Dear editors and reviewers,

Thanks a lot for the comments and suggestions about our research.

Over the last month, we tried our best to fix this manuscript and reply the comments one by one as follows. The useful suggestions help us understand the key and advantages of our prediction method deeply. The key is the instability parameter k, which is also proposed in our method for the first time. At present, there is no other way to detect the transition process at the end of the time sequence, which is actually related to extreme events. Therefore, we hope that the transition process which has just started can be detected and predicted. That is what we did in this manuscript.

Best wishes,

All authors.

==

Reviewer’s Comment 1

General comments

• Section 2.1

1. It is never explicitly stated which equilibrium states are represented by u and v (which is the start and which is the end state.

[REPLY]: Below equation (2), we added “Parameters u and v represent the two equilibrium states respectively. Parameter u represents initial state, and parameter v represents end state. ” to state the meaning of parameters u and v.

2. Figure 2a shows an example for $k = -0.4$, not $k = 0.4$. Adjust figure or text accordingly.

[REPLY]: We check the parameters values, and they should be:

v=1.0, u=2.0, k=-0.4 for the gray line,
v=-1.0, u=1.0, k=0.4 for the gray line,
v=-1.0, u=1.0, k=0.3 for the gray line.

We correct the text and figure 2a as follows:

As shown in figure 2a, parameters u and v being fixed, and setting k as 0.4 (the dash gray line), the system increasing to the new state costs a shorter time than that setting k as 0.3 (the black line). It is noted that if $k<0$ *(as* v=1.0, u=2.0 *and* k=-0.4 *), the system decreases from state 2.0 to state 1.0 as the gray line. This states that if the* *absolute value of k is relatively large, the shorter the transition time of the system, that is, the more unstable(Yan et al, 2016). If parameter k is set large enough, the system collapses and becomes chaotic as shown in figure 2b.*

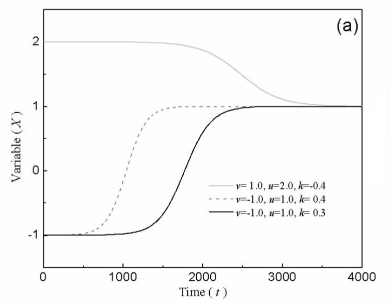

Figure 2. (a)The transition processes of system swinging between different stable states since the parameters are different.

3. Absolute value of *k* had not been previously discussed, which may also relate to my previous comment. Maybe discussing this earlier clears up what is going on in the figure.

[REPLY]: As in the previous reply, we added a discussion about parameter *k*. This result is based on a numerical experiment on the parameter *k*, which is discussed in detail in the previous article (Yan et al, 2016) as follows:

" …and the parameter κ is shown in Fig. 2. In Fig. 2a, the system reached the same state (x=μ) during the evolution by changing the initial variable in threshold ($x_0 \in$ (ν, μ)), when parameter κ (κ=0.1) was positive (black lines). The dashed lines show the condition that the system reached in the state x = ν, when the parameter κ (κ < 0) was negative. In Fig. 2b, the parameter κ. (κ = 105) was larger than before, and the system became bifurcated. If the parameter κ. (κ = 135) is much larger, the system will become chaotic as shown in Fig. 2c. Therefore, the parameter κ is a stability parameter, and the parameters μ and ν are the start and end states before and after the abrupt change…"

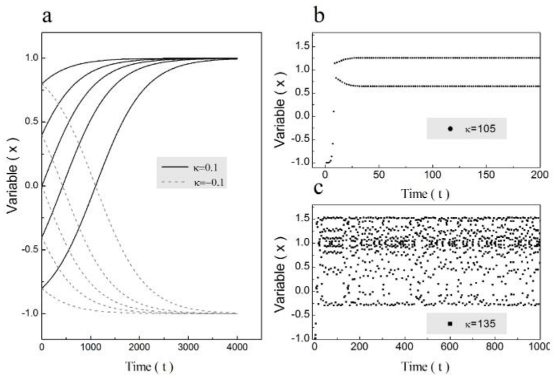

Figure 2. The evolution of the system over time, with different stability parameters: (a) the system reaches to the stable states with a different initial variable when parameter κ = ±0.01; (b) the system becomes bifurcated when the parameter κ = 105; (c) the system becomes chaotic when the parameter κ = 135. [Reference from the previous article (Yan et al, 2016)]

4. (previous comment) The statement "*According to Thom's theory (1972), the system described by a quartic function . . .* " is still unclear. The authors' response cleared up the confusion between quartic and quadratic from the previous version, but did not make this sentence clearer to the reader. According to the theory, if the system's general potential energy is described by a quartic function then the system has a tipping point. The system itself does not need to be described by a quartic function.

