ not onl. ‘discuss their relevance’ it is not clear to me that this has really been achieved satisfactorily. This needs to be improved.

See our answer to both referees in the beginning of this reply

9. Page 4, line 21: I find reference to ‘shear’ somewhat confusing, since power is best understood as the product of a force times displacement by unit time. Why not refer to the wind stress rather than the shear? Moreover, the wind stress is not just due to the shear, it also includes a form stress part due to the wind blow creating pressure positive and negative pressure anomalies on the upwind and downstream sides of sea surface waves.

Yes, the reviewer is right but by writing “wind stress” we are afraid that the reader thinks that we are using the approach where the force is calculated based on the wind only and not the difference between wind and current. This is detailed in section 3. We now replace “shear“ by “shear-stress” in the text.

10. Line 25. May be indicate the value of Cd used for the calculations.

We now added:

Variations of the drag coefficient are not considered and all the results are independent of a constant $C_d$.

11. Page 4, linear 29. ‘goestrophic’ - > ‘geostrophic’

Done.

12. Page 4-5, Lines 31-33. What does it mean physically? Is the power converted into available potential energy or is it dissipated into heat? How does this result justify estimating the wind power input proposed by the authors? Are the overall results sensitive to using the surface velocity or 15 m velocity? The calculations seem easy enough to do that the authors should describe both.

The wind injected at the surface goes into waves or is dissipated locally in the Ekman layer (see Zhai et al.), has no direct significance on the ocean dynamics. This why we did not consider it here.

13. Page 6, Lines 19-20: ‘This indicates the existence of a large deviation principle ’What does that mean? What does that imply? Why is this important or useful?

The relation of FT and large deviation principal is often asked when I communicate about this work and I wanted to clarify the point here. If the LD exists for all z than the normalized symmetry function converges, but not necessarily to a strait line. If this sentence about LD leads to confusion it can be taken away. The rest of the paper is completely independent of it. I would prefer to keep it.

14. Page 8. Lines 6-8. Why is this useful?

If a FT holds we have “half of the pdf” in the case of non-equlibrium stat. mechanics where we do not know the pdf this is the only information we have and it is useful. This is now discussed in more detail in the Conclusions (see answer to both reviewers above).

15. Page 8. Lines 7-8. ‘Extreme events are often key for the system [...]’ What does that mean? To what extent are negative wind power input ‘extreme’ and ‘key’ for the understanding of ocean energetics.

They are extreme because they are in the tails of the pdf. In this events, both, the atmosphere and the ocean loose energy, so large amounts of energy go into small-scale turbulence. We now write:

Extreme negative events lead to strong transfer of energy to small-scale turbulence in the atmospheric and oceanic boundary layers, potentially causing strong mixing in the atmosphere and ocean.

16. Page 9. Lines 14-26. These last three paragraphs are particularly vague and abstract and not really related to any issues pertaining to ocean energetics. Is it possible to link these to ocean energetics in some way? This paper does not contribute to the theory of FT, so it is unclear why it should speculate on it.

We consider if FT is applicable to air-sea interaction and find that is does in some cases. These last three paragraphs are key as they show how FTs can be useful and the last paragraph puts the work in a larger context, it does not speculate. So we would like to keep the paragraphs. We rewrote the last three paragraphs (see answer to both reviewers above).

## References

Ciliberto, S., Garnier, N., Hernandez, S., Lacpatia, C., Pinton, J.-F., and Chavarria, G. R.: Experimental test of the Gallavotti–Cohen fluctuation theorem in turbulent flows, Physica A: Statistical Mechanics and its Applications, 340, 240–250, 2004.

Csanady, G. T.: Air-sea interaction: laws and mechanisms, Cambridge University Press, 2001.

Derrida, B.: Non-equilibrium steady states: fluctuations and large deviations of the density and of the current, Journal of Statistical Mechanics: Theory and Experiment, 2007, P07 023, 2007.

Ferrari, R. and Wunsch, C.: Ocean circulation kinetic energy: Reservoirs, sources, and sinks, Annual Review of Fluid Mechanics, 41, 2009.

Frisch, U.: Turbulence: the legacy of AN Kolmogorov, Cambridge university press, 1995.

Gallavotti, G. and Cohen, E. G.: Dynamical ensembles in nonequilibrium statistical mechanics, Physical Review Letters, 74, 2694, 1995a.

Gallavotti, G. and Cohen, E. G. D.: Dynamical ensembles in stationary states, Journal of Statistical Physics, 80, 931–970, 1995b.

Shang, X.-D., Tong, P., and Xia, K.-Q.: Test of steady-state fluctuation theorem in turbulent Rayleigh-Bénard convection, Physical Review E, 72, 015 301, 2005.

Touchette, H.: The large deviation approach to statistical mechanics, Physics Reports, 478, 1–69, 2009.

Wunsch, C.: The work done by the wind on the oceanic general circulation, Journal of Physical Oceanography, 28, 2332–2340, 1998.

