



1 **A method to predict the uncompleted climate transition** 2 **process**

3 **Pengcheng Yan¹, Guolin Feng², Wei Hou²**

4 [1]{Institute of Arid Meteorology, China Meteorological Administration, Key
5 Laboratory of Arid Climatic Change and Reducing Disaster of Gansu Province, Key
6 Laboratory of Arid Climatic Change and Reducing Disaster of China Meteorological
7 Administration, China}

8 [2]{National Climate Center, China Meteorological Administration, China}

9 [*]Correspondence to: Wei Hou (houwei@cma.gov.cn)

10 **Abstract**

11 Climate change could be expressed as a climate system transiting from the initial state
12 to a new state in a short time. By considering the short period as a continued process,
13 which is called transition process, more details of climate change would be described
14 according to analysis the time sequence self. We had proposed a method to quantify
15 the transition process of the Pacific Decadal Oscillation(PDO) time sequence and
16 global sea surface temperature system. And the quantitative relationships among the
17 parameters characterizing the abrupt changes is revealed during the transition process.
18 In this paper, we develop this method to predict the end moment(state) if the transition
19 process has not been completed. Application of prediction method to the PDO
20 sequences indicates that the PDO index increased from a stable stage before 2011 and
21 gradually evolved to a transition process, and it was likely to end in 2015,which is
22 consistent with observations.

23 **Keywords**

24 Prediction method; Transition process of abrupt change; System stability; Pacific
25 Decadal Oscillation

26 **1. Introduction**

27 A system transiting from one stable state to another in a short period is called



1 abrupt change(Charney and DeVore, 1979; Lorenz, 1963, 1979). The abrupt change
2 system has two or more states(Goldblatt et al, 2006; Alexander et al, 2012), the
3 system swings between these states that is also called attractors in physics. This
4 phenomena is verified in many fields including biology, ecology, climatology(Thom,
5 1972; Overpeck and Cole, 2006), brain science(Sherman et al, 1981), human behavior,
6 etc. The latest famous climate change event is global warming hiatus, which has been
7 studied deeply by many researches(Amaya et al, 2018; Kosaka and Xie, 2013). Seven
8 different kind of abrupt changes are mentioned in Thom's research. And over the last
9 several decades, many methods have been proposed to identify different kinds of
10 abrupt change(Li et al, 1996), like Moving T-Test, Cramer's(Wei, 1999),
11 Mann-Kendall (MK, (Goossens and Berger, 1986)), Fisher (Cabezas and Fath, 2002),
12 etc. It is noticed that most abrupt change detection methods suggests that the abrupt
13 change is only a turning point, and the significant difference of a sequence on the two
14 sides of the turning point is defined as the index to measure the abrupt change. This
15 detection has a drawback. Notably, when the abrupt change occurs at the end of
16 sequence, it is difficult to detect. The filtering process considers the continuity of the
17 time series and provides a very important technique for the detection and prediction of
18 abrupt change. For an abrupt climate change event, the determination of the start
19 moment and end moment for the abrupt change is helpful for better understanding
20 abrupt climate change, and it is also an important supplement for studies of abrupt
21 climate change.

22 Mudelsee (2000) studied the abrupt change of a time sequence and illustrated
23 that abrupt change has a duration, namely, the transitional process. Moreover, it can
24 be quantitatively described with the Ramp Function, which is more advanced than
25 traditional metrics of abrupt change. Yan et al. (2014, 2015) developed the detection
26 method with a Non-linear Function replacing the Ramp Function. The new method
27 can confine the beginning and ending points of abrupt change and quantitatively
28 describes the process of abrupt climate change, and three parameters are introduced.
29 Besides, a quantitative relationship among the parameters is revealed (Yan et al, 2015).
30 The relationship could be used to predict the end moment(state) if the system had left



1 the original state but not yet got reach to the new state.

2 In this paper, several ideal time sequences are tested to study the prediction
3 method. Then, the method is applied to study the abrupt change of the Pacific decadal
4 oscillation (PDO), which is an important signal (Mantua et al, 1997; Zhang et al, 1997)
5 that reveals climatic variability on the decadal timescale. The PDO has not only affect
6 inter-decadal change in the Pacific and surrounding areas but also has a modulation
7 effect on the inter-annual signal (Yang et al, 2004). Previous studies (Lu et al, 2013;
8 Mantua et al, 1997; Trenberth and Hurrell, 1994) have indicated that there have been
9 many oscillations in the PDO over the past 100 years, and in particular, the most
10 famous are the transformations in the 1940s and 1970s. During the 1940s, the PDO
11 transformed from a high state to a low state, and during the 1970s, it did the opposite.
12 All this changes and their processes had been studied in the previous researches (Yan
13 et al, 2015; Yan et al, 2016). However, the research on the prediction of abrupt change
14 process is rare and difficult. Thus, based on the theory of abrupt change processes and
15 the continuity characteristics of the time series, we develop a new method to perform
16 a non-linear extrapolation to predict the end moment (state) of the abrupt change of
17 the PDO, by quantitatively identifying the abrupt change process of the PDO.

18 **2. Methods**

19 **2.1 The detection method of transition process**

20 The real time sequence change abruptly as shown in figure 1a, and the system
21 jumps to a high state in point C. If the period around point C is expanded to a longer
22 period, or the period around point C is observed on a more short time scale, a
23 transition period is obtained in figure 1b, which is a part of the original time sequence.
24 In fact, many abrupt change should be considered to be a transition period much more
25 than a point. Mudelsee(2000) indicated that this kind of abrupt change and the
26 transition period can be expressed with an ramp function. As shown in figure 1c, the
27 time sequence is divided into three segments, including two equilibrium states and
28 one increasing state. Then, the following ramp function was used to fit the transition



1 period.

$$2 \quad x_t = \begin{cases} x_1 & t \leq t_1 \\ x_1 + (t - t_1)(x_2 - x_1)/(t_2 - t_1) & t_1 < t \leq t_2 \\ x_2 & t > t_2 \end{cases} \quad (1)$$