[REPLY]: According to Thom's theory, the quartic function of generalized potential energy characterizes an abrupt change. The characteristic of equation (2) has to be confirmed by checking its generalized potential energy. The expression of external force, which is connected with the acceleration, is the second derivative of the state variable. Then, the external force f(x) is obtained as follows:

$$
\begin{aligned}
f(x) &= \frac{d^2x}{dt^2} = \frac{d[k(x-u)(v-x)]}{dt} = \frac{d(-kx^2+k(u+v)x-kuv]}{dt} \\
&= [-2kx+k(v+u)]\frac{dx}{dt} = 2k^2[x-(u+v)/2](x-u)(x-v) \\
&= k^2[2x^3-2(u+v)x^2+(u^2+v^2+4uv)x-(u+v)uv]
\end{aligned}
.
$$

Then, its generalized potential energy can be expressed as:

$$
\begin{aligned}
V(x) &= -\int_0^x f(x)dx = -\int_0^x k^2[2x^3-2(u+v)x^2+(u^2+v^2+4uv)x-(u+v)uv]dx \\
&= \frac{k^2}{2}[x^4-2(u+v)x^3+(u^2+v^2+4uv)x^2-2(u+v)uvx]
\end{aligned}
,
$$

which means that the logistic model, $\frac{dx}{dt}=k(x-u)(v-x)$, can be used to represent the abrupt change.

5. Variable n_2 is introduced in the text, but n_1 and n_3 are not.

[REPLY]: Variables n_1, n_2, n_3 represent the lengths of the first, the second and the third segment respectively. This part has been corrected.

6. (previous comment) The parameter *h* is defined twice, where one is an approximation of the other. The parameters should be labelled differently to clarify which *h* one is discussing in the rest of the manuscript.

[REPLY]: Actually, the linear trend parameter *h* was only defined to be the ratio of system state change to time change. The value of parameter *h* has different way to obtain. One is to calculate by regression method, and another is to obtain according to the definition, which by using the solution of the equation.

- The regression method to calculate *h* based on equation (6) in the manuscript:

$$
h = \sum_{i=n_1+1}^{n_1+n_2} \bar{i} \cdot \bar{x}_i \Big/ \sum_{i=n_1+1}^{n_1+n_2} \bar{i}^2
.
$$

- Another way to calculate *h* by using the solution,

$$t=\frac{1}{k(u-v)}\ln\left(\frac{x-v}{x-u}\cdot\frac{x_0-u}{x_0-v}\right)+t_0,$$ of equation (2),

$$h=\frac{x_b-x_a}{t_b-t_a}=\frac{(\beta-\alpha)(u-v)}{\frac{1}{k(u-v)}\left(\ln\left(\frac{x_b-v}{x_b-u}\right)-\ln\left(\frac{x_a-v}{x_a-u}\right)\right)}=k(u-v)^2\frac{(\beta-\alpha)}{\ln\frac{\beta(\alpha-1)}{\alpha(\beta-1)}}=k(u-v)^2\chi$$

.

This is the key part of the prediction method we proposed in this manuscript. We can not get the values of parameter k directly. Only by using the above equation can calculate k value on the basis of obtaining the other values of parameters h, v, u, where the χ value is constant. Then, the threshold of parameter k is revealed.

7. (previous comment) Punctuation was added to Eq. 5 and 6, but they were not properly incorporated into a sentence as suggested.

[REPLY]: Thanks a lot for such detailed suggestion, and we correct these mistakes.

8. The variables x_0 and t_0 are used in Eq. 6 and never introduced in the text.

[REPLY]: The variables t_0 and x_0 represent the initial time and the initial state respectively. They are mentioned in the manuscript.

9. (previous comment) Punctuation was added to Eq. 5 and 6, but they were not properly incorporated into a sentence as suggested.

[REPLY]: We correct the mistakes.

10. (previous comment) The relation of α and β to x_a and x_b was explained in the authors' response to my comments but was not clarified in the text of the paper. The mathematical relationship is only seen in Figure 2d but it should also be in the text as it is necessary for the understanding of the mathematical derivation in this section.

[REPLY]: We supplement more equations now, and introduce how to get them step by step.

11. (previous comment) The parameter μ is used in equation 7 but not introduced previously.

[REPLY]: The parameter μ should be u, which is modified in the text.

12. (previous comment) Eq. 7 and 8 are still not clearly integrated into the text as previously suggested. The equations should not be referenced before they are introduced. (This point holds for all equations in the manuscript.)

[REPLY]: We apologize that we did not explain them clearly in the previous responses. Now, more equations are introduced to explain how to work about the prediction method step by step.

