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energy

The Impact of Entrained Air on Wave Dissipation

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Abstract. We make a physical-mathematical analysis of the implications that the presence of a large number of tiny bubbles may have on the thin upper layer of the sea. In our oceanographic example the bubbles are due to an intense rain on an otherwise non-stormy surface; if stormy, other processes would take the role. For the direct effect we have analyzed, the implications are estimated non-significant when compared to other processes of the ocean. However, we hint to the possibility that our analysis may be useful in other areas of research or practical application.

1 Introduction

needs to be more specific

among many other processes.

We describe a methodological approach to analyze the implications of the presence of a large number of gas bubbles on the dynamics of an ~~otherwise~~ liquid s~~ubstance.~~ ~~Granted the general approach, driven by our oceanographic background we focus our attention~~ on the ocean. Air bubbles are ubiquitous in the ocean and are known to be generated by intense stirring, wave breaking, ~~as well as a by-product of organic processes and human activities~~. The residence time of small air bubbles can be longer than the wave time scale because viscous effects will be comparable to buoyant forces on the bubbles. ~~However, the residence time of larger bubbles may be long as well due to intense stirring~~ (see Tkalich and Chan ~~(2002);~~ Restrepo et al. (2015); Moghimi et al. (2018)). ~~For a specific application~~ we zoom on ~~the presence of~~ air bubbles in the thin upper layer of the ocean. This ~~layer is crucial for the interaction between ocean and atmosphere, and it is also extremely complicated because of the large number of there acting processes at a very large range of different scales.~~ ~~Again driven by our professional background, as a specific example~~ we ~~then~~ focus on the attenuation of the shortest (capillary) ~~water~~ waves. ~~Tiny as they are in the immensity of the ocean~~, these waves are crucial for the quantification of all the exchanges between ocean and atmosphere, in particular for the momentum and energy wind transfers to the ocean as input to wind waves, currents and turbulence. ~~Not obvious to the casual observer, but highly effective in practice~~, one of the main reasons for the presence of air bubbles in the upper ocean layer is rain. ~~To the observer~~, the macroscopic effect of rain, if strong enough, is the smoothing of the very short (capillary) waves that characterize the surface in stormy conditions. ~~These ripples are easily seen when wind is blowing also~~

redunda

the wave time scale is from seconds to days /!\

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~~on a small pond or puddle of water.~~ ~~Rain is known to smooth these ripples, and~~ the matter became important with the launch of the first satellite born scatterometers (see Weissmann and Bourassa (2011)). The lack of ripples on the surface strongly affects the return signal to be measured on board, thence the interest in the matter. The associated change of sea surface roughness modifies also the interaction of wind with the sea ~~(the wind tends 'to slide' on the surface)~~, strongly reducing the energy and momentum input by atmosphere to the ocean. (See Cavaleri and L.Bertotti (2018) and Cavaleri et al. (2018) in this respect). ~~This has the macroscopic effect of strongly reducing the number of breakers, so the wrong mariners' interpretation that 'rain calms the sea.' As a matter of fact long waves are hardly affected by the direct impact of rain. Indeed (see, among others, Le Méhauté and Khangaonkar (1990) and Peirson et al. (2013)), extremely large rain rates are required to get a measurable attenuation also for short waves in the short distance of a suitably prepared wave tank.~~

In this paper we focus on a more subtle effect of rain that happens just below the surface, in the upper thin layer of the sea. With a strong dependence on the size, hence falling speed, of the rain drops, small air bubbles are created when each drop reaches the surface. We analyze how the characteristics of this upper layer are modified by the presence of bubbles. One of the parameters to be estimated is $2\nu_e$, the dissipation coefficient in the wave damping term (of the form $\exp[-2\nu_e k^2 t]$), where t is time, and k the wavenumber of the wave. Consistent with classical wave dynamics theory, we will connote $2\nu_e$ as the *wave dissipation*. ~~We specify at once that~~ we do not consider intense oceanic conditions, particularly in the sense of strong wind and breakers at the surface. These breakers have been amply studied (see Deike et al. (2017) and Deike et al. (2016)). However, in this case the bubble size distribution is very wide, with many large bubbles inducing turbulence (on top of the one due to wave orbital motion (see Babanin (2006)), decaying into a larger number of smaller ones, and producing localized white surface patches, used to detect breakers with remote vision or sensing (refer to Buscombe and Carini (2019) and Hwang et al. (2008)).

? Deike or his paper?

