

Letter to the Reviewer

Dear Editor, Dear Reviewer,

Thank you for the comments. The manuscript has been modified following your suggestions. Please, find some details to your major comments. Note that the minor points are corrected in blue in the text, in addition of the added sentences.

Thank you for the additional improvements.

Major comments raised by the reviewer

1/ ... “What I however should request is adding some cautionary remarks regarding the feasibility of using correlations for the purpose for which they are used here.”

R: We agree with the reviewer as already mentioned in the previous round where R2 suggested to add the Anderson-Darling Test for the same reason. Therefore, we add (p. 11 l. 286-288):

“However, we acknowledge that the (Pearson) correlation coefficient could be biased due to inherent normalized sum constraint between the distributions estimated directly from the data. Therefore, in this instance the AD test should be more reliable.”

2/ “.However, in discussing the outcomes of this test, the authors frequently speak about an “accepted null hypothesis”, which is a heavily flawed phrasing.”

R: This term is loosely used in documentation of statistical tests described in various software (e.g. MATLAB). We agree with the reviewer and changed “accepted null hypothesis” by “not rejected null-hypothesis” as suggested in captions of Tab. 3, 4 and A1, text in ll. 279, 287, 329

Minor comments:

1. Separating main text, appendices and supplementary information has greatly improved the readability of the manuscript. Given that a concise paper is most appealing for readers, I would even suggest moving the additional Tab. A1 and Figs. A1-A4 to the supplementary material, since they do not add essential information to the manuscript and in fact just blow up its size (thereby also increasing the page charges to be calculated by the publisher). My recommendation would be keeping only Appendix A in the main document, possibly not even using a numbering then and referring to its content in the main body simply to as “see Appendix” or similar.

R: We have kept only appendix A in the main document. Appendix B is now Section E in the supplementary material. The reference in the main text mentions only “Appendix”.

2. In order to improve the internal referencing of the material, I strongly recommend the authors to revise references to the supplementary material by emphasizing explicitly the part of that document that is referred to, e.g., “see Supplementary Material Sect. A” or “Supplementary Material Fig. S1/Tab. S1” if following my suggestions under the previous point 1.

R : Done

3. Please use consistently the term “Gaussian Lévy” (or “Lévy Gaussian”, but I would tend to prefer the former) instead of mixing both terms.

R : We homogenized the manuscript following the remark (Gaussian Lévy).

4. The block of equations on pp.7-8 misses a few equation numbers, including such that are later referred to in the manuscript (e.g., Eq. (7)). This should be clarified.

R : We have added one number for each block to avoid unnecessary notation. The manuscript is revised accordingly.

5. I experienced some personal confusion with the authors’ use of the term “epoch”. Would you mind explaining its meaning in the context of this work precisely along with its first occurrence (p.3, below Eq. (1))?

R : « epoch » means the time associated with an observation. We have clarified the meaning by replacing “L epochs” with “L observations”, and also using “time step” in the definition of the multivariate noise model below eq. (2).

6. Ll.19-20: As it is written now, one may think that Flicker noise and white noise are examples of band-pass noise, which I would clearly object. Please rephrase.

R : The sentence is rephrased – see in the introduction “To name a few ...”

7. L.177: “nonstationary signal (around the mean)” – do you mean a signal with a non-stationary mean or (as I suppose) a signal with a stationary (zero?) mean and nonstationary variations about this mean?

R : The non-stationary properties of the coloured noise involve that 1/ the mean value varies with time, and 2/ there are variations around the mean. In other words, depending on the power-law exponent, the phenomena of aggregation or intermittency can exist. However, due to the small amplitude of the coloured noise in the GNSS time series, we “do” exclude large variations around the mean, therefore we use “non-stationary (around the mean)”. This is explained in the supplementary material Section A – at the end. We have added the reference in the main text.

8. L.215: “see also the discussion on the Hector Software (Bos et al., 2013) in the Supplementary Material Sect. A).

R : Done

9. L.231: p and q are NOT lags, but the AR and MA model orders of the considered model.

R : it is replaced: lags => model orders

10. L.372: “uncorrelated assumption” => “assumption of uncorrelated components”

R : Done

11. Eq. (A.2), first line: the meaning of the brackets is not clear If placed as now, they are not necessary.

R : We are not sure what the reviewer means. This is the same definition as Montillet and Bos, 2019 – Chapter 2.

In Eq. (A.3), the brackets however appear necessary, but you should use `\left(` and `\right)` to enforce brackets of a proper size for the enclosed content.