13. Parameter ω is never defined in the text but is used in Eq. 8.

[REPLY]: The parameter ω is defined to be the difference between the initial state v and the end state u, which is called the amplitude of change. Now, we delete the parameter ω, and describe the amplitude of change with u-v directly in text.

• Section 2.2

1. Eq. 9 seems out of place and not incorporated into any text where it is introduced.

[REPLY]: We used the equation to build the ideal time sequence. The description is corrected as follows:

In order to test the prediction method,an ideal time sequence is constructed by using equation (12), which is the sum of the logistic model and random numbers, where η_t *represents the random number,*

$$\begin{cases} x_t = x_{t-1} + kt(x_t - u)(v - x_t) \\ x'_t = x_t + \eta_t \end{cases} . \quad (12)$$

2. The statement "*The end moment and the end state of the prediction result match the presetting lines.*" is not entirely accurate. The two predictions using 250 and 260 moments can be argued to match the truth, but the first using 240 moments appears to obviously overestimate the end state.

[REPLY]: We add calculation about the prediction errors for the three time sequences, and they are 0.37, 0.27 and 0.26 respectively, which state that the prediction error is small when the transition process experienced is longer. We give more discussion about the prediction error as follows in the text:

"The prediction end moment and the prediction end state are basically consistent with the original time sequences. However, the average absolute prediction errors of the three time sequences are 0.37, 0.27 and 0.26 respectively. When the length of the sequence is 240, the prediction state is overestimated, and the average absolute prediction error is 0.37. With the length of the system experienced expanding, the prediction error decreases. The prediction states are very close to the original states when the length is 260. Therefore, in the actual prediction, we hope that the transition process has been experienced for a long enough time, which will help to predict accurately."

• Section 3.1

1. (previous comment) The phrase "*transition change*" (or "*transition changes*") is still used in this section and subsequent sections.

[REPLY]: The mistake is corrected.

2. The text states that Figure 6b has a variation of 20-60 years of subsequence lengths, yet the figure appears to show from 15-60 years.

[REPLY]: The mistake is corrected to be 15-60 years in the text.

3. (previous comment) When Figure 7 is first discussed in the text, it is unclear which sub-sequence results are being discussed. I assume its the 10-year sub-sequence, but it

is never specified. Also, a quantitative definition of a peak is never introduced in the text, nor in the authors' previous response (the authors' mention "*extremely high frequency*" without an actual threshold on what defines a frequency to be "*extremely high*".

[REPLY]: Thanks a lot for the comment about how to quantify the peak, which is also confuse us. Now, the reference line is set as 5%. Only the frequency which is bigger than 5% can be considered to be one mainly peak. Besides, we also recalculate the frequencies of *k* values, the values in the previous figure 7 (a) were truly wrong. There is one mainly peak for that the length of sub-sequence is less than or equal to 30 years, and there are two mainly peaks for that the length of sub-sequence is greater than 30 years. **There is a "continuous relationship" as mentioned in the previous GENERAL COMMENT #6.** Without this comment, the mistake about the *k* values for that the length is 10 years would not be found. Therefore, we are very grateful to the reviewers for insisting on this comment.

More discussion is supplemented in text as follows:

"The small figure in figure 7 (a) shows that the k values (marked with green dots) are more that 100 during 1960~1970 when the length of sub-sequence is 20 years. The frequencies of parameter k values when the length of sub-sequence are 10, 20, 20, 40, 50, and 60 years respectively are displayed in figure 7. It is noted that some of the values of parameter k are so large that their frequencies are almost zero which are not necessary to be counted. The frequencies of k values which belongs to -10 to 10 are shown in figure 7. By considering the frequency which is bigger than 5% as a peak, there is only one peak when the length of sub-sequence is 10 years. Most of the k values are concentrated around zero, which means that the transition processes detected are stable. For the situations which the lengths of sub-sequences are 20 and 30 years, there is one mainly peak, and it is near zero. When the lengths of sub-sequences are more than 30 years, there are two mainly peaks. One is near zero, and another is much less than zero. It states that the much more unstable transition processes are detected when the lengths of sub-sequences are large. From the perspective of the k threshold values, the k values in the range of (-10, 10) accounts for 63.90%, 70.64%, 77.00%, 90.05%, 93.69%, and 89.90% of all k values for that the lengths of sub-sequences are 10, 20, 30, 40, 50, 60 years respectively. They are 55.64%, 67.52%, 73.99%, 83.45%, 85.46%, 84.82% for the range of (-5, 5) and 35.64%, 62.22%, 59.36%, 68.28%, 66.25%, 47.55% for the range of (-2, 2). In the following studies, the k values are mainly considered to be in the range of (-2, 2)."

• Section 3.2

1. (previous comment) The phrase "*transition change*" (or "*transition changes*") is still used in this section.