Our target has a reduced scale and physical impact. We consider the case of rain on a non-stormy sea, in practice either swell, or calm sea, or under a limited wind speed. Our analysis becomes meaningful, at least for sea waves, when the rain is not composed mainly of large drops. ~~Were this the case, at least for the aspect of our analysis dealing with the attenuation of surface ripples, the kinematic, hence mechanical, effect of the falling drops would be more significant of the physical aspect we consider.~~ ~~In practice~~ for the analysis ~~of our example the image~~ we have ~~is of~~ an intense rain with not too large, hence many, drops impinging on a non-stormy sea surface. Just to frame the order of magnitude, under an intense rain storm, *e.g.*, 100 mm h^{-1}, ~~the rainwater flux is about 3×10^{-6} m^3 m^{-2} s^{-1}~~. Assuming 4 mm diameter rain drops (volume 3×10^{-8}m^3), this translates into 100 drops per m^2 per second ~~(*i.e.*, 1 per 10 × 10 cm^2 s^{-1}).~~ In Woolf (2001) it is suggested that under these conditions air injected into the sea can be a few cm^3 m^{-2} s^{-1}. We make a physical-mathematical analysis of the ensuing processes. As expected, compared to other wave attenuation processes, the rain induced bubbles have a minor effect on the surface waves. ~~However, although stimulated by an oceanographic process, the general aspect of the proposed physical and mathematical approach could be used in other areas of science or practical application. Without a specific analysis and with just some thoughts, examples could be the physics of sea surface with buoyant organisms or droplets of oil, the free surface dynamics of fizzy liquids and free surfaces populated by foam, or, mutatis mutandis, the injection of fuel in a combustion chamber.~~

too speculative

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With this background the paper is structured as follows. In Section 2, we propose a model that suggests how the entrainment of air leads to an increase of the effective viscosity of the upper ocean. We then use results from ocean acoustics (see Oguz and Prosperetti (1990), Prosperetti and Oguz (1993)) to compute the distribution of air bubbles for a given rain rate (Section 3). How changes of effective viscosity affect gravity waves is taken up in Section 4. It is found that under vigorous rains the air concentration due to air bubbles near the surface is capable of producing a damping effect akin to (idealized) contamination of a free surface. Discussion of the results and conclusions and possible extensions of the present work appear in Sections 5 and 6.

2 The Dynamical Approach And The Presence Of Bubbles

Homogeneity can only be assumed in the upper layer were bubbles reside. This implies the need of a two layers model.

To determine the impact of air bubbles on the thin upper level of the ocean, we perform homogenization on the Navier-Stokes equations. Homogeneization is a well-established technique in modeling transport in complex media endowed with statistical homogeneity in the media, as viewed at large scales (see Babuska (1976); Bensousan et al. (1978); Cioranescu and Donato (1999)). Einstein's PhD thesis, derived expressions for enhanced diffusion based upon averaging the small scales (see Einstein (1906)). In many circumstances there is a nuanced difference in these two approaches. The homogeneization averaging process is performed on the dynamics, whereas in the Einstein averaging it is performed on the material itself. We assume that air

scale?

bubbles are distributed uniformly in the transverse direction, within the upper ocean. For our specific example we explore how the presence of bubbles affects the effective viscosity of the fluid averaged over a cell Ω, of size ℓ^3 (sub-wave scale), over which the distributions of the density and viscosity of the combined water and air bubble mixture are statistically stationary. The typical sub-wave speed is u_Ω. The ratio of the bubble radii to the averaging ℓ, defines for us a small parameter $\epsilon \ll 1$.

Velocity and position are denoted by $\mathbf{u} = (u, v, w)$, $\mathbf{r} = (x, y, z)$, respectively. The free surface is denoted by η and $z = 0$ corresponds to the quiescent sea level. Gravity is $g\hat{\mathbf{z}}$ and, irrelevant and for pure convenience, the vector $\hat{\mathbf{z}}$ points upwards, along increasing z. Velocity is scaled by u_Ω, length by ℓ, time by ℓ/u_Ω, density by ρ_w, the density of water, pressure by $\rho_w u_\Omega^2$. We then define a Reynolds number $\alpha = u_\Omega \ell/\nu_w$, where ν_w is the kinematic viscosity of water. We also define a Froude number $1/\sqrt{\gamma} = u_\Omega/\sqrt{g\ell}$. The scaling leads to

$$\alpha \mathbf{u}_t + \alpha \mathbf{u} \cdot \nabla \mathbf{u} = -\alpha\gamma \nabla \Pi + \nabla \cdot [D(\mathbf{r})\Xi], \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

where the stress tensor $\Xi = \nabla \mathbf{u} + [\nabla \mathbf{u}]^\top$. D is the (linear) proportionality tensor.