3 t represents time, and x_t represents its state. Based on the linear fitting method, x_t
4 was obtained to describe the system's behavior. By referring this work, a novel
5 method(Yan et al, 2015) was proposed to detect the transition period by using the
6 logistic model(May, 1976). The logistic model was created to study the population of
7 Humans or insects. The population changed(increased or decreased) looks like the
8 expanded of abrupt change, which is also like the change in figure 1c. The modified
9 logistic model is expressed as follows and its image is shown in figure 1d:

$$10 \quad \dot{x} = k(x - u)(v - x) \quad (2)$$

11 Parameters u and v represent the two equilibrium states respectively. Parameter k
12 represents the speed of switching between different states. As shown in figure 2a,
13 parameters u and v being fixed, and setting k as 0.5, the system transiting to the new
14 state costs a shorter time than that setting k as 0.4. If parameter k is set large enough,
15 the system collapses and becomes chaotic(as shown in figure 2b). When parameter k
16 is set to different values, more situations have been discussed in detail in the previous
17 research(Yan et al, 2016). The result shows that parameter k characterizes the stability
18 of the system (the larger the absolute value, the more unstable the system). Besides,
19 the system does not always evolve to one of the two stable states. If the initial state of
20 the system is between two stable states, the system will converge to one of states;
21 otherwise, the system will crush.

22 Mathematical derivation is an effective method to learn the characteristics of the
23 system of the logistic model. The left side of Eq.(2) represents the change of variation
24 with time, and it can be considered to be the general velocity, which is also the
25 derivative of variation to time. We continue to calculate the derivative of velocity to
26 time, and we have the general force as follows:

$$27 \quad \ddot{x} = 2k^2[x - (u + v)/2](x - u)(x - v) \quad (3)$$



1 Calculating the spatial integral of general force (the system itself), the general
 2 potential energy is obtained as follows:

$$\begin{aligned}
 V_{(x)} &= -\int_0^x \ddot{x} dx = -\int_0^x 2k^2 [x - (u+v)/2] (x-u)(x-v) dx \\
 &= \frac{k^2}{2} [x^4 - 2(u+v)x^3 + (u^2 + v^2 + 4uv)x^2 - 2(u+v)uvx]
 \end{aligned}
 \tag{4}$$

4 According to Thom's(1972) theory, the system described by a quadratic function
 5 would exhibit tipping-point abrupt change, which the system jumps from one state to
 6 a new state abruptly. In figure 2c, the potential energy of Eq.(4) is verified to have two
 7 states with the lowest energy, which are stable, and the system is able to transit
 8 between them. This bistable structure is common in the climate system (Goldblatt et
 9 al, 2006). Therefore, Eq.(2) can be used to describe the abrupt change system, and the
 10 parameters represent different key factors of the transition period during abrupt
 11 change.

12 In figure 2d, the smooth continuous time sequence is divided into three segments.
 13 In each segment, the parameters can be obtained by regression. For the first segment,
 14 the system is stable, and the average value of system states is v . Similarly, in the third
 15 segment, the average value of system state is u . Parameters v and u are the two stable
 16 states of the system, which their values can be calculated by Eq.(5), where n_1 and n_3
 17 are the length of first segment and the third segment respectively. For the second
 18 segment, the transition period can be considered linear approximately. Then, the linear
 19 trend h is calculated by regression (Huang, 1990) in Eq.(6), where i, x_i denote the time
 20 and the state of the system at this time, and \bar{i}, \bar{x}_i are their averages respectively. And,
 21 n_2 is the length of second segment.

$$\begin{cases}
 v = \sum_{i=1}^{n_1} x_i / n_1 \\
 u = \sum_{i=n_1+n_2+1}^n x_i / n_3
 \end{cases}
 \tag{5}$$

$$h = \frac{\sum_{i=n_1+1}^{n_1+n_2} \bar{i} \cdot \bar{x}_i}{\sum_{i=n_1+1}^{n_1+n_2} \bar{i}^2}
 \tag{6}$$

24 Additionally, the linear trend h can be expressed with two points on the curve



1 approximately as follows, where the two points are represented by $A(x_a, t_a)$ and $B(x_b,$
 2 $t_b)$ (figure 2d).

$$3 \quad h = \frac{x_a - x_b}{t_a - t_b} \quad (7)$$

4 Then, we do integration for both sides of Eq.(2). In order to simplify Eq.(8), an
 5 intermediate variable(Eq.(9)) is introduced, and Eq.(8) is rewritten as Eq.(10).

$$\begin{aligned} \frac{dx}{dt} &= \kappa(x - \mu)(v - x) \\ \Rightarrow \frac{1}{\mu - v} \int_{x_0}^x \left(\frac{1}{x - v} - \frac{1}{x - \mu} \right) dx &= \kappa \int_{t_0}^t dt \\ 6 \quad \Rightarrow \ln \left(\frac{x - v}{x - \mu} \right) \left(\frac{x_0 - \mu}{x_0 - v} \right) &= \kappa(\mu - v)(t - t_0) \quad (8) \\ \Rightarrow \frac{x - v}{x - \mu} &= \frac{x_0 - v}{x_0 - \mu} e^{\kappa(\mu - v)(t - t_0)} \end{aligned}$$

$$7 \quad \xi(t) = \frac{x_0 - v}{x_0 - \mu} e^{\kappa(\mu - v)(t - t_0)} = \frac{x - v}{x - \mu} \quad (9)$$

$$8 \quad t = t_0 + \frac{1}{(\mu - v)\kappa} \ln \frac{x_0 - \mu}{x_0 - v} \xi(t) \quad (10)$$

9 The transition period including points A and B is approximately linear. Thus, the
 10 following relationship is established by defining the location parameters α, β .

$$11 \quad \begin{cases} \alpha = \frac{x_a - v}{\mu - v} \\ \beta = \frac{x_b - v}{\mu - v} \end{cases} \Rightarrow \begin{cases} x_a = \alpha(\mu - v) + v \\ x_b = \beta(\mu - v) + v \end{cases} \quad (11)$$

12 Then, Eq.(10) and Eq.(11) are introduced into Eq.(7), and the difference between
 13 the two stable states(u and v) is defined as the amplitude of change(w). A relationship
 14 among the linear trend(h), stability parameter(k), amplitude of change(w), and the
 15 location parameters(α, β) are confirmed.