# Empirical evidence of a fluctuation theorem for the wind mechanical power input into the ocean

Achim Wirth[1] and Bertrand Chapron[2]

[1]Univ. Grenoble Alpes, CNRS, Grenoble INP, LEGI, 38000 Grenoble, France
[2]LOPS, Ifremer, Plouzané, France

**Correspondence:** achim.wirth@legi.cnrs.fr

**Abstract.** The ocean dynamics is predominantly driven by the shear-stress between the atmospheric winds and ocean currents. The mechanical power input to the ocean is fluctuating in space and time and the atmospheric wind sometimes decelerates the ocean currents. Building on 24-years of global satellite observations, the input of mechanical power to the ocean is analysed. A Fluctuation Theorem (FT) holds when the logarithm of the ratio between the occurrence of positive and negative events, of a certain magnitude of the power input, is a linear function of this magnitude and the averaging period. The flux of mechanical power to the ocean shows evidence of a FT, for regions within the recirculation area of the subtropical gyre, but not over extensions of western boundary currents. A FT puts a strong constraint on the temporal distribution of fluctuations of power input, connects variables obtained with different length of temporal averaging, guides the temporal down- and up-scaling and constrains the episodes of extreme events.



## 1 Introduction

The exchange of heat, momentum and matter between the atmosphere and the ocean has a strong influence on our climate (Stocker et al. (2013)). Recent advances in satellite and in-situ based global Earth Observation (EO) systems and platforms, have significantly improved our ability to monitor ocean-atmosphere interactions. In the present work the exchange of momentum is considered, which is described by the fluxes of mechanical power at the ocean surface. It is caused by the shear-stress at the surface due to the difference between the atmospheric winds and the ocean currents near the surface, in the corresponding planetary boundary layers. In the present work we do not discuss the various physical processes occurring at the air-sea interface which are important for the momentum transfer. For a general discussion on air-sea interaction we refer to Csanady (2001), for ocean energetics to Ferrari and Wunsch (2009) and for wind work to Wunsch (1998).xs The atmospheric winds are usually stronger than the ocean currents and therefore the atmosphere mostly loses energy at the interface by friction and the ocean mostly gains energy. As a feedback mechanism, the presence of surface currents will then modulate the air-sea transfer of momentum (Bye (1985), Renault et al. (2017)). The energy exchange is not conservative and most of the mechanical energy is dissipated (Duhaut and Straub (2006), Wirth (2018), Wirth (2019)). In the present work we are not concerned with the details of the exchange in the respective boundary layers (see e.g. Veron (2015)) but suppose that it is well represented through bulk formulas of air-sea interaction (Fairall et al. (1996)). In those models the power input is estimated based on theshear-stress at the surface and the ocean current near the surface and also depends on the sea state and the density stratification in the atmosphere and the ocean.

More precisely, we consider the mechanical energy exchange between the atmosphere and the ocean at a time, $t$, over a fixed surface area $A$ of the ocean (the area which spans $10^o$ in the longitudinal and the latitudinal direction). For this area we evaluate the mechanical power the ocean gains at the interface $\mathcal{P}(t)$. Due to the turbulent dynamics in the atmosphere and the ocean the quantities are fluctuating over a large range of scales in time and space.

We focus on two properties of the mechanical power input to the ocean at the surface: (i) on average the ocean gains energy at the interface $\langle \mathcal{P}(t) \rangle > 0$ (where $\langle . \rangle$ represents an average over the observation period and several surface areas $A_i$) and (ii) the power input is fluctuating, in time and space, due to the turbulent motion in the atmosphere and the ocean and negative events, with $\mathcal{P}(t) < 0$, occur.

Today, fluctuations are the focus of research in statistical mechanics, which was traditionally concerned with averages. Fluctuations in a thermodynamic system usually appear at spatial scales which are small enough so that thermal, molecular, motion leaves an imprint on the dynamics as first noted by Einstein (1906) (see also Einstein (1956) and Perrin (2014)). The importance of fluctuations is, however, not restricted to small systems. Fluctuations can leave their imprint on the dynamics at all scales when (not necessarily thermal) fluctuations are strong enough.

Turbulent fluid motions are typical examples (i.e Frisch (1995)), for which average motions can not be understood or modelled without some knowledge about the turbulent fluctuations, and vice versa. Turbulent fluctuations can be especially pronounced in geophysical flows, which are highly anisotropic due to the influence of gravity and rotation. This leads to a quasi two-dimensional dynamics and an energy cascade from small to large scales and strong fluctuations (see i.e. Boffetta and Ecke (2012) for a review on 2D turbulence). Likewise, the description of air-sea interactions on large time scales may not be understood without some knowledge of the fluctuations at smaller and faster scales. Furthermore, the research interest in many natural systems lies mostly in the fluctuations rather than in an average state, weather and climate dynamics are examples where we focus on the fluctuations of the same system on different time scales. For the weather the time scale of interest is from roughly an hour to a week, for the climate the focus is from tenths to thousands of years and beyond.

A recent concept which is presently subject of growing attention in non-equilibrium statistical mechanics are Fluctuation Theorems (FT) (see e.g. Evans et al. (1993), Gallavotti and Cohen (1995a), Gallavotti and Cohen (1995b), Ciliberto et al. (2004), Shang et al. (2005) and Seifert (2012)). Not only the average values of quantities like entropy, work, heat or other, are studied, but their fluctuating properties are scrutinised. There are different forms of FTs, reviewed in detail by Seifert (2012). In the present paper we focus on the FT put forward in Gallavotti and Cohen (1995a), Gallavotti and Cohen (1995b) and Gallavotti and Lucarini (2014), corresponding to the detailed fluctuation theorem in the limit of large averaging times. When the FT applies to a fluctuating quantity, as i.e. $\mathcal{P}(t)$ in the present study, it relates the probability to have a negative event, i.e. the ocean loses energy, to the probability of a positive event, i.e. the ocean gains energy, of the same magnitude. The FT is not concerned with instantaneous values but considers the fluctuations of temporal averages over varying averaging time. The FT, which is stated precisely in the next section, thus puts a strong constraint on the fluctuations of the quantity considered and its temporal averages of varying length.