Let $\mathbf{R} = (X, Y, Z)$ be the large-scale space variable, such that $\nabla = \nabla + \epsilon \nabla_{\mathbf{R}}$, and assume slow time $\partial_t = \delta \partial_T$. Also, assume that $\Pi(\mathbf{r}, \mathbf{R}, T) = p - z + \epsilon^2 p_0 + \epsilon^4 p_1 + \ldots$, $\mathbf{u} = \epsilon(\mathbf{u}_0 + \epsilon \mathbf{u}_1 + \epsilon^2 \mathbf{u}_2 \ldots)$ and $\eta = \alpha(\eta_0 + \epsilon\eta_1 + \epsilon^2\eta_2 \ldots)$. The orders are $\alpha = \mathcal{O}(\epsilon^2)$, $\delta = \mathcal{O}(\epsilon)$, and $\gamma = \mathcal{O}(\epsilon^{-1})$.

In the absence of bubbles and in the smallest of continuity scales, D would be equal to ν_w, the kinematic viscosity of water ~~the ocean.~~ In what follows, we will derive a homogeneized version (see Caflisch et al. (1985)) of the dynamics equations, appropriate at (larger) scales, relevant to wave dynamics, that shows that in the presence of bubbles the divergence of the stress

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tensor is modified. In the averaged variables, the term in the momentum associated with the stress tensor carries an effective viscosity K. We explore if this change in the momentum balance comes with the presence of (rain induced in our example) gas bubbles in the near-surface ocean. ~~The specific possibility we have explored is that the gas voids are created by the impact of the rain drops on the ocean surface~~. If the rain is intense enough, the voids can cause a significant change in viscous forces, ~~within the momentum balance~~. Under the assumption that the subsurface air bubble distribution is spatially homogeneous, we can then propose a periodic arrangement of sub-wave cells.

redundant

up to what depth?

Collecting by orders in ϵ, the momentum equation is,

– $\mathcal{O}(\epsilon)$:

$$\nabla \cdot [D(\mathbf{r})\Xi_0] = 0.$$

At this microscale, pressure gradients in the cell are negligible, as are variations in velocity within the cell. We are also assuming stationary conditions. Upon integrating in $\mathbf{r}$ over and invoking periodicity ~~it is clear that~~ $\Xi_0 = \Xi_0(\mathbf{R}, t, T)$ and thus $\mathbf{u}_0 = \mathbf{u}_0(\mathbf{R}, t, T)$.

For readability, pls. explain the physical meaning of each term.

– $\mathcal{O}(\epsilon^2)$:

$$\nabla \cdot [D(\mathbf{r})\Xi_1] + \nabla \cdot [D(\mathbf{r})\Xi_0] + \nabla_{\mathbf{R}} \cdot [D(\mathbf{r})\Xi_0] = 0. \tag{3}$$

The last term above is zero (we make use of $\nabla \cdot (\nabla \mathbf{u})^T = \nabla(\nabla \cdot \mathbf{u})$). Integration by parts of (3) and using periodicity, $\Xi_1(\mathbf{r}, \mathbf{R}, t, T) = -\Xi_0 + D^{-1}(\mathbf{r})C(\mathbf{R}, t, T)$, where C is a tensor that is independent of $\mathbf{r}$. Periodicity and integration in $\mathbf{r}$ over Ω of every differential term in Ξ_1 implies that

$$0 = -\Omega\Xi_0 + \int_\Omega D^{-1}(\mathbf{r})d\mathbf{r}\, C(\mathbf{R}, t, T). \tag{4}$$

Hence, the tensor

$$C(\mathbf{R}, t, T) = \left[\frac{1}{\Omega}\int_\Omega D^{-1}(\mathbf{r})d\mathbf{r}\right]^{-1} \Xi_0.$$

Define the tensor

$$\chi = \left[\frac{1}{\Omega}\int_\Omega D^{-1}(\mathbf{r})d\mathbf{r}\right]^{-1}. \tag{5}$$

Returning to (3),

$$\Xi_1 = -\Xi_0 + D^{-1}(\mathbf{r})\chi\Xi_0, \tag{6}$$

thus,

$$\nabla_{\mathbf{R}} \cdot (D\Xi_1) = -\nabla_{\mathbf{R}} \cdot (D(\mathbf{r})\Xi_0) + \nabla_{\mathbf{R}} \cdot (\chi\Xi_0). \tag{7}$$

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unreadable!