$$\begin{aligned}
 h &= \frac{[\beta(\mu-\nu)+\nu]-[\alpha(\mu-\nu)+\nu]}{\frac{1}{(\mu-\nu)\kappa} \left(\ln \frac{x_0-\mu}{x_0-\nu} \xi(t_\beta) - \ln \frac{x_0-\mu}{x_0-\nu} \xi(t_\alpha) \right)} \\
 &= \frac{\kappa(\mu-\nu)^2(\beta-\alpha)}{\ln \frac{\xi(t_\beta)}{\xi(t_\alpha)}} = \kappa\omega^2 \frac{(\beta-\alpha)}{\ln \frac{\beta(\alpha-1)}{\alpha(\beta-1)}}
 \end{aligned}
 \tag{12}$$

The part $(\beta-\alpha)/\ln((\beta(\alpha-1))/(\alpha(\beta-1)))$ in Eq.(12) is only related to the location parameters, then let it be χ , and the relationship among χ , α , β is displayed in figure 3a. Then, the relationship of Eq.(12) is rewritten as Eq.(13):

$$h = \kappa\omega^2 \chi \tag{13}$$

According to the numerical experiment, the relationship between parameter χ and location parameters is shown in figure 3a and figure 3b, and figure 3a is the profile of the diagonal in figure 3b, which indicates that the sum of α and β is 1. Parameter χ changes little when the location parameter varies in a certain range as shown in figure 3b. It is obvious that the closer the points(A and B) are to the turning points, the more the process between point A and point B can represent the whole transition process as shown in figure 3c. However, the process between point A and point B is linear when the two points are located to the top of the segment. Let the sum of α and β be 1, then the change of parameter χ is only related to parameter α (or parameter β), as shown in the diagonals in figure 3b(also in figure 3a). Parameter χ changes little when parameter α is about 0.2 or larger. In figure 3c, three ideal experiments were carried out. In each experiment, points(A and B) were set to be different positions, and their parameters were calculated respectively as table 1. The parameters α are set as 0.20/0.25/0.15 respectively in three different tests. For test 2 and test 3, both of the percentages of α changing to test 1 are 25%, while the percentages of χ changing are only 4.07% and 7.73% respectively, which means the percentage change of χ is much less than α . In addition, linear trends of these three ideal models are calculated according to the points and by regression method, and the values are shown in table 1. It is noted that although the positions of points are different, the trend obtained according to the points is almost the same as that



1 obtained by regression method. The error percentages are 2.36%, 2.25%, 1.38%
2 respectively, which means that when the position of the points(the values of
3 parameters α and β) changes slightly, there is little influence on the parameters.

4 **2.2 The prediction method of transition process**

5 Eq.(13) shows the quantitative relationship among linear trend, stability
6 parameter, and amplitude of change. There is a linear relationship between linear
7 trend and stability parameter; and there is the quartic function relationship between
8 linear trend and amplitude of change. We did reveal this quantitative relationship
9 much more than in theory but in real time series by studying the sea surface
10 temperature (Yan et al, 2016). Based on this relationship, we are going to improve a
11 method to deal with the problem which the transition process is not finished. During
12 the real time sequence, the system transits away from the original state, but it has not
13 been in a new state as shown in figure 4. The red line represents the period which has
14 been experienced, while the gray line represents the period which hasn't been
15 experienced. Based on the system states which is far away from the original state, a
16 quasi linear extension of the transition process is established(dash line). Then the
17 parameters v and h are obtained by the method of Eq.(5) and Eq.(6). Assuming that
18 the parameter k satisfies the statistics in the history of the system, the parameter u can
19 be predicted on the basis of Eq.(13), and the end moment of this transition process is
20 also predicted apparently. An ideal time sequence is constructed by using the logistic
21 model and random numbers to achieve the prediction as shown in figure 5a, and three
22 uncompleted changes are shown in figures 5b, 5c and 5d respectively. The parameters
23 v , u and k of the logistic model are set as -1.0, 2.0, 0.1, for the ideal time sequence,
24 and the random number range is 0-1. The parameters v , h are obtained by regression
25 method when making prediction. There is no way to obtain the parameter k from the
26 historical data for an ideal sequence, thus it is given directly, and the prediction of the
27 end state(moment) is drawn in the graph(represented by blue line). The results show
28 that this prediction method performs well even the system stays in different position
29 of the transition process.



1 **3. Prediction studies of the parameter threshold and the index for the** 2 **PDO abrupt change process**

3 In order to test the validity of this prediction method in a real climate system, the
4 Pacific Decadal Oscillation(PDO, which represent the interdecadal variability of the
5 Pacific, (Newman et al, 2016)) index is used to detect the transition process over the
6 past 100 years. The PDO index data used is from website of the University of
7 Washington (<http://research.jisao.washington.edu/pdo/>). It's found that PDO index
8 gradually increased in 2011. Therefore, it is necessary to study the end moment(state)
9 of this transition process. The time period from January of 1900 to November of 2015
10 is studied as the training data, and the time period from December of 2015 to April of
11 2017 is used as the test data.

12 **3.1 Threshold of stability parameter k**

13 The histogram in Figure 6a shows the PDO time sequence from January of 1900
14 to November of 2015, and it shows that the PDO went through several changes. The
15 green dots in Figure 6a are parameter k when the sub-sequence length takes 20 years.
16 In the early 1940s and late 1970s, there are two abrupt changes of the PDO, and the
17 absolute value of the stability parameter k is large, which means that the system is not
18 stable during transition periods. In the 1940s, the PDO transits from a positive phase
19 to a negative phase, and the $k < 0$, whereas the situation in the 1970s is the opposite.
20 Figure 6b shows more k values corresponding to the different sub-sequence lengths
21 (as indicated by X-axis, the variation range of the sub-sequence is 20-60 years, with
22 an interval of 1 year). The Y-axis is the start moment of abrupt change, and the
23 locations of the dots indicate the start moments of detected abrupt changes for the
24 corresponding sub-sequence lengths. In particular, the blue dots represent that
25 parameter k is negative, and the red dots represent that it is positive. The dots in the
26 left side region are clearly denser than in the right side region. This is because when
27 the length of sub-sequence is short, the magnitude of abrupt change is also often small.