FTs have been established analytically for Langevin type problems with thermal fluctuations (Seifert (2012)). Most experimental data comes also from micro systems subject to thermal fluctuations. The thermodynamic frame of the quantities considered, as entropy, heat and work is not necessary to establish FTs. Examples of non-thermal fluctuations are the experimental data of the drag-force exerted by a turbulent flow (Ciliberto et al. (2004)) and the local entropy production in Rayleigh-Bénard convection (Shang et al. (2005)). For these non-Gaussian quantities the existence of a FT was suggested empirically. Our work is strongly inspired by these investigations of the FT in data from laboratory experiments of turbulent flows.

In Wirth (2019) the FT was investigated for three parameterizations of air-sea interaction and we refer the reader to this work for the theory and analytical solutions on fluctuating air-sea interaction in these idealised models. In that publication the concept of FT is also placed in a broader context of fluctuating dynamics and the relation to the fluctuation-dissipation-relation and the fluctuation-dissipation-theorem is given (see also Seifert (2012) for a general discussion). Here we extrapolate the research of Wirth (2019) by applying the concept of FTs to data derived from satellite measurements and discuss their relevance. It is important to notice that even in the case of the idealised models the FT was not established by analytical calculation, but it was confirmed numerically that the FT is obtained asymptotically, in the long-time limit, when the averaging time is larger than the characteristic time-scale of the slow ocean-dynamics.

## 2 The Fluctuation Theorem

We are interested in the mechanical power, $\mathcal{P}(t)$, absorbed by the ocean over a given surface area, $A$, of the ocean surface and an observation period $t_{\mathrm{obs}}$. We suppose that $\mathcal{P}(t)$ is a statistically stationary random variable, meaning that its statistical properties (mean value, moments and temporal correlations) do not change when shifted in time. Its statistical properties, at every instance of time, are completely described by its probability density function (pdf), $p(z)$, which gives the probability that $\mathcal{P}(t)$ takes values between $z_1$ and $z_2$ by integration: $\Pr[z_1 < \mathcal{P}(t) < z_2] = \int_{z_1}^{z_2} p(z)dz$. The symmetry function is:

$$S(z) = \ln\left(\frac{p(z)}{p(-z)}\right). \tag{1}$$

It compares the occurrence of events when the ocean receives power of magnitude $z$ to the occurrence when the ocean loses power of the same magnitude. We further denote the normalised energy received during an interval $\tau$ starting at time $t_0$, by:

$$\overline{E(t_0)}^{\tau} = \frac{\int_{t_0}^{t_0+\tau} \mathcal{P}(\tau')d\tau'}{\int_0^{t_{\mathrm{obs}}-\tau} \int_{t_0}^{t_0+\tau} \mathcal{P}(\tau')d\tau' dt_0/(t_{\mathrm{obs}}-\tau)}, \tag{2}$$

where $t_{\mathrm{obs}}$ is the total length of the available data record. The corresponding pdf is denoted by $p(z,\tau)$ and the symmetry function by $S(z,\tau)$. Note that the averaging starts at time $t_0$ and extends over the interval $\tau$.

The Galavotti-Cohen fluctuation theorem (called FT in the sequel for brevity) holds for $\mathcal{P}$ if, for averaging times larger than a characteristic time scale of the system ($\tau \gg \tau_0$), two conditions are satisfied: (i) the symmetry function depends linearly on the variable $z$, and (ii) on $\tau$:

$$S(z,\tau) = \sigma\tau z, \tag{3}$$

where $\sigma$ is called the contraction rate. The contraction rate $\sigma > 0$ (see Gallavotti and Cohen (1995a), Gallavotti and Cohen (1995b), Ciliberto et al. (2004) and Shang et al. (2005)) depends on the problem considered. We did not manage to determine it from observed quantities. In systems where the fluctuation is due to thermal motion its value is related to the thermal energy, that is the product of the Boltzmann constant and temperature, $k_B T$. When fluctuations arise from turbulent motion the temperature has (almost) no influence on the fluctuations and the contraction rate $\sigma$ depends on the turbulence. Indeed, in the incompressible Navier-Stokes equations temperature does not appear explicitly and only the kinematic viscosity has a slight dependence on temperature. There is, therefore no reason why $k_B T$ is a governing parameter of the problem.

If the FT holds it is sufficient to know the probability for either $z > 0$ or $z < 0$ to obtain the whole pdf, when $\sigma$ is known. The FT therefore constraints "half" of the pdf, a strong constraint in the absence of an equivalent of the Boltzmann distribution. This property also allows to calculate the probability of the rare events of $z < 0$ from frequent events $z > 0$.

For a dynamical system the FT may or may not hold and it might only be valid for a range of values. It was already noted in Gallavotti and Cohen (1995a) and Gallavotti and Cohen (1995b) that the FT might only be valid for values $z < z^*$, when the large deviation function (see i.e. Touchette (2009)) diverges outside the interval $[-z^*, z^*]$. More recently it was recognised that boundary conditions, that is the value $\mathcal{P}(t)$ at $t = t_0$ and $t = t_0 + \tau$, can leave their signature in the symmetry function $S(z,\tau)$, even when the limit of $\tau \to \infty$ is taken, whenever the pdf $p(z)$ has tails which are exponential or less steep than exponential (see Farago (2002), Van Zon and Cohen (2004) and Rákos and Harris (2008)). In such case an extended FT (EFT) should be expected, which shows a linear scaling of the symmetry function near the origin with a transition to a flatter curve for larger values. An analytic expression of the symmetry function, or the value of $z^*$ is obtained only for very idealised cases and the results presented here are empirical.