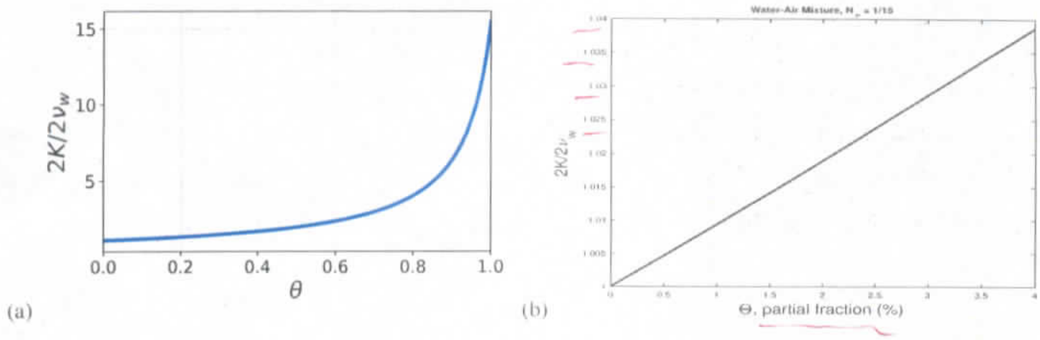

Figure 1. (a) Full range, and (b) physical range, relative effective wave dissipation $2K/2\nu_w$, as a function of volumetric ratio of air to water Θ, using (11) . Assumed here, the ratio $N_\nu := \nu_w/\nu_a = 1/15$.

energy?

– $\mathcal{O}(\epsilon^3)$: the momentum balance reads

$$\begin{aligned}\frac{\partial \mathbf{u}_0}{\partial T} + \mathbf{u}_0 \cdot \nabla_{\mathbf{R}} \mathbf{u}_0 + \mathbf{u}_0 \cdot \nabla \mathbf{u}_1 + \nabla_{\mathbf{R}} p_0 = \\ + \nabla \cdot [D(\mathbf{r})\Xi_1] + \nabla_{\mathbf{R}} \cdot [D(\mathbf{r})\Xi_0] \\ + \nabla \cdot [D(\mathbf{r})\Xi_2] + \nabla_{\mathbf{R}} \cdot [D(\mathbf{r})\Xi_1].\end{aligned} \tag{8}$$

Using (7) in the last term in (8) and averaging all quantities over Ω, leads to

$$\frac{\partial \mathbf{u}_0}{\partial T} + \mathbf{u}_0 \cdot \nabla_{\mathbf{R}} \mathbf{u}_0 + \nabla_{\mathbf{R}} p_0 = \nabla_{\mathbf{R}} \cdot \left\{ \chi \left(\nabla_{\mathbf{R}} \mathbf{u}_0 + [\nabla_{\mathbf{R}} \mathbf{u}_0]^\top \right) \right\}. \tag{9}$$

The homogenized incompressibility condition is

$$\nabla_{\mathbf{R}} \cdot \mathbf{u}_0 = 0. \tag{10}$$

meaning what? viscosity of water?

The tensor χ takes the value of the ocean, changing it when bubbles appear for whatever reason. In what follows we limit ourselves to the simplest possible case: $\chi = \nu_w \delta_{ij}$, when bubbles are not present, where δ_{ij} is the 3-space dimensional Kronecker Delta. For our specific example we use (5) assuming a matrix consisting of a homogeneous concentration of air bubbles, with diffusion constant ν_a, in a background ~~ocean~~ fluid. In this very simple case, the enhanced value of χ becomes

$$\chi = K\delta_{ij}, \qquad K = \frac{\nu_w}{1 - \Theta(1 - N_\nu)}, \tag{11}$$

where $N_\nu := \frac{\nu_w}{\nu_a}$, and Θ connotes the volumetric ratio of air to water. For a given $\Theta > 0$, $N_\nu < 1$ leads to an increased effective diffusion, and, when $N_\nu > 1$, the effective dissipation is lower. For air or oil bubbles, $N_\nu < 1$.

With a fixed ratio $N_\nu = 1/15$, which is roughly the ratio of viscosities of water and air, Figure 1 shows how the effective dissipation coefficient K changes, as a function of the volumetric ratio Θ. The relationship is nearly linear for the small volume fractions of the oceanic case. Equation (11) suggests that the effective viscosity of water with air bubbles will be different from

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highly counter-intuitive !

(actually larger than) ν_w. If we adopt the specific form of χ as in (11), the dissipation term in (9) becomes $K\nabla_{\mathbf{R}}^2\mathbf{u}_0$ when bubbles are present and $\nu_w\nabla_{\mathbf{R}}^2\mathbf{u}_0$, when there are no air bubbles.