[REPLY]: This mistake is corrected.

• Section 4

1. Along with my comment #2 for Section 2.2, not all of the ideal experiments accurately predict the end state. This first (using 240 moments) seems to overestimate

the state. This should be noted and discussed.
[REPLY]: We discuss more about the overestimate when the length is 240, and the prediction error is 0.37. With the length of the system experienced expanding, the prediction error decreases. The prediction states are very close to the original states when the length is 260. This reveals that we will predict the turning point (end state and end moment) well if the transition process has been experienced for a long period.

2. (previous comment) The phrase "*transition change*" is still used in this section.
[REPLY]: This mistake is corrected.

• Figures
1. In Figure 2a the legend appears to be wrong. The lines do not correspond to their starting (v) and ending (u) values
[REPLY]: The mistakes are corrected for the legend of figure 2a as the reply to comment #2 for Section 2.1 in page 1.

2. Caption of Figure 6 is wrong. The X-axis of 6b shows sub-sequence length in years, not months.
[REPLY]: This mistake is modified.

3. Caption of Figure 7 has not been adjusted to reflect the new figure.
[REPLY]: The caption is changed to be "Figure 7. Statistical results of instability parameters for different sub-sequences lengths. The X-axis is the value of parameter k, and the Y-axis is the statistical frequency for the sub-sequence length of 10a, 20a 30a 40a 50a and 60a".

Reviewer's Comment 2
Major comments:
1. According to the Thom's theory (1972) there are at least 4 different types of dynamics which describe different physical systems. In page 5, line 2 the authors state "It means that Eq. (2) describes a system with tipping-point abrupt change". This is not correct since the tipping-point picture proposed by Thom (1972) is associated with a thirth-order polynomial function, generally known as "fold catastrophe". The authors refer to a quartic polynomial function which is related to the so-called "cusp catastrophe", that can be related to different kinds of bifurcation (e.g., the pitchfork). The bistable function the authors found is indeed very common for the climate system (as for simple energy-balance models, see a lot of papers by Ghil and coauthors, or the famous stochastic resonance mechanism, see Benzi et al. papers) but the equation governing the dynamics of the system should be characterized by a third-order polynomial in the righthand term. Indeed, given a forcing F(x) the dynamics of the system can be described by dx = F(x) dt and assuming that F(x) is conservative then there exists a potential function V(x) such that F(x) = -dV(x)/dx. This means that if F(x) is a polynomial function of order n, then V(x) is a polynomial function of order

n+1. By looking at Eqs.(1) and (2) of the present paper there is a discrepancy between the quadratic forcing term of Eq. (1) and the quartic potential function of Eq. (2). The authors need to carefully consider this point.

[REPLY]: Thanks a lot for the suggestion. The logistic model represents the first derivation of the state variable with respect to time,

$$\frac{dx}{dt} = k(x-u)(v-x).$$

The second derivative with respect to time is the acceleration, which is proportional to the external forcing marked with *f(x)*. Then, we have

$$\begin{aligned} f(x) &= \frac{d^2x}{dt^2} = \frac{d[k(x-u)(v-x)]}{dt} = \frac{d(-kx^2+k(u+v)x-kuv]}{dt} \\ &= [-2kx+k(v+u)]\frac{dx}{dt} = 2k^2[x-(u+v)/2](x-u)(x-v) \\ &= k^2[2x^3-2(u+v)x^2+(u^2+v^2+4uv)x-(u+v)uv]. \end{aligned}$$

The above formula was omitted from the original manuscript, now we supplement it in the current version. The generalized potential energy (Benzi et al, 1982) is expressed as the integral of the external force to the state,

$$\begin{aligned} V(x) &= -\int_0^x f(x)dx = -\int_0^x k^2[2x^3-2(u+v)x^2+(u^2+v^2+4uv)x-(u+v)uv]dx \\ &= \frac{k^2}{2}[x^4-2(u+v)x^3+(u^2+v^2+4uv)x^2-2(u+v)uvx], \end{aligned}$$

And it is a quartic potential function.

We hope that such a supplement in the manuscript can make this part easier to understand.

2. Fig. 6: in my opinion the authors should revise this figure, also concering comments raised by previous reviewers. There is still a discrepancy between the different form of k, it is not simply readable, and it seems to me that there is something wrong in plotting the green dots. Please revise.

[REPLY]: Thanks a lot about this. The letters are so similar that it is difficult to distinguish them. We apologize for this and modify the mistakes in the manuscript now.

3. Fig. 7: as for Fig. 6 in my opinion the authors should revise this figure. Where is the gray region in the upper-right corner stated in the caption? Moreover, the authors should also insert

[REPLY]: This is a mistake. The caption has been corrected as:

"Figure 7. Statistical results of instability parameters for different sub-sequences lengths. The X-axis is the value of parameter k, and the Y-axis is the statistical frequency for the sub-sequence length of 10a, 20a 30a 40a 50a and 60a."