## 3 Power Input

The calculations of the power input to the ocean are based on the shear-stress at the surface and the ocean velocity. The shear-stress is usually evaluated, based on the difference between the horizontal wind velocity $\mathbf{u_a^s}$, usually taken at 10m above the ocean surface and the horizontal ocean surface-current $\mathbf{u_o^s}$, using the quadratic drag law (see i.e. Renault et al. (2017)):

$$\mathbf{F} = C_d \sqrt{(\mathbf{u_a^s} - \mathbf{u_o^s})^2)}(\mathbf{u_a^s} - \mathbf{u_o^s}). \tag{4}$$

The drag coefficient $C_d$ depends on the sea-state and the stratification in the atmosphere and the ocean, it is obtained using bulk formulas (Fairall et al. (1996)). Variations of the drag coefficient are not considered and all the results are independent of a constant $C_d$.

To obtain the power input, the vector product between the shear-stress and the ocean current-velocity is taken:

$$\mathcal{P}(t) = \mathbf{F} \cdot \mathbf{u_o}. \tag{5}$$

For the work done on the large-scale geostrophic-circulation, Wunsch (1998) and Zhai et al. (2012) used the surface geostrophic velocity estimates from altimetry for $\mathbf{u_o}$. Using model data, Rimac et al. (2016) chose the velocity at the surface to calculate the total power input, to then evaluate that only a fraction of this power is transmitted to the interior ocean at the base of the mixed layer. In the present work, largely building on 15-m drogued drifter velocities (Rio et al. (2014)), we use for $\mathbf{u_o}$ the estimation of the current velocity at 15m depth.

## 4 Data

In this study, we build on the newly released GlobCurrent products, now available via the Copernicus Marine Environment Monitoring Service (CMEMS, http://marine.copernicus.eu/services-portfolio). Essentially building on the quantitative estimation of ocean surface currents from satellite sensor synergy, the production has been performed of a 25 years reanalysis of global, $1/4^o$ maps of ocean currents at two levels, the surface and 15m depth. The data is obtained from the combination of altimetry, GOCE, wind and in-situ data (largely building on 15-m drogued drifter velocities) (Rio et al. (2014)).

Strongly based on altimeter data, this global ocean surface current product, and also similar global observation-based products (Bonjean and Lagerloef (2002), Sudre et al. (2013)), suffer from well-known limitations. The full spatio-temporal ocean dynamics is certainly not well captured, possibly missing part of the geostrophic component and a number of dominant ageostrophic signals (e.g. inertial oscillations). Also, accuracy is strongly reduced in the Equatorial Band where the geostrophic approximation fails, in coastal areas where altimetry accuracy decreases and where ageostrophic currents often dominate, and in the seasonally ice-covered Polar Seas. Nevertheless, this global ocean surface current product provides a consistent data set covering the last 25 years.

Satellite winds are from the Copernicus project (http://marine.copernicus.eu/services-portfolio/access-to-products/). They are from scatterometer and radiometer wind observations. It is a blended product based on the different missions (ERS-1, ERS-2, QuikSCAT, and ASCAT) available at $1/4^o$ spatial resolution and every 6 hours and is described in Bentamy et al. (2017) and Desbiolles et al. (2017). The data record for which wind and current data is available extends over 24 years, 1993–2016, at the same resolution in space and time.

The FT is a property that concerns the tails of a pdf, and it is necessary to consider a large amount of data, as provided by the GlobCurrent products. Still, a time record of 24 years of data coverage at a single location is too small for empirically suggesting or refuting the existence of a FT. To increase the amount of data, we use different tiles $A_i$ that obey similar statistical properties. The tiles represent an effective area of $0.5^o$ in the longitudinal and latitudinal direction. For a trade-off between ensemble size and similar statistical properties, we choose to consider domains extending $10^o$ in the longitudinal and latitudinal direction, composed of $20 \times 20$ non-overlapping tiles each.

Four domains are considered, the first is in the recirculation area of the subtropical gyre ($20^o - 30^oN$, $20^o - 30^oW$) of the North Atlantic (case: ASG), the second in the Gulf Stream extension ($35^o - 45^oN$, $35^o - 45^oW$)(case: GSE). The third is in the recirculation area of the subtropical gyre of the North Pacific ($15^o - 25^oN$, $150^o - 160^oE$)(case: PSG) and the fourth in the Kuroshio extension ($30^o - 40^oN$, $150^o - 160^oE$) (case: KUE). The data record from which wind and current data is available extends over the 24 years, 1993–2016, at a resolution of 6h. At four occasions in time data was missing. The gaps were filled by linear interpolation.

The pdfs of $\overline{E(t)}^{\tau}$ are calculated for an interval that spans at least twice the variance of each pdf from the origin. Note, that the average is unity by definition. The pdfs are calculated with three different resolutions (bin sizes). The interval is separated into 21, 31 and 41 bins of equal size and the pdfs are obtained by counting the number of occurrences for each bin. The symmetry function is only calculated when probabilities are lager than $10^{-3}$ per bin, this led to an omission of bins in exp. 1, only.