At this point we have developed a model for mass and momentum conservation at large spatio-temporal scales with a homogeneous microdiffusivity diffusivity tensor $D(\mathbf{r})$. One can imagine that this model is relevant to phenomena that involve a high density of trapped air, such as ocean foams. In the foam case the primary challenge is to apply homogeneization ideas to derive stress conditions for a bubbly interface. For our example we are focusing on air entrainment due to rain and in the following section we show how we couple the air entrainment to the rain.

in such a case more certainly a two-layers model will be needed. I do not agree the presented model will be still valid

3 Air Entrainment due to Rain

3.1 Rain Distribution

The density of rain drops is assumed to follow the Marshall-Palmer distribution (see Marshall (1948)). The density N_r of rain drops of radius r [m] per unit volume [number m^{-3} m^{-1}] is

$$N_r(r,R) = 2N_0\exp(-\Lambda(R)r), \tag{12}$$

where $N_0 = 8\times10^6$ m^{-3} m^{-1}, $\Lambda(R) = 8.2R^{-0.21}\times10^3$ [m^{-1}], and R [mm h^{-1}] is the rain rate. Using N_r we can then define the drop rate density (DRD), which describes the rate of falling rain drops of radius r per surface area [number m^{-2} s^{-1} m^{-1}]. Namely,

$$\mathrm{DRD}(r,R) = w_r(r)N_r(r,R), \tag{13}$$

where w_r is the terminal velocity of drops in the air (see Snyder (1990)). The terminal velocity is computed following the third order polynomial estimates of Dingle and Lee (1972).

3.2 Bubble Production

To complete our quantitative example we need to relate the number of bubbles in the upper layer of the ocean to rain. Falling rain drops generate subsurface air bubbles in the neighborhood of the sea surface (see Prosperetti and Oguz (1993) for a review). Small rain drops produce small air bubbles with a very narrow distribution of radii. Intermediate rain drops, with a radius between 0.55 - 1.1 mm, were shown not to produce air bubbles (see Medwin et al. (1992)). For these the impinging rain drops do not have the kinetic energy necessary to produce the requisite conical crater and jetting of the sea surface that engulfs air. Large rain drops, with radii larger than 1.1 mm, create a crater and a canopy on the sea surface, which by collapsing produces a downward liquid jet at the bottom of the crater, followed by the generation of an air bubble. At high rain rates the larger rain drops are responsible for the bulk of the gas injection, creating bubbles with a varied distribution of radii.

The production of air bubbles by rain can be measured by acoustic means (see Prosperetti and Oguz (1993)). Oguz and Prosperetti (1990) classify air bubble production by falling rain drops in two regimes. Small rain drops, of radii between 0.41

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and 0.55 mm, create *type I* air bubbles. These have a radius of 0.22 mm and are created at the bottom of a conical crater created on the ocean surface by the impinging rain drop. These air bubbles have a narrow acoustic spectrum with a spectral peak at 14 Hz. The acoustic spectrum for these type I air bubbles was found to be insensitive to the rain rate. Medwin et al. (1990) observed that when rain hits the ocean surface at an angle, owing to strong winds or very steep wave conditions, the acoustic spectrum peak shifts downward and there is a broadening of the spectrum at higher frequencies). The type I air bubbles do not contribute significantly to total submerged gas volume and their contribution will be ignored in what follows.

Rain drops with a radius greater than 1.1 mm produce *type II* air resonant bubbles of varying radius. The relationship between the rain drop radius r [m] and the peak acoustic emission frequency f_0 [kHz] due to trapped air bubbles was empirically determined (see Medwin et al. (1992)) as

$$f_0 = \frac{160}{8r^3 \times 10^9} + 0.6. \tag{14}$$

The relation between the near surface air bubble radius a [m] and the peak acoustic emission follows from the Rayleigh-Plesset equation (Leighton, 1994, p. 306). For large bubbles considered here, it can be simplified to Minnaert's formula (see Minnaert (1933); Plesset and Prosperetti (1977)),

$$f_0 = \frac{1}{2\pi a \times 10^3}\sqrt{\frac{3\gamma P}{\rho_0}}, \tag{15}$$

where $\gamma = 1.4$ is the ratio of specific heats of the bubble gas, P is the ambient pressure (surrounding the bubble), and $\rho_0 = 1030$ kg m^{-3} the density of water. Close to the surface, the ambient pressure is approximately the surface pressure $P = 1.01 \times 10^5$ Pa and Equation (15) further simplified as $f_0 = 3.25 \times 10^{-3}/a$ (see (Medwin and Clay, 1997)).

The relation between the entrained air bubble radius $a(r)$ [m] and the incident rain drop radius r, for large rain drops, thus reads, for $r > 1.1 \times 10^{-3}$ m,

$$a(r) = 3.25 \times 10^{-3}\left(\frac{160}{8r^3 \times 10^9} + 0.6\right)^{-1}. \tag{16}$$

As an example, rain drops of radius 1.1-2.3 mm produce *type II* air bubbles of radius 0.2-1.3 mm, respectively.

