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Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "Application of Lévy Processes in Modelling (Geodetic) Time Series With Mixed Spectra" *by* Jean-Philippe Montillet et al.

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Dear Reviewer,

Thank you for your comments. We will emphasize some key points that perhaps have not been sufficiently underlined in the manuscript (and will be further emphasized in the next revised version).

1- "The mathematical model is really messily written, and it is hard to understand it.Take all the definitions from the appendix, and put them in the text, and remove all the speculative/descriptive material."

[R] Here, we have rewritten the model and the explanation in order to avoid the same issue as in the previous version. We want to emphasize that we are working on residual time series which are defined in Appendix A:

"to obtain the residual time series x(t) (t the epoch), we subtract the functional model "s0(t)" to the GNSS observations "s(t)".The functional model of the geophysical signals is based on the polynomial trigonometric method (Li et al. , 2000; Williams ,2003; Tregoning and Watson , 2009) . . . "

In addition, we have added a comprehensive discussion on how to obtain the residual time series. It includes the ML estimator (using Hector) which is used to jointly estimate the stochastic noise model and the geophysical parameters. The geophysical parameters are described with the equation A.1 (Appendix). Now, in the residual time series described in Eq. 1, we state that "s_r" is the residual geophysical signal. Therefore, it includes 1/ the residual signal from the estimation of the model in equation A.1 2/ the missing signals which have been omitted in the geophysical models such as small short-time duration transient signals, small amplitude exponential relaxation, small offsets (undetectable by eyes) . . . We must emphasize that this mismodelled geophysical signals (especially the small offsets) can be modelled/assimilated as random-walk in the stochastic noise model. That is one of the justifications behind using three stochastic processes (instead of 2) in modelling the stochastic noise component of the GNSS time series (see L. 135 ".. It is similar to the approach used in previous works (Langbein , 2008; Davis et al. , 2012;Langbein and Svarc , 2019; He et al. , 2019) looking at the presence of a random-walk component in the stochastic model, hence adding a third covariance matrix in Eq. (3). . . ."). This section will be revised accordingly with most of the Appendix A moved to section 2.1. Note that to formulate the model of the residual time series, we use the model of the GNSS observations defined in (Montillet and Bos, 2019: Chapter 1 and 2). Also the covariance matrix defined in Eq. (3) is also based on (Montillet and Bos, 2019: Chapter 2). That will be clarified in the next version of the article.

All the material is here to give the background of this work, and justification of the assumptions (especially the consideration of the third stochastic process). It is worth mentioning that this material has been substantively revised and superfluous discussion/descriptions have been deleted from the previous submitted article.

2- " After you have done the basic model, you can construct an a posteriori distribution of the unknown parameters, and then the target is to sample these parameters with MCMC methods or obtain optimisation-based MAP/ML estimators. Define explicitly your estimators with respect to posterior distribution. Of course you can use some other constructions as well, but define your estimators explicitly. The N-step method is totally heuristic and should not be included in the manuscript. Please come up with some mathematics/stats-based parameter estimation algorithm."

[R] The goal of this work is to understand/investigate the use of three stochastic processes instead of 2. To do that, we use the Levy processes which can characterise this third stochastic process. Table 1 displays the three cases (Gaussian, Fractional and Stable Levy) together with the assumptions. Basically, there is nothing heuristic in the choice of these 3 cases and the statistical assumptions. We have comprehensively documented past works which lead to these assumptions in Section 2.3.

The justification behind the N-step method is that we want to investigate the variations in the stochastic noise models in order to classify the third stochastic process. Thus, we make the assumptions that the properties of the noise can vary with the length of the time series, because the coloured noise is not stationary. We cannot vary the time series with a length of more than 1 year, because this value is a trade-off between too short and too long, meaning that with only a few months the noise property remains stationary, and at the opposite with longer time we risk including more geophysical signals due to events affecting the stations (Earthquake, land subsidence ...). The N-step method is carried out using Hector (based on ML) in order to estimate jointly the stochastic noise parameters and the geophysical signals. However, the characterisation of the third stochastic process is not only based on the results with the N-step

method, but also by fitting the residual time series with the Levy alpha stable distribution, and testing which model ARMA vs FARIMA fits the residuals. Note that the fitting of the Levy alpha stable distribution is done by using a ML approach (see line 220). The fitting of the ARMA and FARIMA models follow the method (based on ML) developed in Montillet and Yu (2015). Thus, the characterisation of this third stochastic process is not heuristic and it is based on testing the residual time series where the N-step method is the backbone of our methodology.

Thank you for the minor comments, it will help to improve the manuscript further. All the necessary corrections will be included in the next draft.

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