## 5 Results

The pdfs $p(z,\tau)$ for the four domains and for different values of averaging times $\tau$ are presented in the left panels of figs. 1, 2, 3 and 4. All clearly display non-Gaussianity. With increasing averaging period, the pdfs become more centred around unity (which is the average value, see eq. (2)), a consequence of the central limit theorem and occurrences of negative values become less likely. In the right panels of figs. 1, 2, 3 and 4 the symmetry function divided by the averaging period is plotted. These plots are similar to those in Gallavotti and Lucarini (2014), who verified the FT in an idealised numerical model.

The verification of the FT, that is of eq. (3), is two-fold. First, we verify the linear dependence of the symmetry function on $z$ for different averaging periods $\tau$, and determine the slope. Second, we verify that the slope is a linear function of $\tau$ for times larger than the characteristic time, $\tau > \tau_0$, of the system. That is, we first have to confirm that the lines in the right panels of figs. 1, 2, 3 and 4 converge towards straight lines for increasing averaging periods and second we see if the lines superpose when increasing averaging periods. This is demanding, and a large amount of data is necessary. For the first point, we have to consider the pdf for an extended range in $z$, including the tails, asking for ensemble sizes (number of intervals of length $\tau$) large enough so that we can observe a clear scaling behaviour. For the second point, we have to increase $\tau$ to verify convergence. Furthermore, for larger $\tau$ the pdfs are more and more peaked around unity and negative events become less and less likely.

For the four domains, we observed a convergence of the normalised symmetry function with increasing averaging time. This indicates the existence of a large deviation principle (see i.e. Touchette (2009)). For the domains within the recirculation area (ASG and PSG) of the subtropical gyre a convergence towards a linear variation with $z$ is observed in less than $t_0 \approx 1$ year. This points towards the existence of a FT, as both points put forward at the beginning of the previous paragraph are observed. For the extensions of the western boundary currents (GSE and KUE), the convergence does not achieve a linear behaviour of the normalised symmetry function. This shows that a FT does not hold, as the first point put forward at the beginning of the previous paragraph is not satisfied.

The contraction rate $\sigma$ is the slope of the curves in the right panels of figs. 1, 2, 3 and 4. To estimate the alignment of the points for $\tau = 1250$ days, we constructed an index $\gamma$: the slope of the normalised symmetry function from the origin to the first bin divided by the slope from the origin to the last bin. A value $\gamma = 1$ indicates a perfect alignment of the first bin with the last. The index is presented in table 1 for the four different domains and three different resolutions of the pdf. For the recirculation

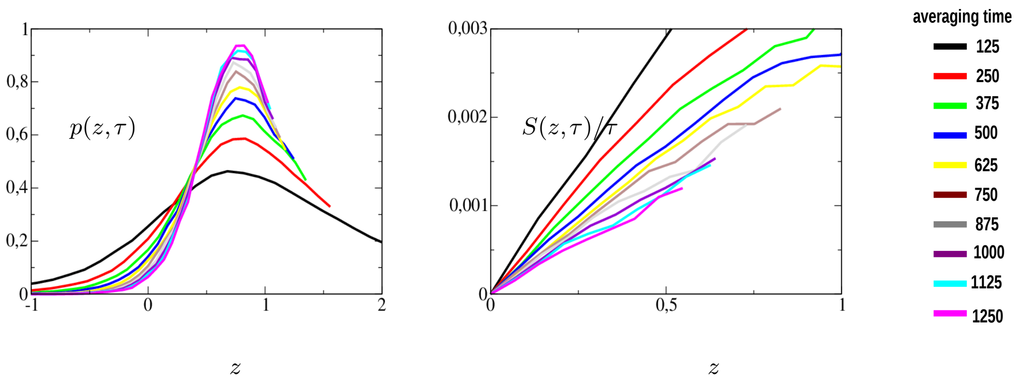


**Figure 1.** The pdf $p(z,\tau)$ (left) and the symmetry function normalised by the averaging time $S(z,\tau)/\tau$ (right) as a function of $z$ for different $\tau$ (see caption). The variable $\tau$ gives the length of the averaging interval in terms of observations done every 6 hours. Data are for case ASG, res $0.5^o$, 1993–2016, res 6h

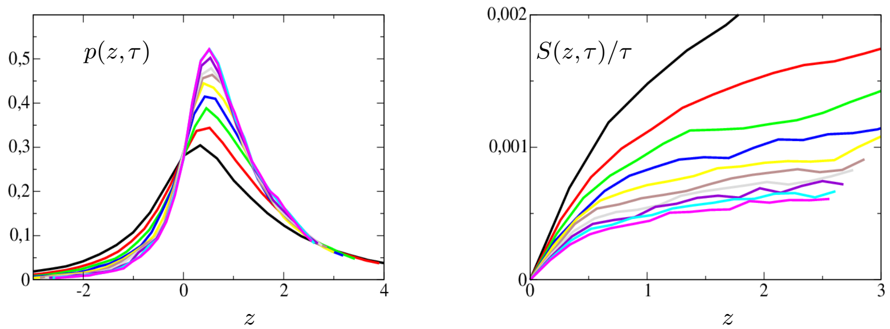


**Figure 2.** The pdf $p(z,\tau)$ (left) and the symmetry function normalised by the averaging time $S(z,\tau)/\tau$ (right) as a function of $z$ for different averaging times $\tau$ (see caption and legend of Fig. 1); data are from GSE, res $0.5^o$, 1993–2016, res 6h