Small rain drops ($r < 0.55$ mm) almost always produce type I air bubbles. However, this is not the case for the larger rain drops. The distribution of the number of air bubbles produced, as a function of the rain drop radius r, was found by Medwin et al. (1992), and is depicted in Figure 2 as a probability distribution.

3.3 Bubble Distribution in the Wave Boundary Layer

Connecting us with what presented in Section II, by finding the bubble distribution in the wave boundary layer, we can estimate the dimensionless volume fraction Θ. More precisely, the volume fraction reads $\Theta = \int V(a)N(a)da$, for bubbles of radius a with density $N(a)$ [number m^{-3} m^{-1}] that occupy a volume $V(a) = (4/3)\pi a^3$. The aim of the final part of this section is to link Θ to the rain precipitation rate.

The bubble distribution $N(a)$ can be described by an advection-diffusion equation (see Woolf and Thorpe (1991) and references therein, and a similar case for oil drops, see Moghimi et al. (2018)). This equation describes a balance between advection

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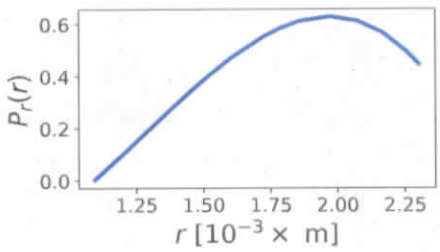

Figure 2. Probability of air bubble creation by large rain drops as a function of rain drop radius r (from Medwin et al. (1992)).

of bubbles by the vertical fluid velocity, attenuation of bubble density (i.e. diffusion, which includes the shrinking of bubble radius during their lifetime, see Merlivat and Memerly (1983)), the production of bubbles by rain and a sink term that captures the loss of air bubbles bursting at the surface or dispersed by oceanic turbulence. Solutions of the advection-diffusion equation necessitate specification of the velocity and the dispersion, generating non-stationary descriptions that might also include mixing due to wave turbulence (see Restrepo et al. (2015); Moghimi et al. (2018)).

In the following, the advection-diffusion equation is simplified to estimate the air bubble concentration in a thin layer close to the surface, similarly to those in Keeling (1993). First, we assume that the air bubble concentration can be linearly related to a bulk bubble distribution (i.e. averaged over the thin layer), which we thus define as $c_e N(a)$. Second, a steady state is considered, in which the bulk bubble distribution follows from a balance between the incoming bubble flux due to rain and the outgoing bubble flux due to upward bubble advection and bursting at the surface:

$$w_r(r)P_r(r)N_r(r) = c_e W[a(r)]N[a(r)], \tag{17}$$

where $P_r(r)$ is the probability of production of an air bubble by a rain drop of radius r (see Figure 2), and $W(a)$ is the upward vertical ascent speed of bubbles of radius a. From this equation, the bubble distribution reads $N(a) = w_r N_r(r,R)P_r/[c_e W(a)]$. We assume as an estimate for the bubble upward speed $W(a)$ the form

$$W(a) = \sqrt{\frac{\tau}{a\rho_0} + ga} \tag{18}$$

where $\tau = 72.8 \times 10^{-3}$ N m^{-1} is the air-sea surface tension (see Clift et al. (1978), p. 172). This form, predicted in Mendelson (1967), is valid for ellipsoidal bubbles with radius greater than 0.65 mm, whose upward path is not rectilinear.

We have introduced a free positive parameter $c_e \leq 1$ which accounts for an increase in bubble density with respect to the bulk estimate of (17), due to two factors. Firstly, bubbles are injected at a finite depth by rain drops, and this depth may vary with bubble size (see Ho et al. (2000)). Secondly, a homogeneous air bubble distribution in the layer between the depth of injection and the surface is a coarse approximation, as accumulation of bubbles near the surface can be highly complex due to buoyant forces, damping, and the background turbulence (see e.g. Merlivat and Memerly (1983)). Hence the factor c_e can first be interpreted as a depth scaling parameter accounting from differences between the bulk estimate and the non-homogeneous bubble distribution N. Second, the effective vertical velocity of bubbles can be significantly lower than (18), up to 60% due to

how much

Such explanation would be greatly appreciated if introduced or hinted before (Introduction?)