1 Therefore, for the entire sub-sequence, there are many detected abrupt changes. When
2 the length of the sub-sequence reaches or exceeds 50 years, the detected abrupt
3 change mainly begins in the 1940s and 1970s. This is also the abrupt change that has
4 been investigated in other research (Shi et al, 2014). The abrupt changes in these two
5 periods correspond to large k values, which means that these two abrupt changes are
6 more unstable than others. More statistical results indicate that the threshold
7 distribution of parameter k values in historical abrupt change processes exhibit
8 multiple peaks (Figure 7). Specifically, the peak with the largest probability is located
9 near to 0. The k value in the distribution is small, which indicates that the abrupt
10 changes that correspond to these k values are stable. The k values are distributed in the
11 peaks on the left side and right side of the origin. When $k < 0$, the PDO time sequence
12 transits from the positive phase to the negative phase, when the threshold of the k
13 peak is wide and the probability is small; when $k > 0$, the PDO time sequence
14 transforms from the negative phase to the positive phase, when the threshold of the k
15 value is narrow and the probability is large. This indicates that the two transitions,
16 which one of them is that the system changes from the positive phase to the negative
17 phase, and the other is that the system changes from the negative phase to the positive
18 phase, are not symmetric, and the latter is more stable. Because there is a difference in
19 parameter k when the selected sub-sequence length is different, the gray region in the
20 upper right corner of Figure 7 also shows the statistical properties of parameter k
21 when the sub-sequence length is 20, 30, 40, 50, or 60 years. When the length of the
22 sub-sequence is 20 years and 30 years, there is only one peak in the distribution of k
23 values, and the parameter k value of the peak is about 0, which means that the abrupt
24 change is stable. That is, when the time sequence is short, the detected amplitude of
25 the abrupt change is small and stable. It is difficult to detect an abrupt change with
26 huge amplitude if the time scale is tiny. When the length of the sub-sequence is 40, 50,
27 or 60 years, the peak value on the side of $k > 0$ is not considerably different, which
28 indicates that the stability degree of the abrupt change from negative to positive is
29 consistent; the location of the peak value on the side of $k < 0$ moves to the left as the
30 sub-sequence length increases, which means that the sub-sequence is longer, the



1 amplitude of detected abrupt change is larger, and it is more unstable. From the
2 perspective of the value, a k value in the range of $(-10, 10)$ accounts for 80.2%, a k
3 value in the range of $(-5, 5)$ accounts for 74.2%, and a k value in the range of $(-2, 2)$
4 accounts for 58.6%. In the following studies, the k value is mainly set in the range of
5 $(-2, 2)$.

6 **3.2 Determination of abrupt change and the threshold for the initial** 7 **state v and linear trend h**

8 We use the method to analyze the abrupt changes occurred in the past 10 years.
9 In particular, the abrupt changes that began in 2007 and 2011 have been identified.
10 Parameters v and h are obtained with sub-sequences of 10, 20, 30, or 40 years for two
11 abrupt changes (Table 1). Specifically, on the scales of 10 and 20 years, we first
12 detected an abrupt change beginning in 2011, and the states were -0.45 and -0.03 ,
13 respectively, with an linear trend of $1.054/a$. On the scales of 30 and 40 years, the
14 earliest detected abrupt change began in 2007, and the states before the abrupt change
15 were 0.36 and 0.41 , respectively, with an linear trend of $0.227/a$. The abrupt change
16 detected on the scale of 50 and 60 years began in 1976, and therefore, it is not
17 investigated. The results above validate that if we select sub-sequences of different
18 lengths, the detected abrupt changes are also different. This is because if the length of
19 the sub-sequence is different, the probability distribution of the magnitude of detected
20 abrupt change process in the sequence is different. Therefore, the abrupt change
21 determined through the percentile threshold method is also different, which indicates
22 that the abrupt change has some scaling properties.

23 Figure 8 shows the segment of PDO around the abrupt change points, which
24 contains the different stages of the abrupt change process. The time sequence has a
25 period of steady state before the abrupt change begins and then enters the
26 development state and gradually transforms to the new steady state. We first analyse
27 the abrupt change that began in 2007. As shown in Figures 8c and 8d, before and after
28 2007, the time sequence transforms from the stable to an augmented abrupt change. It



1 is worth noting that due to the different sub-sequence selections, the abrupt change
2 began during the previous stable state in 2007, and Figures 8a and 8b show different
3 results. However, after the abrupt change begins, the rate of abrupt change during the
4 persistent process of abrupt change is consistent, which further supports the detection
5 result that is shown in Figure 9. For the abrupt changes beginning before and after
6 2011, before the abrupt change begins, the sequence maintains stability over some
7 time and then begins to gradually increase (Figures 8a and b). The abrupt climate
8 changes of two PDO sequences beginning in 2007 and 2011 both belong to the
9 augmented abrupt change.

10 In the three abrupt changes that were obtained by the aforementioned analysis,
11 the sub-sequence length interval is 10 years, and we further select the interval of the
12 sub-sequence to be 1 year. That is, we respectively study the abrupt change detected
13 under the situation when the sub-sequence length is selected as 10, 11, 12, ..., and 40
14 years, and we study the initial stage v of these abrupt changes and the linear trend h .
15 Figure 9 shows that although the interval of the sub-sequence length is shorter, the
16 detected abrupt change only has two abrupt changes. One began in 2007, and the
17 other began in 2011. Moreover, every linear trend h is the same (the specific values
18 are shown in Table 1), whereas the v values of the initial stage are different. In
19 particular, the abrupt change that began in 2007 is detected for the sub-sequence of
20 30-40 years, and the value of parameter v is in the range of (0.28, 0.45). The abrupt
21 change that began in 2011 is detected for the sub-sequence of 10-30 years, and the
22 value of parameter v increases as the length of the sub-sequence increases, whereas
23 the variation range of threshold is (-0.48, 0.12), which is significantly different from
24 the situation in 2007.

25 **3.3 Prediction study on the turning of the abrupt change beginning in** 26 **2011**

27 After the threshold ranges for parameters k , v , and h are determined, according to
28 the quantitative relationship, we can calculate the stable state and the end moment of



1 the abrupt change after the abrupt change of the system, and we therefore predict the
2 turning of a sequence. Using the abrupt change in 2011 as an example, we study the
3 ending state and end moment for the PDO index abrupt change. According to the
4 research results that are presented in Sections 3.1 and 3.2, the parameter is $h=1.054/a$
5 in this abrupt change, and the threshold range of parameter k is determined to be $(0, 2)$.
6 The range of parameter v is determined to be $(-0.48, 0.12)$, and the variation situation
7 of parameter μ and end moment with parameters k and v are shown in Figure 10. The
8 results indicate that the threshold range of parameter u for the ending state is $(1, 7)$,
9 and the time range of the ending moment is $(2013a, 2017a)$. Because of the
10 probability distribution function of stability parameter k , the probability for this abrupt
11 change to end after 2015 is large, and after 2015, the sequence stops the augmented
12 transformation, approaching stability.