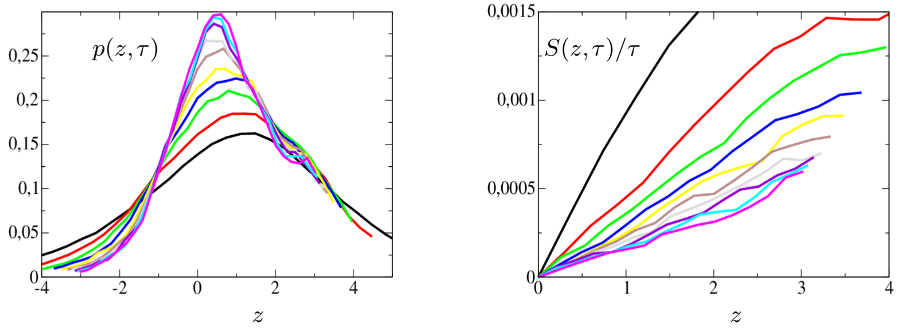


**Figure 3.** The pdf $p(z,\tau)$ (left) and the symmetry function normalised by the averaging time $S(z,\tau)/\tau$ (right) as a function of $z$ for different averaging times $\tau$ (see caption and legend of Fig. 1); data are for case PSG, res $0.5^o$, 1993–2016, res 6h

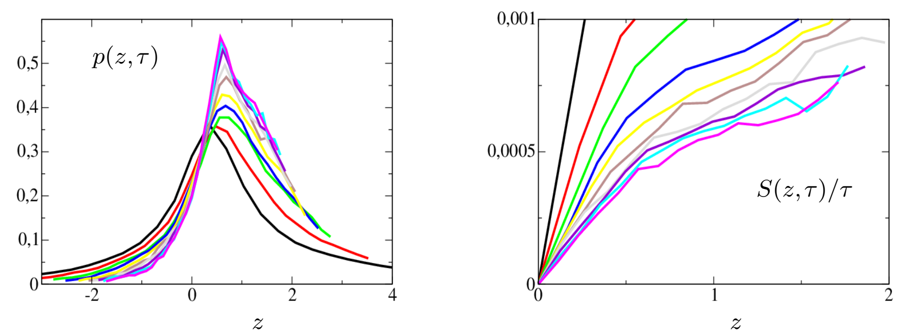


**Figure 4.** The pdf $p(z,\tau)$ (left) and the symmetry function normalised by the averaging time $S(z,\tau)/\tau$ (right) as a function of $z$ for different averaging times $\tau$ (see caption and legend of Fig. 1); data are for case KUE, res $0.5^o$, 1993–2016, res 6h

| exp. | ASG | GSE | PSG | KUE |
|---|---|---|---|---|
| $\gamma$ (21 bins) | 1.11 | 3.55 | 0.90 | 1.83 |
| $\gamma$ (31 bins) | 1.02 | 3.63 | 0.92 | 1.71 |
| $\gamma$ (41 bins) | 0.92 | 3.54 | 1.03 | 1.75 |

**Table 1.** Index $\gamma$ measuring the alignment of the normalised symmetry function for the four experiments and different resolutions of the pdfs (number of bins).

area of the subtropical gyre cases, the index varies around unity for the different bin sizes. It is significantly greater than unity in the Gulf Stream and the Kuroshio extension for all bin sizes considered.

We did not attempt to present error-bars in the figures and numbers in the tables, as uncertainties depend on the number of statistically independent events, that is the correlation time. In the case of air-sea interaction there are correlations due to the atmospheric dynamics (mostly synoptic), the ocean dynamics, the annual cycle, interannual variability and a climatic trend. How these processes contribute to the tails of the pdf's, to extreme events, is currently a hot topic in climate science (see *i.e.* Ragone et al. (2018)).

## 6 Discussion

We obtain clear evidence that a FT applies to data within the recirculation area of the subtropical gyre in the Atlantic and the Pacific Ocean. In these cases the FT can be used to estimate the occurrence of rare negative events from frequent positive events of the same magnitude for all averaging periods $\tau$ (measured in days). If the FT applies, the probability of the rare extreme negative events can be calculated from frequent positive events. Extreme negative events lead to strong transfer of energy to small-scale turbulence in the atmospheric and oceanic boundary layers, potentially causing strong mixing in the atmosphere and ocean. Extreme events are often key for the system in a variety of applications and are the focus of recent research in climate science (Ragone et al. (2018), Seneviratne et al. (2012)). As an example: in the Atlantic subtropical gyre (ASG) case the slope of the symmetry function is $S(z,\tau) = 2. \cdot 10^{-2} \tau z$, this means that an event of the magnitude $z = -1$ is $p = \exp(-2. \cdot 10^{-2}\tau)$ less likely than an event having the average value ($z = 1$) and an event of the magnitude $z = -2$ is $p = \exp(-4. \cdot 10^{-2}\tau)$ less likely than an event having twice the average value ($z = 2$). The variable $\tau$ gives the length of the averaging interval in terms of observations done every 6 hours, that is $\tau = 400$ corresponds to a period of 100 days. A FT represents a tool to obtain the rare negative events from frequent positive events for all averaging times $\tau > \tau_0 \approx 1$ year. and demonstrates that, to leading order, the probability of negative events vanishes exponentially with the averaging time.