4. I'm not sure I completely understood the main purposes of the manuscript as written in the present form. Indeed, it seems to me that the authors state that "By using a piece-wise function, the transition process is stated approximately" (lines 14-15 in the Abstract) and that "Thus, we had proposed a method (Yan et al, 2015, 2016) to study the transition process by using a continuous function" (lines 17-18 in the Abstract) but they are using a piecewise prediction method which is based on a linearization of the logistic equation. So, what would be the main benefit? Why not to directly use the logistic fit for investigating the transition?

[REPLY]: This is a quality comment. We also hope that we can obtain the parameters of the logistic model by fitting the climate time sequence with the logistic model directly. Unfortunately, this is not easy.

The transition process also can be obtained by fitting the climate time sequence with the ramp function which is also called piece-wise function. However, there is no dynamics of the piece-wise function, which means that there is not quantitative relationship during the transition process. The logistic model can provide enough parameters to describe the dynamic process. Next, we analyze how to obtain the parameters of the logistic model.

For a climate time sequence, we don't know when the abrupt change happens, which means that any length of the entire sequence must be taken out to study. That's what we did.

We use the solution, $x=\frac{u-v}{1-e^{k(u-v)(t-t_0)}}+u$, of logistic model to fit the segment of the entire time sequence as shown in figure 2.1d. We have the values of variables x and t, but it is not easy to know the values of parameters u, v and k. Assuming $t_0=0$, we can obtain the value on the right side of the above equation by given values of parameters u, v and k. And the Root Mean Square Error Function with x is established,

$$\begin{cases} x'_t=\frac{u_m-v_n}{1-e^{k_o(u_m-v_n)t}}+u_m \\ \xi_{m,n,o}=\left(\frac{1}{T}\sum_{t=1,T}(x_t-x'_t)^2\right)^{\frac{1}{2}} \end{cases},$$

where the parameter T represents the total time of the segment. The parameter u_m represents for any value in the range of the parameter u. The parameters v_n and k_o represent for any values in the range of the parameters v and k respectively.

In order to simplify the calculation, the range of parameters needs to be determined in advance. It is obvious that the values of parameters u and v should be between the maximum and minimum values of the sequence. However, the range of the parameter k is unclear. Even if we give the k value a large enough range, it is still difficult for us to obtain enough precision for the parameters. Because if we want to improve the accuracy of the parameters by 10 times, the amount of calculation will be increased by 1000 times.

In figure 2.2, we had proposed a method to divide the segment into three parts. They are two equilibrium states and one transition process. We only need to change the length of each part and perform linear fitting to obtain parameters' values with more precision by using the following equation,

$$
\begin{cases}
v = \sum_{i=1}^{n_1} x_i / n_1 \\
u = \sum_{i=n_1+n_2+1}^{n} x_i / n_3 \\
h = \sum_{i=n_1+1}^{n_1+n_2} \bar{i} \cdot \bar{x}_i / \sum_{i=n_1+1}^{n_1+n_2} \bar{i}^2
\end{cases}
.
$$

