Nonlin. Processes Geophys. Discuss.,
https://doi.org/10.5194/npg-2020-23-RC2, 2020

Nonlinear Processes in Geophysics
Discussions

Interactive comment on "Application of Lévy Processes in Modelling (Geodetic) Time Series With Mixed Spectra" *by* Jean-Philippe Montillet et al.

Reik Donner (Referee)

reik.donner@pik-potsdam.de

Received and published: 20 September 2020

The authors discuss the stochastic model based description of time series with mixed spectra motivated by the typical properties of geodetic (GNSS) time series. They consider this problem in terms of an additive model with two or three components representing different types of stochastic processes, among one is taken from the family of Lévy processes in three possible flavors. The authors describe a procedure for the step-wise iterative estimation of the associated process parameters and apply their approach to both, artificial data and three real-world GNSS series.

While I find that the overall topic is relevant and appropriate for the readership of Non-

linear Processes in Geophysics, my impression is that the specific background of the models considered here needs to be more clearly presented for a broader audience. Generally, the overall model structure should be motivated in a more systematic and more transparent way than done in the present version of the manuscript. Some specific questions I came across when working through this discussion paper, which I suggest the authors to briefly address in the process of revising especially Sections 1 and 2.1 of the manuscript, include the following:

Can you explain a bit more systematically how (and why) different types of non-stationarity are associated to the different components of your stochastic model? While I think that this is relatively clear for "deterministic" (monotonic) trends and seasonality, abrupt offsets due to jumps in the series could either be associated with the functional (deterministic offsets due to seismic events or instrumentation changes) or the stochastic part (in the latter case, I would expect that they can be considered as Lévy flights, but I am not sure if such effects commonly occur in GNSS series). Moreover, I am wondering about non-stationarity beyond just the mean, i.e., possible time-dependence in the variance (or even higher-order distributional characteristics), which I believe cannot be captured by the present model setup but would require either some multiplicative component (e.g., functional model times noise of a certain type) or a stochastic model component beyond ARMA/FARIMA that inherets the property of conditional heteroscedasticity (e.g., some ARCH/GARCH type model). Again: I do neither claim that those characteristics are necessarily typical for GNSS series (but the manuscript title points to a wider class of time series with mixed spectra where this could arise), nor do request the authors to provide a solution for any possible type of situation that could arise. What I however would appreciate to see more transparently is the list of assumptions that underlie the discussed class of stochastic models, and some brief discussion on if and how possible phenomenological findings (like heteroscedasticity of the noise component, intermittency,. . .) are included in the general model structure studied in the present work.

You mention the use of the Hector software at several, in my opinion not necessarily relevant places. Since you do not provide any specific information that is unique to this software, I recommend mentioning this only in the acknowledgements and removing corresponding unnecessary statements elsewhere (e.g. ll. 200-201, 390, 402-403, 417-418).

Regarding the analysis of the modelling results, I am not convinced that it makes sense statistically to use correlations as a measure for the goodness-of-fit/matching between an empirical and a theoretical distribution. For the purpose of this study (Tab. 3/4 and associated text), some two-sample statistical test like Kolmogorov-Smirnov or Anderson-Darling (probably rather the latter since heavy-tailed distributions are included) appear more reasonable, while it – one the one hand - is unclear how the correlations have been computed and – on the other hand – the mean correlation values are trivially very high while the provided uncertainty margins (Tab. 3/4) clearly exceed the possible range of correlation values (bounded from above by one) and are therefore meaningless.

Regarding the general presentation, the manuscript is well readable but contains a quite relevant number of minor grammatical errors, mainly confusion of singular and plural forms, missing or wrong prepositions, and other minor things that I do not wish to list here explicitly. Thorough proofreading during the revision process is recommended.

Some more specific suggestions on helpful additions and minor modifications to the text are listed below:

Ll.19/20: “ground motion” sounds to me more appropriate than “soil motion”, unless you explicitly aim to focus on just the uppermost soil layer of the ground

L.75: I don’t quite see that it is relevant to mention the length of the real-world series at this point – to me this rather fits in the results section.

Ll.78-80: Can you explain the relationship between slowly varying mean of colored

noise and the Gauss-Markov assumption a bit more explicitly?

Ll.89-90 or below: Maybe you could add a brief comment on the relevance of H for processes with infinite variance (e.g. Cauchy-class processes)?

Ll.101-107: In the context of this brief discussion of FARIMA models, you might also briefly recall the relationship between the fractional model order d and Hurst exponent H.

L.114: The definition includes a variable k that I don’t find appearing anywhere before.

Ll.154-155: This sentence would benefit for some further explanation for non-specialists.

L.171: In which sense do you consider colored noise to be non-stationary?

Figures 1 and 2: emphasize on the different ranges on the y axes somewhere in the figure caption

L.243: The list of values given in the text is inconsistent with that shown in the figure.

L.246: I would not speak of “earlier” here due to the low number of data points in the figure, but rather refer to the overall values of the variance.

L.262: What do you mean by “driving parameters”?

Ll.271-274: I understand this as that heavy tails in the series can either be attributed to the residuals or to the third component. Can such an attribution be actually unique?

L.325 and several times later: There are quite a few cases of equations spanning over different lines with duplicate left-hand sides indicating that those are in fact different equations even though they are not.

L.348: “colored noise can generate long-memory processes” seems a bit odd to me; rather, colored noise commonly constitutes a long-memory process

L.349: Doesn’t the mentioned varying amplitude of the colored noise rather call for

multiplicative/heteroscedastic models? (See main comment above.)

L.373: “long-term correlations” (remove “processes”)

You should make use of the full functionality of the LATEX template; e.g. use \appendix followed by \section to generate individual appendices. Note that the references must not appear as an appendix. Moreover, also the use of \citet versus \citep in the text could be improved.

Ll.417-430: Please emphasize that the expressions in Z denote composite operators in terms of the backshift operator Z applied to x(t) and b(t), respectively.

L.434: instead of the term “hyperbolic”, “algebraic” seems more commonly used

L.436-439: The link to fBm is a bit unclear here. Since you consider stationary processes (FARIMA class), it would be more reasonable in my opinion to link this to fractional Gaussian noise (fGn) the aggregation of which than provides sample paths for fBm.

L.451: extend the equation by an expression including f_t(u) as the latter is used in the text below

Appendix F: Did you consider putting those materials into an Electronic Supplementary Material instead of an Appendix in the main paper?

Interactive comment on Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2020-23, 2020.