R : It is revised – see Eq. (A.3) and (A.4)

12. Supplementary Material Section A: The text in ll.16-22 is a literal repetition of ll.84-90 of the main manuscript. Just shorten this to one sentence with reference to the corresponding Eq. (3).

R : the paragraph is shortened accordingly

R: Please, note that all the following corrections are written in blue in the manuscript when it is appropriate

- L.8: “The fractional Lévy process...”
- L.9: “The stable process...”
- L.10: “Therefore, it implies...” – what does “it” refer to? **R : the model**
- L.15: “phenomena”
- L.15: “earthquakes”
- Ll.16-17: “These time series provide the estimated daily positions of...”
- L.19: “describe”
- L.20: “cyclic”
- L.20: “referred to”
- Ll.27-28: “To name a few, previous choices include the...”
- L.39: “distributions”
- L.40: “from a Gaussian distribution”
- L.47: “to the residual...”

- L.77: “a multivariate continuous-time stochastic process”
- L.80: “transposition operator”
- L.81: “ $K \in [0,2]$ ” **R: “0” is excluded as it is the case of white noise, which is defined by the identity matrix in the first term.**

- L.88: “represents”
- L.89: “model for x” **R: better “also called X in Eq ...”**
- L.93: “correctly” might better read “properly” (there is not ground truth...)
- L.98: “bursts”
- L.107: “a random walk”
- L.109: use \citet instead of \citep for reference Eke et al.(2002)
- L.114: “based on different”
- L.123: close bracket not after “2018”, but after “1.6” **R: we added [..]**
- L.127: “where sign(...)”
- L.128: “sets”
- L.132: use \citet instead of \citep for reference Weron et al. (2005)
- L.133: “with H being”
- L.140: “similar approach as used”
- L.144: “Gaussian Lévy”
- L.146: “This is the special case”
- L.152: “ratio between the amplitudes of the coloured and white noise components determines...”
- L.153: “most suitable among the FARIMA...”
- L.157: “modelled”
- L.167: “as a random walk”
- L.220: “Gaussian Lévy”
- L.233: “selection (ARMA, FARIMA)”
- L.234: “(e.g., Panahi (2016))”
- L.238: “than that of the white noise... should be preferred de facto....”
- L.245: “power-law noise”
- L.250: “between 1 and 3” **R: we stick with the maths’ notations.**

• L1.254-256: don’t put physical units in math mode (no italics). **R: That was asked by previous reviewer**

- L1.257, 269: “in scenario C”
- L.272: “the fits”
- L.273: “over 50 simulations”
- L.274: “the average correlation”
- L.275: I suggest not to capitalize “normal” (also in what follows from here) **-R: we want to be consistent with the whole text**
- L.282: do not capitalized “intermediate” **-R: we want to be consistent with the tables**

- L.282: “However, scenario C....”
- L.283: “introduce”
- L.284: “performs”
- L.286: “displayed”
- L.287: “...in Table 2 give the probability...”
- L.297: “of the three GNSS stations DRAO,...” (note that I would also suggest not to put the station abbreviations in italics, but this is up to the authors...)
- L.309: “dependence on the...”

- L.320: “explained by...”
- L.321: use \citep instead of \citet for reference to He et al. (2019)
- L.323: “series of ASCO” **R: We do not see the issue**
- L.326: “of the noise that is much larger...”
- L.327: “is approximately the same”
- Ll.332,333: “Up coordinate”
- L.341: “develop closed-form expressions for the mean...”
- L.344: “equal to 1”
- Fig. 3, caption: “function of the residual” (wo times, also in the captions of the other similar figures currently placed in the appendix)
- L.351: “intersect” => “intercept”?
- L.356: remove “Note that” (not necessary) and “if” (grammatically useless)
- L.358: the inline equation seems to miss brackets around the content that is summed up
- L.368: “where Cross...”
- L.371: “with the residual signals”
- L.392: remove “exhibiting”
- L.393: “to using”
- L.398: “an N-step”
- L.417: “in the presence of remaining offsets can be written as”
- L.431: remove “note that”
- SI, 1.3: “from the GNSS...”
- SI, 1.8: what do you mean by “nominated time”? **R : changed with “ at time t..”**
- SI, 1.14: remove “Furthermore”
- SI, 1.33: remove “Note that”
- SI, 1.53: “aggregation of fractional...”
- SI, 1.69: remove “Note that”
- SI, 1.73: “The representation of fLsm in Eq. (C.7) is similar...”
- SI, ll.101-102: The sum... can be written as...” – this sentence is incomplete. **R: the sentence is deleted**