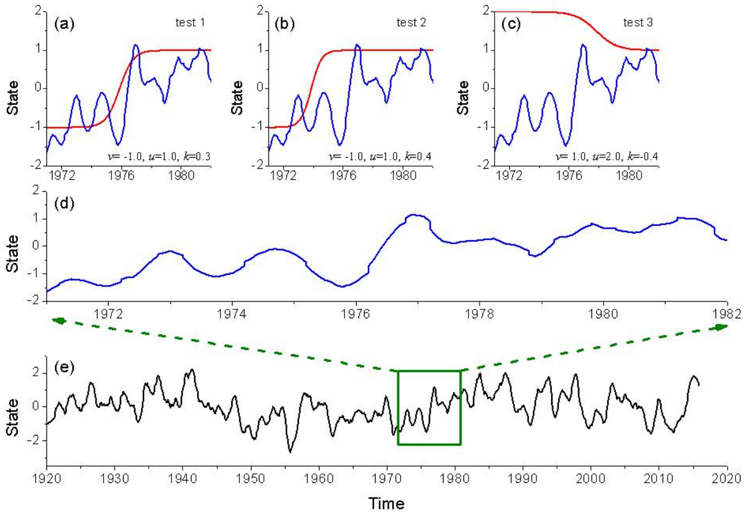

Figure 2.1 Schematic diagram of direct fitting method

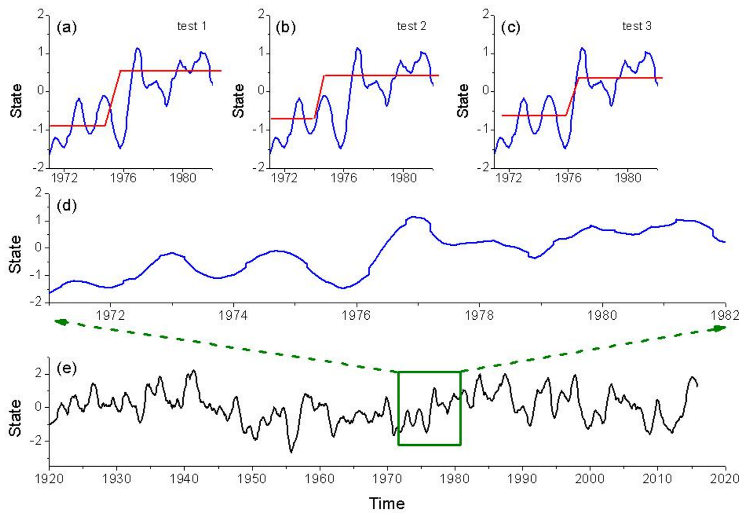

Figure 2.2 Schematic diagram of the method we had proposed

Moreover, the authors use the term "tipping point" but they are not considering/discussing implications of tipping points as well as comparing their results with established methods for tipping point evaluation (variance,

autocorrelation, ..., see papers by Ditlevsen, Lenton, and colleagues). I urge the authors reorganize both the introduction and the conclusion sections to take into account these aspects.
[REPLY]: Thanks for the suggestion about the references. We have added some researches on cusp catastrophe and its early warning in the text.

Minor comments:
- Page 2, line 6: the term "attractors" is too wide in this context, probably it is better to use "fixed points" or "equilibrium states".
[REPLY]: Thanks for the suggestion, we replace “attractors” to be “equilibrium states”.

- Page 4, line 9: the term "discontinuous" seems not to be appropriate, should be better "discrete".
[REPLY]: Thanks for the suggestion, we correct the mistake.

- Page 4, line 19: "k as 0.4" should be "k as 0.5". Indeed, in Fig. 1 being u and v fixed then k values are chosen as 0.3 and 0.5, when k=-0.4 is chosen different values for u and v are considered.
[REPLY]: Thanks for noticing these small differences. And they are mistakes. After checked the original data, we find that the k values are 0.4 (the dash gray line) and 0.3 (the black line)for v=-1, u=2. The k value is -0.4 for v=1.0, u=2.0. We correct this part as:
“ As shown in figure 2a, parameters u and v being fixed, and setting k as 0.4 (the dash gray line), the system increasing to the new state costs a shorter time than that setting k as 0.3 (the black line). It is noted that if k<0 (as v=1.0, u=2.0 and k=-0.4), the system decreases from state 2.0 to state 1.0 as the gray line. This states that if the absolute value of k is relatively large, the shorter the transition time of the system, that is, the more unstable(Yan et al, 2016). ”

- Fig. 5: are the authors showing x_t' and not x_t? Please confirm.
[REPLY]: We add the caption of figure 5 as:
“Figure5. The ideal time sequence constructed by the logistic model and random numbers. The X-axis represents time, and the Y-axis represents variable xt’. (a) Completed transition process with 500 moments, Uncompleted transition processes (the gray lines) and their prediction result (the blue lines) with (b) 240 moments, (c) 250 moments, and (d) 260 moments, the light gray lines are the original entire ideal time sequences.”

- Check some English forms through the text.
[REPLY]: We checked English forms through the text.

A method to predict the uncompleted climate transition process

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Abstract

Climate change is expressed as a climate system transiting from the initial state to a new state in a short time. The period between the initial state and the new state is defined as transition process, which is the key part to connect the two states. By using a piece-wise function, the transition process is stated approximately (Mudelsee, 2000). However, the dynamic processes are not included in the piece-wise function. Thus, we had proposed a method (Yan et al, 2015, 2016) to ~~study~~fit the transition process by using a continuous function. In this manuscript, this method is developed to predict the uncompleted transition process based on the dynamic characteristics of the continuous function. We introduce this prediction method in details and apply it to three ideal time sequences and the Pacific Decadal Oscillation (PDO). The PDO is a long-lived El Niño-like pattern of Pacific climate variability (Barnett et al, 1999). ~~This method reveals a~~A new quantitative relationship during the transition process has been revealed, and it~~which~~ explores a nonlinear relationship between the linear trend and the amplitude (difference) between the initial state and the end state. As the transition process begins, the initial state and the linear trend are estimated. Then, according to the relationship, the end state and end moment of the uncompleted transition process are~~is~~ predicted.