The FT does not seem to apply in the highly non-linear Gulf Stream extension for $z \gtrapprox 0.3$ and Kuroshio extension $z \gtrapprox 0.5$. For these regions, the symmetry function follows a FT for small values of $z$, before the curve flattens. This resembles the behaviour found in the EFT (see section 2). Indeed, in these two cases (GSE & KUE) the tails of the pdf of $\mathcal{P}$ show pronounced super exponential tails and boundary values might be important leading to a behaviour predicted by an EFT. Nevertheless, a similar change of slope was also found using highly idealised models of air-sea interactions (discussed in Wirth (2019)), in which a friction term was added to the ocean. This suggests that an increased energy cascade, in the extension of boundary currents, might be responsible for the departure from a FT. When the scaling of the symmetry function flattens for higher power-input, the manifestation of a negative extreme event, versus a positive event of the same magnitude, becomes more likely.

During data analysis, we also found that a FT does not apply when islands or coastlines are present (not shown here). Departure from a FT for the power input to the ocean is found where horizontal dynamics dominates over the vertical ocean-atmosphere momentum exchanges. The influence of the horizontal transport of energy with respect to the injection of energy through the surface decreases with domain size considered, as the circumference of a domain grows linearly, whereas its surface growth is quadratic. Yet, determining the existence of a FT for larger ocean domains asks for more data, which is currently not available. Our results are purely empirical, a theory explaining why the power input follows a FT in some cases and not in others, is still missing.

Finally, we put the theory of FTs in the more general context of climate dynamics. A measurement, especially when coming from satellites always contains some averaging in space and time. A FT, when it applies, will help to relate averages over varying periods and is a powerful tool to guide the up and down-scaling of observational data in time and obtain the statistical information on shorter and longer time scales, which are not explicitly observed. More precisely, when the pdf of the power supply, and therefore also the symmetry function is known form observations for given averaging times the symmetry function can be calculated for shorter and larger averaging times and constrains “half” of the pdf. This is useful in down-scaling and the construction of statistical parameterizations of not directly observed dynamics over shorter time scales. On the other hand, the information can be useful for developing models for the persistence of events over large time-scales not yet observed. A FT can help to decide if the persistence in time of a phenomena is within the likeliness of the statistically stationary dynamics or due to external influences. Furthermore, when data from observations follow (or not) a FT, model data should do likewise. As such, the FT becomes a tool of investigating the fidelity of models.

We conclude by looking at our results from the stand point of dynamical systems. Statistical mechanics of systems in equilibrium are described by the Boltzmann distribution, which is completely determined by the temperature. In non-equilibrium statistical mechanics no such universal distribution is known (see i.e. Derrida (2007), Touchette (2009) and Frisch (1995)), but some quantities in some processes seem to follow a FT which constraints the pdf and might indicate some universality. The mechanical power-input to the ocean by air-sea interactions, as a forced and dissipative dynamical system, may thus belong to a class of particular non-equilibrium systems exhibiting a FT symmetry property and offer guidance for climate studies.

*Author contributions.* AW has performed the coding, writing was shared by both authors

*Competing interests.* No competing interest

*Data availability.* Data is available under: http://marine.copernicus.eu/services-portfolio/access-to-products/ and http://marine.copernicus.eu/services-portfolio)

*Acknowledgements.* This work was funded by Labex OASUG@2020 (Investissement d'avenir - ANR10 LABX56). These data were provided by the Centre de Recherche et d Exploitation Satellitaire (CERSAT), at IFREMER, Plouzane (France) and CMEMS. Part of this work was performed when AW visited LOPS, Brest. We are grateful to Abderrahim Bentamy for explanations concerning the data and Mickael Accensi and Jean-Fancois Piolle for help with the data analysis.

*Data availability.* Data is available under: http://marine.copernicus.eu/services-portfolio/access-to-products/ and http://marine.copernicus.eu/services-portfolio)

## References


Bentamy, A., Grodsky, S. A., Elyouncha, A., Chapron, B., and Desbiolles, F.: Homogenization of scatterometer wind retrievals, International Journal of Climatology, 37, 870–889, 2017.

Boffetta, G. and Ecke, R. E.: Two-dimensional turbulence, Annual Review of Fluid Mechanics, 44, 427–451, 2012.

Bonjean, F. and Lagerloef, G. S.: Diagnostic model and analysis of the surface currents in the tropical Pacific Ocean, Journal of Physical Oceanography, 32, 2938–2954, 2002.

Bye, J. A.: Large-scale momentum exchange in the coupled atmosphere-ocean, in: Elsevier oceanography series, vol. 40, pp. 51–61, Elsevier, 1985.

Ciliberto, S., Garnier, N., Hernandez, S., Lacpatia, C., Pinton, J.-F., and Chavarria, G. R.: Experimental test of the Gallavotti–Cohen fluctuation theorem in turbulent flows, Physica A: Statistical Mechanics and its Applications, 340, 240–250, 2004.

Csanady, G. T.: Air-sea interaction: laws and mechanisms, Cambridge University Press, 2001.

Derrida, B.: Non-equilibrium steady states: fluctuations and large deviations of the density and of the current, Journal of Statistical Mechanics: Theory and Experiment, 2007, P07 023, 2007.

Desbiolles, F., Bentamy, A., Blanke, B., Roy, C., Mestas-Nuñez, A. M., Grodsky, S. A., Herbette, S., Cambon, G., and Maes, C.: Two decades [1992–2012] of surface wind analyses based on satellite scatterometer observations, Journal of Marine Systems, 168, 38–56, 2017.