13 Figure 11 shows the PDO time sequence(black lines) and the prediction
14 result(black dot dash lines). The trend of the PDO time sequence during 2006-2011 is
15 almost 0, which means that the period belongs to the stable stage prior before the
16 abrupt change point. After 2011, the sequence increases significantly, and it is not able
17 to be known whether the increasing process has been completed or not. The prediction
18 result show that the abrupt change process has been completed in 2015. And during
19 2016-2017, the trend of the index is almost 0 too, which means that the system stops
20 to increase and it reaches to a new stable state. The real PDO sequence(star points) is
21 consistent with the prediction results.

22 **4. Conclusion and discussion**

23 In this paper, we develop a method to study the uncompleted transition process.
24 The method is applied to predict the ideal time sequences and the PDO time sequence.
25 The quantitative relationship between the parameters characterizing the transition
26 process plays an important role. For the PDO time sequence, the abrupt change started
27 began in 2011, and the end moment is predicted to be 2015, which is consistent with
28 the real time series. The study indicates that the PDO system over the past 100 years



1 exhibits a bistable structure, which is also be mentioned in the previous findings(Yan
2 et al, 2016). It means that the sea surface temperature system in the Pacific often
3 changes between different states. Besides, the detection of abrupt change being
4 consistent with timescale is revealed, namely, there are some differences in the abrupt
5 changes that are detected on different timescales in which the most recent abrupt
6 change on the 10-20-year scale began in 2011. In traditional researches of time
7 sequence abrupt change, time scale is rarely mentioned, while the same abrupt change
8 is detected as different forms. The abrupt change with smaller time scales has a
9 continuous process, and the abrupt change with larger time scales becomes abrupt
10 change point.

11 In this paper, a detected abrupt change beginning in 2011 appears relatively close
12 to the end of the 115-year sequence, and it is difficult to identify by using other
13 methods. However, the method we proposed not only detects abrupt changes that
14 occur at the end of the sequence but can also do prediction. The findings increases the
15 possibility of resolving the problem associated with difficult processing at the end of a
16 time sequence, and it also provides new thoughts and a new method for studies
17 regarding the prediction of abrupt changes.

18 **Acknowledgements**

19 This study was jointly sponsored by National Key Research and Development
20 Program of China (Grant No. 2018YFE0109600), National Natural Science
21 Foundation of China (Grant Nos. 41875096, 41775078, 41675092), Northwest
22 Regional Numerical Forecasting Innovation Team (GSQXCXTD-2017-02).

23 **References**

- 24 Alexander R, Reinhard C, Andrey G. Multistability and critical thresholds of the Greenland ice sheet. *Nature*
25 *Climate Change* 2012; 429-432
- 26 Amaya D, Siler N, Xie S, Miller A. The interplay of internal and forced modes of Hadley Cell expansion: lessons
27 from the global warming hiatus. *Climate Dyn* 2018; 51, 305–319, doi:10.1007/s00382-017-3921-5
- 28 Cabezas H, Fath BD. Towards a theory of sustainable systems. *Fluid Phase Equilibria* 2002; 194–197 3,



- 1 doi:10.1016/S0378-3812(01)00677-X
- 2 Charney JG, DeVore JG. Multiple flow equilibria in the atmosphere and blocking, *J. Atmos. Sci* 1979; 36,
3 1205–1216, doi: 10.1175/1520-0469(1979)036<1205:CO;2
- 4 Goldblatt C, Lenton TM, Watson AJ. Bistability of atmospheric oxygen and the Great Oxidation. *Nature* 2006;
5 443:683-686, doi: 10.1038/nature05169
- 6 Goossens C, Berger A. Annual and Seasonal Climatic Variations over the Northern Hemisphere and Europe during
7 the Last Century. *Annals of Geophysics* 1986; 4: 385, doi: 10.1016/0040-1951(86)90317-3
- 8 Huang JY. *Meteorological Statistical Analysis and Prediction*, Beijing: China Meteorological Press 1990; 28–30
- 9 Kosaka Y, Xie SP. Recent global-warming hiatus tied to equatorial Pacific surface cooling. *Nature* 2013; 501:
10 403–407, doi: 10.1038/nature12534
- 11 Li JP, Chou JF, Shi JE. Complete detection and types of abrupt climatic change. *Journal of Beijing Meteorological*
12 *college* 1996; 1:7-12
- 13 Liu TZ, Rong PPg, Liu SD, Zheng ZG, Liu SK. Wavelet analysis of climate jump. *Acta Geophysica Sinica* 1995;
14 38(2):158-162
- 15 Lorenz EN. Deterministic nonperiodic flow. *J. Atmos. Sci* 1963; 20:130, doi:
16 10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2
- 17 Lorenz EN. Nondeterministic theories of climatic change. *Quaternary Research* 1976; 6(4):495-506, doi:
18 10.1016/0033-5894(76)90022-3
- 19 Lu CH, Guan ZY, Li YH, Bai YY. Interdecadal linkages between Pacific decadal oscillation and interhemispheric
20 oscillation and their possible connections with East Asian Monsoon. *Chinese J. Geophys* 2013; 56(4):1084-1094,
21 doi: 10.1002/cjg2.20012
- 22 Mantua NJ, Hare S, Zhang Y, John W, Robert F. A Pacific Interdecadal Climate Oscillation with Impacts on
23 Salmon Production PDO. *Bull.amer.meteor.soc* 1997; 78(6):1069-1079, doi:
24 10.1175/1520-0477(1997)078<1069:APICOW>2.0.CO;2
- 25 May RM. Simple mathematical models with very complicated dynamics. *Nature* 1976, 261:459–467, doi:
26 10.1201/9780203734636-5
- 27 Mudelsee M. Ramp function regression: a tool for quantifying climate Transitions. *Comput. Geosci* 2000,
28 26:293–307, 10.1016/S0098-3004(99)00141-7
- 29 Newman M, Alexander MA, Ault TR, Cobb KM. The Pacific Decadal Oscillation, Revisited. *J. Climate* 2016; 29:
30 4399–4427, doi: 10.1175/JCLI-D-15-0508.1
- 31 Overpeck JT, Cole JE. Abrupt change in earth's climate system. *Annu. Rev. Environ. Resour* 2006; 31:1-31 doi:
32 10.1146/annurev.energy.30.050504.144308
- 33 Sherman DG, Hart RG, Easton JD. Abrupt change in head position and cerebral infarction. *Stroke* 1981; 12(1):2,
34 doi: 10.1161/01.STR.12.1.2
- 35 Shi WJ, Tao FL, Liu JY, Xu XL, Kuang WH, Dong JW, Shi XL. Has climate change driven spatio-temporal
36 changes of cropland in northern China since the 1970s? *Climatic Change* 2014; 124:163-177, doi:



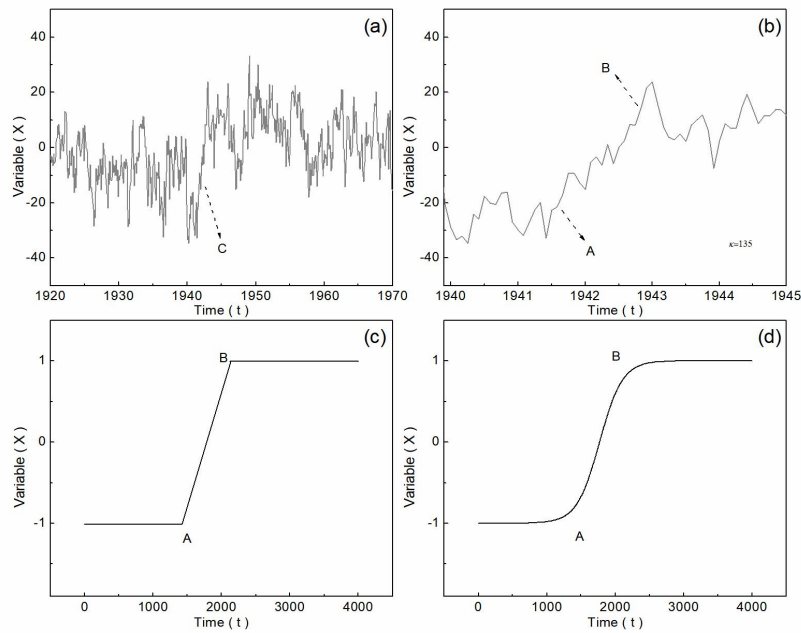
- 1 10.1007/s10584-014-1088-1
- 2 Thom R. Stability Structural and Morphogenesis. Sichuan:Sichuan Education Press, 1972
- 3 Trenberth KE, Hurrell JW. Decadal atmosphere-ocean variations in the Pacific. *Clim. Dyn* 1994; 9:303-319, doi:
4 10.1007/BF00204745
- 5 Wei FY. Modern Climatic Statistical Diagnosis and Forecasting Technology, eijing: China Meteorological Press,
6 1999
- 7 Yan PC, Feng GL, Hou W, Wu H Statistical characteristics on decadal abrupt change process of time sequence in
8 500 hPa temperature field. *Chinese Journal of Atmospheric Sciences* 2014; 38(5): 861–873
- 9 Yan PC, Feng GL, Hou W. A novel method for analyzing the process of abrupt climate change. *Nonlinear*
10 *Processes in Geophysics* 2015; 22:249-258, doi: 10.5194/npg-22-249-2015
- 11 Yan PC, Hou W, Feng GL Transition process of abrupt climate change based on global sea surface temperature
12 over the past century, *Nonlinear Processes in Geophysics* 2016; 23:115–126, doi:10.5194/npg-23-115-2016
- 13 Yang XQ, Zhu YM, Xie Q, Ren XJ. Advances in studies of Pacific Decadal Oscillation. *Chinese Journal of*
14 *Atmospheric Sciences* 2004; 28(6):979-992
- 15 Zhang YJ, Wallace M, Battisti DS. ENSO-like interdecadal variability :1900-93. *J .Climate* 1997; 10:1004-1020,
16 doi: 10.1175/1520-0442(1997)010<1004:ELIV>2.0.CO;2
- 17
- 18



1 Table 1. The parameters of ideal models

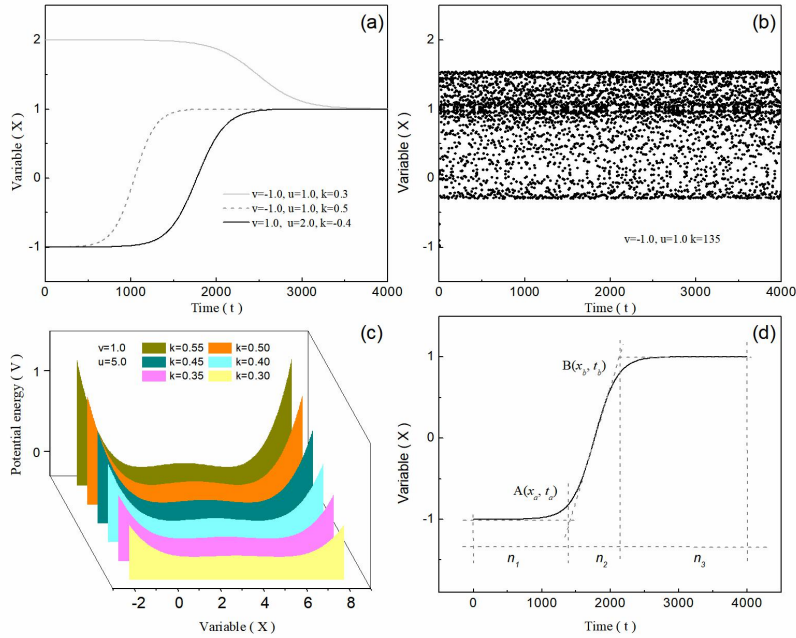
	α	χ	h_0	h	$ h_0-h /h$
test 1	0.20	21.87E-2	12.99E-4	12.69E-4	2.36%
test 2	0.25	22.76E-2	9.10E-4	8.90E-4	2.25%
test 3	0.15	20.18E-2	32.27E-4	32.72E-4	1.38%