Keywords

Prediction method; Transition process of abrupt change; System stability; Pacific Decadal Oscillation

1. Introduction

A system transiting from one stable state to another in a short period is called abrupt change (Charney and DeVore, 1979; Lorenz, 1963, 1979). The abrupt change system has two or more states (Goldblatt et al, 2006; Alexander et al, 2012), the system swings between these states that are also called equilibrium states~~attractors~~ in physics. This phenomena is verified in many fields including biology (Nozaki, 2001), ecology (Osterkamp et al, 2001), climatology (Thom, 1972; Overpeck and Cole, 2006; Yang et al, 2013a, 2013b), brain science (Sherman et al, 1981), etc. The cusp catastrophe has been detected in climatology widely. Many researches studied the characteristics and early warning signals of the cusp catastrophe (Lenton, 2012; Pierini, 2012; Livina et al, 2012). The latest observed climate change event ~~i~~was global warning hiatus~~, which has been studied deeply by many researchers~~ (Amaya et al, 2018; Kosaka and Xie, 2013; Yang et al, 2017). In Thom's research(1972), ~~S~~seven different kinds of abrupt changes ~~a~~were mentioned~~ in Thom's research(1972)~~. Over the last several decades, many methods have been proposed to identify different kinds of abrupt change (Li et al, 1996), such as Moving T-Test, Cramer's (Wei, 1999), Mann-Kendall (MK, Goossens and Berger, 1986), Fisher (Cabezas and Fath, 2002), etc. It is noticed that most abrupt change detection methods suggest that the abrupt change is around a turning point. The significant difference between the average values of the two sequences on both two sides of the turning point is defined as the index to measure the abrupt change. It is difficult for ~~T~~th~~i~~ese kinds of ~~detection~~ methods~~ has a drawback. It is difficult~~ to detect the transition period of the abrupt change, and it is difficult to identify the abrupt change that occurs at the end of sequence.

Mudelsee (2000) studied the abrupt change of a time sequence and illustrated that abrupt change has a duration, which can be quantitatively described with a piece-wise (ramp) function. We developed the detection method by using a continuous function to replace the ramp function(Yan et al, 2014, 2015). The new method can confine the beginning and ending points of abrupt change and quantitatively describes the process of abrupt climate change, and three parameters are introduced. A quantitative relationship among the parameters is revealed (Yan et al, 2015). The relationship could be used to predict the end moment (state) if the system had left the original state but not yet reached to the new state, which is defined as an uncompleted transition process.

In this manuscript, three ideal time sequences are tested to study the prediction method. The prediction method is also applied to study the climate transition process of the PDO, which is an important signal that reveals climatic variability on the decadal timescale (Mantua et al, 1997; Barnett et al, 1999; Zhang et al, 1997; Yang et al, 2004). Previous studies (Lu et al, 2013; Trenberth and Hurrell, 1994) have indicated that there are many abrupt changes in the PDO over the past 100 years. Most researches mentioned the climate changes happened in the 1940s and 1970s. During the 1940s, the PDO transited from a high state to a low state, while during the 1970s, it did the opposite. All of these changes and their processes had been studied in our previous research (Yan et al, 2015 2016). The climate transition processes were explored clearly. However, we still can not know when the transition processes finish their increasing or decreasing to a stable state if the transition process has begun. We develop a new method to predict the end state and the end moment of a transition process based on the quantitative relationship.

2. Methods

It is necessary to describe the transition process quantitatively before the prediction of the uncompleted climate transition process. ~~We~~The ~~had proposed a~~ detection method by using the logistic model to obtain a transition process ~~. In section 2.1, the method~~ is introduced in section 2.1 ~~briefly~~. On the basis of the detection method, the prediction method for studying the uncompleted transition process is further developed in section 2.2.

2.1 The detection method of transition process

The real time sequence changes abruptly as shown in figure 1a, and the system jumps to a high state in point C. If the period around point C is observed on a shorter time scale (as shown figure 1b), a transition period is obtained, and it is a part of the original time sequence. In fact, many abrupt changes could be considered to be a transition period with a more detailed view. The transition period was expressed with a ramp function in Mudelsee's research (2000) as shown in figure 1c, and the time sequence is divided into three segments, including two equilibrium states and one increasing state. The ramp function is as follows:

$$x_t = \begin{cases} x_1 & t \le t_1 \\ x_1 + (t - t_1)(x_2 - x_1)/(t_2 - t_1) & t_1 < t \le t_2 \\ x_2 & t > t_2 \end{cases} \quad \text{——} \qquad (1)$$