Duhaut, T. H. and Straub, D. N.: Wind stress dependence on ocean surface velocity: Implications for mechanical energy input to ocean circulation, Journal of physical oceanography, 36, 202–211, 2006.

Einstein, A.: Zur theorie der Brownschen Bewegung, Annalen der Physik, 324, 371–381, 1906.

Einstein, A.: Investigations on the Theory of the Brownian Movement, Courier Corporation, 1956.

Evans, D. J., Cohen, E. G., and Morriss, G. P.: Probability of second law violations in shearing steady states, Physical review letters, 71, 2401, 1993.

Fairall, C. W., Bradley, E. F., Rogers, D. P., Edson, J. B., and Young, G. S.: Bulk parameterization of air-sea fluxes for tropical ocean-global atmosphere coupled-ocean atmosphere response experiment, Journal of Geophysical Research: Oceans, 101, 3747–3764, 1996.

Farago, J.: Injected power fluctuations in Langevin equation, Journal of statistical physics, 107, 781–803, 2002.

Ferrari, R. and Wunsch, C.: Ocean circulation kinetic energy: Reservoirs, sources, and sinks, Annual Review of Fluid Mechanics, 41, 2009.

Frisch, U.: Turbulence: the legacy of AN Kolmogorov, Cambridge university press, 1995.

Gallavotti, G. and Cohen, E. G.: Dynamical ensembles in nonequilibrium statistical mechanics, Physical Review Letters, 74, 2694, 1995a.

Gallavotti, G. and Cohen, E. G. D.: Dynamical ensembles in stationary states, Journal of Statistical Physics, 80, 931–970, 1995b.

Gallavotti, G. and Lucarini, V.: Equivalence of non-equilibrium ensembles and representation of friction in turbulent flows: the Lorenz 96 model, Journal of Statistical Physics, 156, 1027–1065, 2014.

Perrin, J.: Atomes (Les), CNRS Editions, Paris, ISBN: 978-2-271-08260-2, 2014.

Ragone, F., Wouters, J., and Bouchet, F.: Computation of extreme heat waves in climate models using a large deviation algorithm, Proceedings of the National Academy of Sciences, 115, 24–29, 2018.

Rákos, A. and Harris, R.: On the range of validity of the fluctuation theorem for stochastic Markovian dynamics, Journal of Statistical Mechanics: Theory and Experiment, 2008, P05 005, 2008.

Renault, L., McWilliams, J. C., and Masson, S.: Satellite Observations of Imprint of Oceanic Current on Wind Stress by Air-Sea Coupling, Scientific reports, 7, 17 747, 2017.

Rimac, A., Storch, J.-S. v., and Eden, C.: The total energy flux leaving the ocean's mixed layer, Journal of Physical Oceanography, 46, 1885–1900, 2016.

Rio, M.-H., Mulet, S., and Picot, N.: Beyond GOCE for the ocean circulation estimate: Synergetic use of altimetry, gravimetry, and in situ data provides new insight into geostrophic and Ekman currents, Geophysical Research Letters, 41, 8918–8925, 2014.

Seifert, U.: Stochastic thermodynamics, fluctuation theorems and molecular machines, Reports on progress in physics, 75, 126 001, 2012.

Seneviratne, S. I., Nicholls, N., Easterling, D., Goodess, C. M., Kanae, S., Kossin, J., Luo, Y., Marengo, J., Mc Innes, K., Rahimi, M., et al.: Changes in climate extremes and their impacts on the natural physical environment, in: Managing the risks of extreme events and disasters to advance climate change adaptation: Special report of the Intergovernmental Panel on Climate Change, pp. 109–230, Cambridge University Press, 2012.

Shang, X.-D., Tong, P., and Xia, K.-Q.: Test of steady-state fluctuation theorem in turbulent Rayleigh-Bénard convection, Physical Review E, 72, 015 301, 2005.

Stocker, T. F., Qin, D., Plattner, G., Tignor, M., Allen, S., Boschung, J., Nauels, A., Xia, Y., Bex, V., and Midgley, P.: Climate change 2013: the physical science basis. Intergovernmental panel on climate change, working group I contribution to the IPCC fifth assessment report (AR5), New York, 2013.

Sudre, J., Maes, C., and Garçon, V.: On the global estimates of geostrophic and Ekman surface currents, Limnology and Oceanography: Fluids and Environments, 3, 1–20, 2013.

Touchette, H.: The large deviation approach to statistical mechanics, Physics Reports, 478, 1–69, 2009.

Van Zon, R. and Cohen, E.: Extended heat-fluctuation theorems for a system with deterministic and stochastic forces, Physical Review E, 69, 056 121, 2004.

Veron, F.: Ocean spray, Annual Review of Fluid Mechanics, 47, 507–538, 2015.

Wirth, A.: A Fluctuation–Dissipation Relation for the Ocean Subject to Turbulent Atmospheric Forcing, Journal of Physical Oceanography, 48, 831–843, 2018.

Wirth, A.: On fluctuating momentum exchange in idealised models of air sea interaction, Nonlinear Processes in Geophysics, 26, 457–477, 2019.

Wunsch, C.: The work done by the wind on the oceanic general circulation, Journal of Physical Oceanography, 28, 2332–2340, 1998.

Zhai, X., Johnson, H. L., Marshall, D. P., and Wunsch, C.: On the wind power input to the ocean general circulation, Journal of Physical Oceanography, 42, 1357–1365, 2012.