2



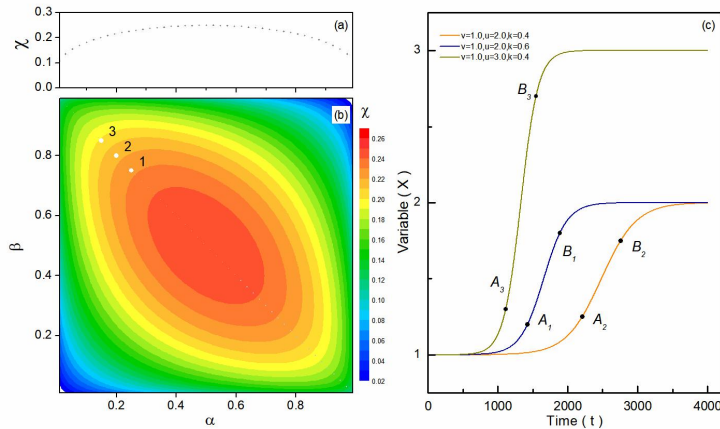
1

2 Figure 1. Transition process of abrupt change in real time sequence and ideal time
3 sequence. (a) The PDO time sequence during 1920 to 1970; (b) The PDO time
4 sequence during 1940 to 1945; (c) The transition process presented by linear function;
5 (d) The transition process presented by nonlinear function



1

2 Figure 2. The system presented by Eq.(2). (a)The system stays in stable states since
 3 the parameters are different; (b)The system stays in unstable states since the
 4 parameters are set as some values; (c)The generalized potential energy function of
 5 system performs differently since the parameters are different; (d)Different segments
 6 of the transition process in the ideal time sequence

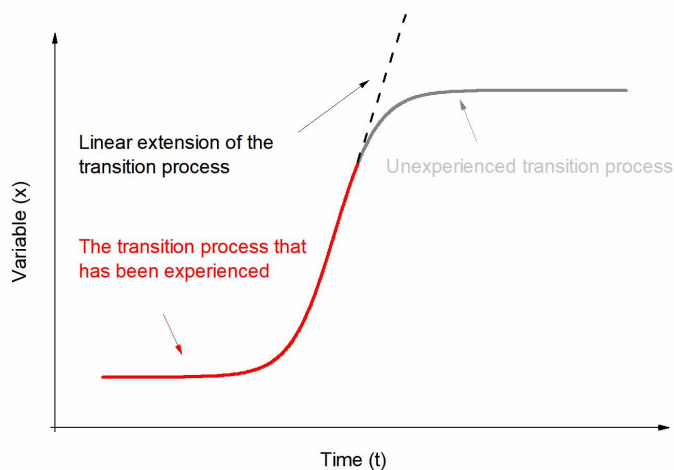


7

8 Figure 3. The influence of different value of parameters α and β on parameter χ and
 9 parameter h . (a) Diagonal section of parameter χ in figure b; (b) Parameter χ with

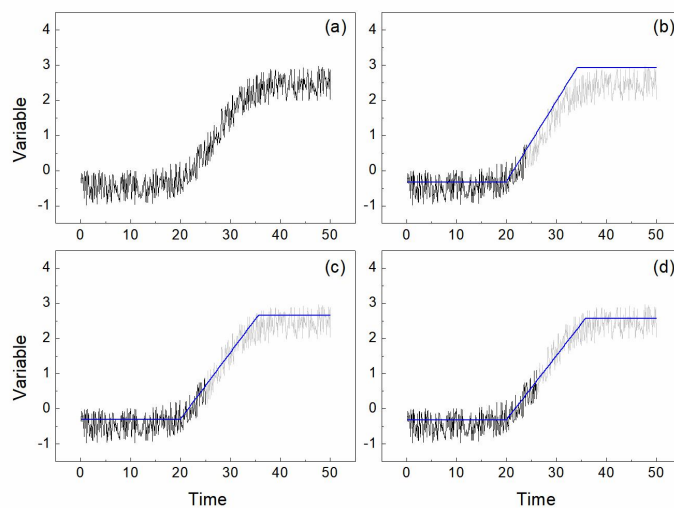


1 parameters α and β ; (c) Points A and B stay in different positions in three tests.



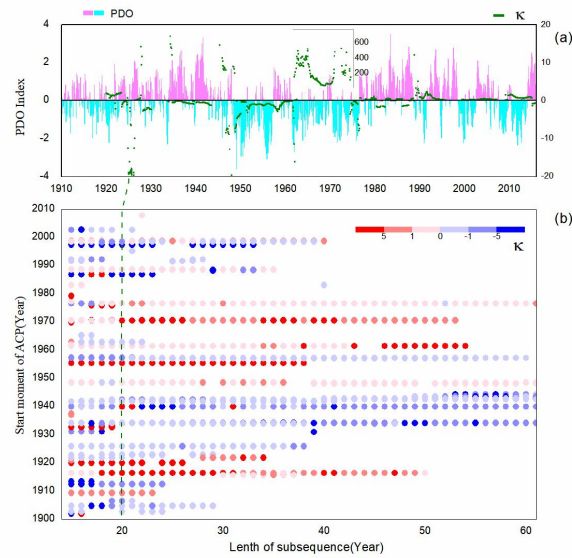
2

3 Figure 4. The schematic diagram of prediction method

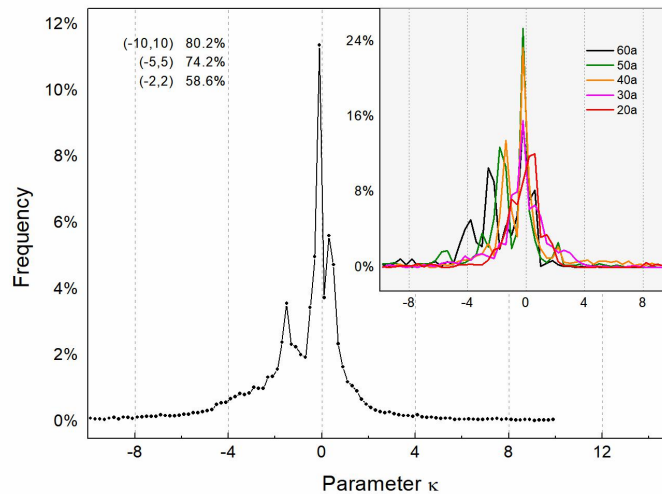


4

5 Figure 5. The ideal time sequence constructed by the logistic model and random
 6 numbers. (a) Completed transition process with length of 50, Uncompleted transition
 7 processes (the gray lines) and their prediction result (the blue lines) with length of
 8 25 (b), 26 (c), and 27 (d)