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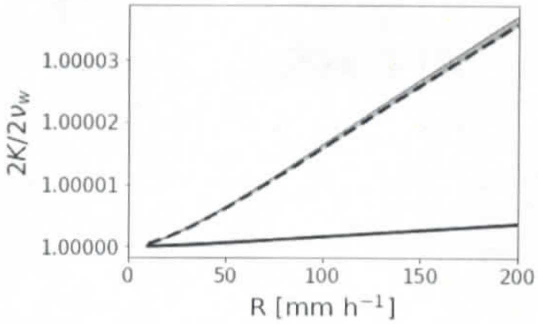

Figure 3. Relative effective wave dissipation $2K/2\nu_w$, as a function of rain rate R for two values of the unconstrained parameter c_e: 1 (solid) and 0.1 (dashed). Grey shadings correspond to a variation of air diffusivity between 10^{-5} m s^{-2} (dashed line) and 10^{-4} m s^{-2}, for a fixed $c_e = 0.1$. The figure highlights that the model outcomes are more sensitive to the parameter c_e than to the value of the diffusivity ν_a.

the presence of contaminants at the bubble surface (see Clift et al. (1978), Fig. 7.3). Hence the factor c_e is also to be interpreted as a decrease in the bubble vertical velocity, which then reads $c_e W(a)$.

Making use of (13), $N(a) = (\mathrm{DRD} \times \mathrm{P}_r)(r)/c_e W(a)$. The volumetric ratio $\Theta = \int V(a)N(a)da$, for bubbles of radius a with density $N(a)$ that occupy volume $V(a) = (4/3)\pi a^3$ then reads

$$\Theta(R) = \int\limits_{1.1\times10^{-3}}^{2.3\times10^{-3}} \frac{4\pi}{3} a(r)^3 \frac{(DRD \times P_r)(r)}{c_e W(a)} dr, \tag{19}$$

where the limits of integration encompass the volumetric contributions of type II bubbles.

Figure 3 shows that the sensitivity of the model to changes in ν_w/ν_a is much lower than to changes in the bubbles volume fraction, i.e. the tuning parameter c_e. The grey shadings are the intervals obtained by varying the air viscosity inside air bubbles (ν_a) from 10^{-5} to 10^{-4} (the lower and upper part of the shadings respectively), for a fixed c_e of 0.1. The resulting sensitivity of the relative wave dissipation is much lower, by one order of magnitude, than the one obtained varying c_e (the full line corresponds to $c_e = 1$ and $\nu_a = 10^{-5}$). (~~Obviously,~~ for this estimate we consider the actual kinematic viscosities of water and air). The obtained values of volume fraction are consistent with data from laboratory experiments of Ho et al. (2000) (see their Figure 7), which suggests that for rain rates of 114 mm h^{-1} the volume fraction near the air/water interface is between 10^{-6} and 10^{-5}, corresponding to c_e of the order of 0.1. The increase in effective wave damping due to the injection of air, at least as estimated by simple considerations, is small compared to the values reported in the experiments in Peirson et al. (2013) (and in Tsimplis (1992)) a further proof of what stated in the Introduction, i.e. that, for large rain rates and, as in the wave tank experiments, large rain drops, the mechanical effect is dominant for the immediate, albeit limited, attenuation of water waves. In these, using very high rain rates, the effective dissipation was found to increase by 3-10 times when rain is present, as compared to without rain.

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4 The Effect of Rain on Gravity and Capillary Waves

The derivation of infinitesimal amplitude gravity waves dynamics, starting from Navier Stokes, appears in Lamb (1916), Article 349, hence we will be brief. We revert back to dimensional quantities in what follows. The solution $\mathbf{u}_0$ satisfies the linear version of (9) and (10) and the two stress conditions at the surface. Namely,

$$K[\partial_X v_0 + \partial_Y u_0] = 0, \quad \text{at } z = 0, \tag{20}$$

$$-\partial_T \phi + (g - \frac{\tau}{\rho_w}\partial_{XX})\eta_0 + 2K\partial_Y v_0 = 0, \quad \text{at } z = 0, \tag{21}$$

where ϕ is the velocity potential and ρ_w is the density of water, and time (at wave scales) is T. The solution of the system, assuming a vanishing velocity at depth, is

$$\begin{aligned}
\phi &= Ae^{kZ+i(kX-\sigma T)}\exp[-2Kk^2T],\\
\psi &= \frac{2Kk^2A}{\sigma}e^{kZ+i(kX-\sigma T)}\exp[-2Kk^2T],\\
\eta &= \frac{kA}{\sigma}e^{i(kX-\sigma T)}\exp[-2Kk^2T],
\end{aligned} \tag{22}$$

where ψ is the streamfunction and σ the angular frequency. The solution (22) represents infinitesimal progressive waves traveling in the X direction. The dispersion relation for the waves is

$$\sigma^2 = gk + \frac{\tau}{\rho_w}k^3.$$

The effective wave dissipation $2K$ appears in the exponential factor $\exp[-2Kk^2T]$. When the rainstorm is sufficiently intense (but the rain drops not too large), the wave dissipation $2K$ will change from $2\nu_w$ to a higher value, depending on how much air is injected by the impinging rain drops (see Figure 1). As it can be surmised from (22), increases in the wave dissipation $2K$ lead to higher wave attenuation. The effect is prominent in the high frequency components. The dependence of the effective wave dissipation on the rain rate, via (11), is depicted in Figure 3.