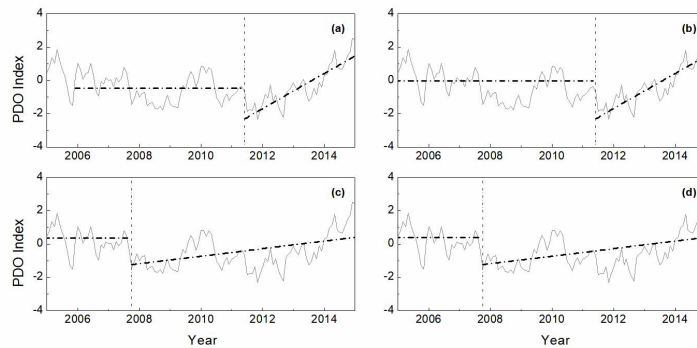
1
 2 Figure 6. Identification of the PDO time sequence and stability parameter κ during an
 3 abrupt change under different sub-sequence lengths. (a) The X-axis is the year, the
 4 histogram in the figure shows the PDO time sequence (left panel), and the green dots
 5 indicate the magnitude of parameter κ when the sub-sequence is 20 years (right panel);
 6 (b) the identification of abrupt change and the stability parameter identified for
 7 different sub-sequence lengths (pink indicates augmented abrupt change, and blue
 8 indicates decreasing abrupt change, with deeper colors representing higher values).
 9 The X-axis is the sub-sequence length (month), and the Y-axis is the beginning time
 10 of abrupt change (year).



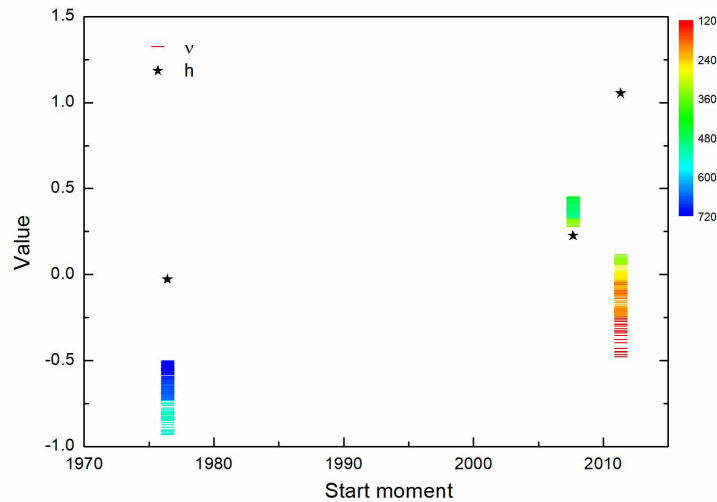
11



- 1 Figure 7. Statistical results of stability parameters for different sub-sequence lengths.
- 2 The X-axis is the value of the parameter, and the Y-axis is the statistical frequency
- 3 with a sub-sequence length of 10 years. The gray region in the upper-right corner is
- 4 for a sub-sequence of 20-60 years.



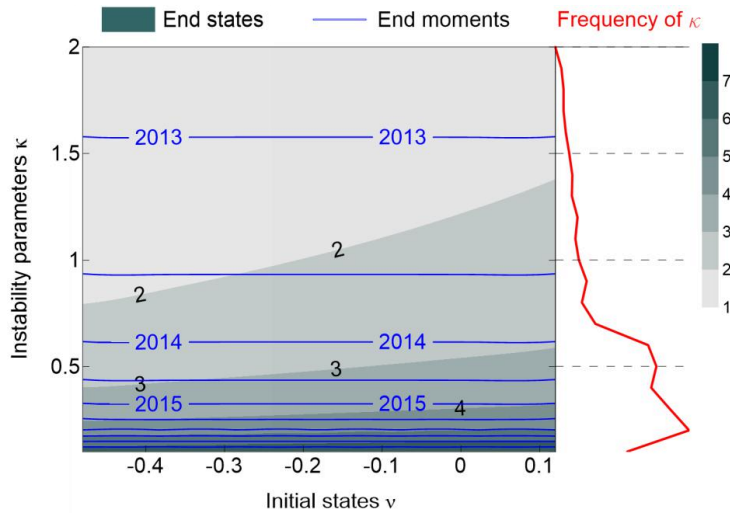
- 5
- 6 Figure 8. The PDO time sequences and the detection of parameters v and h when the
- 7 sub-sequence was set at (a)10 years, (b)20 years, (c)30 years, (d)40 years. The gray
- 8 lines is PDO time sequences. The horizontal dash lines are stable states before climate
- 9 change point, the slope dash lines represent transition processes of climate change,
- 10 and vertical dotted line is the start moment of climate change.



- 11
- 12 Figure 9. Several abrupt changes detected when the sub-sequence was set at different
- 13 lengths, as well as the values of the initial state v and linear trend h for the abrupt
- 14 changes. The black asterisk represents parameter h , and the colourful short bar

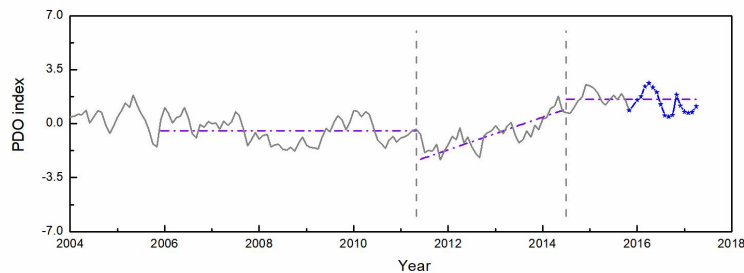


1 represents parameter v . The colour bar represents the sub-sequence length.



2

3 Figure 10. Variation ending state and ending time for a system with the initial state
 4 parameter v (horizontal ordinate) and stability parameter k (vertical coordinate). The
 5 red line on the right side shows the probability distribution of stability parameter k .



6

7 Figure 11. Prediction of the PDO index turning point. The gray solid line is the actual
 8 PDO index used in the study until November of 2015; the solid line with an asterisk is
 9 the actual PDO index from December 2015 to April 2017; the gray vertical dashed
 10 line is a diagram for the beginning and ending times of abrupt change, which are 2011
 11 and 2015, respectively; the dot-dashed line represents the system state at different
 12 stages.