5 Discussion and conclusions

Stimulated by an oceanographic problem, we have developed a physical-mathematical approach to analyze how the presence of a large number of small bubbles affects the characteristics of the containing liquid. ~~Granted the general method and having specified the necessary conditions for its~~ application, we need to focus ~~on a specific example. As expected,~~ given also our ~~background~~, this has been found in the presence of air bubbles in the thin upper layer of the ocean. ~~Following a logical sequence of arguments~~, we first derived the (upscaled) equations for mass and momentum conservation, along with the equations for wave motions for averaged material and dynamic quantities. The upscaled velocity and pressure are averages over subwave scales over which the material with heterogeneities assume a stationary distribution.

The effective diffusivity calculation requires that we know the kinematic diffusivity tensor, $D(\mathbf{r})$ and, although we feature the role played by air bubbles in water, this diffusivity tensor should account for all sources of small scale heterogeneity. Under

do you hope for this? or is it so?

pls. be consistent and more rigorous with the terminology. you call this: diffusivity. viscosity ocean value
it makes the reading confusing!

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Not so obvious
Pls. provide all the orders of magnitude
i.e. wave size, period, stage (wave age)
(scope)

This sentence is unnecessary if the terminology was consistent.

homogeneization conditions, the model yields a dynamic equation that has a higher effective diffusion constant. In particular we looked at the increased kinematic viscosity of the air-water mixture and how this can affect the damping of surface waves. Again as expected from obvious general considerations, the matter can be of any significance only for the very shortest waves, in practice in the capillary regime. ~~Of course we need~~ a source for ~~the~~ tiny (~~not large~~) uniformly distributed bubbles. ~~This~~ is rain, ~~for our purpose relevant when abundant, but as many relatively small drops~~ (large drops lead to splashing and turbulence in the upper layer, a different process). Pls. explain better. (scope)

move paragraph up ↑

Many simplifications were made in the formulation of the rain/wave model. The model has three critical parameters: the rain rate R, the air-to-water volume fraction Θ (and the flux strength parameter associated with the fluxes of air bubbles in the layer which in turn affects the volume fraction) and the ratio of diffusivities of water and air, N_ν. Given the approximations, the basic result is that the volume fraction of air to water would have to be exceptionally large for the effect to be significant, when compared to other damping mechanisms, such as turbulence induced dissipation due to the impact of large rain drops. However, granted the limited effect in the analyzed case, larger amounts of bubbles or, mutatis mutandis, of buoyant organic or hydrocarbons, are certainly possible. In these cases, that we have not explored, the damping effect can indeed be relevant and, within the specified approximations, quantifiable following the procedure we have outlined. very unclea

There are several processes where the mixture of a liquid containing a large number of bubbles or droplets has different characteristics. Examples are a frothy ocean, whatever the content, cavitating flows as it happens in ship propellers or in pressurized flows. One example we came across while preparing this article describes how foams are used to ameliorate unwanted ship motion due to sloshing of their holding tank contents (see Denkov et al. (2005); Kim et al. (2007)), as well as the stabilizing effect of free surface sloshing of bubbly drinks (see Cappello et al. (2015)). These foamy cases are not trivial extensions of the homogeneisation procedure we present in this paper. Consideration of chemistry, compressible effects, topology, is required and the intricate formulation of stress conditions at the interfaces would need to be derived. Nevertheless the procedure may play a constructive role in formulating a mean field description of the dissipation and consequent attenuation of the foam on sloshing motions.

highly speculative
consider removing.

6 Summary

A discussion on the counter-intuitive effect of air intrusion (lower viscosity) would be better appreciated.

We itemize here below our main conclusions:

- 1. The presence of tiny air bubbles, that we assume uniformly distributed, changes the physical characteristics of the containing liquid, affecting momentum balances.
- 2. A general methodology has been developed to approach this, or similar, kind of problems.
- 3. We have applied the method to estimate the increased effective kinematic viscosity of the upper tiny layer of the sea in presence of rain, hence derived the changes to the effective wave damping due to tiny air bubbles in the upper sea.
- 4. The increased wave damping affects mainly capillary waves. This specific aspect turns out negligible compared to other processes affecting wave damping.

not shown quantitatively.