,

where t represents time, and x_t represents the system state~~s~~. Before t_1 and after t_2, the system stays in the two equilibrium states x_1, and x_2. Between t_1 and t_2, the system's states are a straight line. ~~, which is obtained by the linear regression method.~~ It is noted that the climate system ~~is~~should be smooth and continuous; it is even the sampling sequence that makes it is discrete~~discontinuous~~. We used a continuous function to express this transition period approximately, and we also created a novel method to detect the transition period (Yan et al, 2015). ~~Here, the detection method is introduced briefly.~~ As shown in figure 1d, ~~T~~the transition process ~~continuous evolution of the logistic model~~ is consistent with the continuous evolution of the logistic model, ~~transition process~~ which was created to describe the evolution of population model(May, 1976)~~, which is shown in figure 1d~~. The modified logistic model with two changeable equilibrium states~~–~~ is expressed as ~~follows~~equation (2), which x represents variable of system state, ~~:–~~

$$\frac{dx}{dt} = k(x-u)(v-x). \qquad (2)$$

Parameters u and v represent the two equilibrium states respectively. Parameter u represents insitial state, and parameter v represents end state. Parameter k represents the switching between different states, and it is defined as the instability parameter. As shown in figure 2a, parameters u and v being fixed, and setting k as 0.54 (the dash gray line), the system transitincreasing to the new state costs a shorter time than that setting k as 0.43 (the black line).– It is noted that if k<0 (as v=1.0, u=2.0 and k=-0.4), the system decreases from state 2.0 to state 1.0 as the gray line. This states that if the absolute value of k is relatively large, the shorter the transition time of the system, that is, the more unstable(Yan et al, 2016). If parameter k is set large enough, the system collapses and becomes chaotic (as shown in figure 2b). When parameter k is set to different values, more situations have been discussed in detail in the previous research (Yan et al, 2016). The result shows that parameter k characterizes the stability of the system (the larger the absolute value, the more unstable the system). .

In equation (2), Tthe first derivative of state variable to time is given, and it is regarded as velocity,. then tThe acceleration $\frac{d^2x}{dt^2}$ g –of state variable is expressed as the derivative of velocity to time, as shown in Eq.equation (3). The acceleration of the system is considered to come from the external forceforced force $f(x)$, which is expressed as $f(x)=m\frac{d^2x}{dt^2}$. Assuming that the coefficient m is 1, the generalized potential energy (Benzi et al, 1982) is expressed as the integral of the generalized force to the system state, as shonwn in Eq.equation(4),.

$$
\begin{aligned}
f(x) &= \frac{d^2x}{dt^2} = \frac{d[k(x-u)(v-x)]}{dt} = \frac{d(-kx^2+k(u+v)x-kuv]}{dt} \\
&= [-2kx+k(v+u)]\frac{dx}{dt} = 2k^2[x-(u+v)/2](x-u)(x-v), \\
&= k^2[2x^3-2(u+v)x^2+(u^2+v^2+4uv)x-(u+v)uv]
\end{aligned}
$$

(3)

$$V(x) = -\int_0^x f(x)dx = -\int_0^x k^2[2x^3 - 2(u+v)x^2 + (u^2+v^2+4uv)x - (u+v)uv]dx$$
$$= \frac{k^2}{2}[x^4 - 2(u+v)x^3 + (u^2+v^2+4uv)x^2 - 2(u+v)uvx] \quad .\quad (4)$$

According to Thom's theory (1972), the generalized potential energy of the system described by a quartic function would exhibit cusp catastrophe in which the system jumps from one state to a new state abruptly. Thus, we did some mathematical derivation to Eq. (2), and the general potential energy is obtained as follows:

$$V_{(x)} = -\int_0^x \ddot{x}dx = -\int_0^x 2k^2[x-(u+v)/2](x-u)(x-v)dx$$
$$= \frac{k^2}{2}[x^4 - 2(u+v)x^3 + (u^2+v^2+4uv)x^2 - 2(u+v)uvx] \quad (3)$$

It means that Eq. (2) describes a system with tipping-point abrupt change. In figure 2c, the potential energy of Eq.equation (34) is verified to have two states with the lowest energy, and both of them are stable. This bistable structure is common in the climate system (Goldblatt et al, 2006). Therefore, Eq.equation (2) can be used to describe the abrupt change system, and the parameters of equation (2) represent different key factors of the transition period during abrupt change. In order to obtain the values of parameters, the time sequence is divided into three segments. The first and the third segment represent the two equilibrium states, while the second segment represents the transition process. During the transition process, we define a new parameter h to represent the ratio of system state change to time, and it is called linear trend. As equation (5), the linear trend h can be expressed by two points on the curve approximately, where the two points are A (x_a, t_a) and B (x_b, t_b) which are displayed in figure 2d,

$$h = \frac{x_a - x_b}{t_a - t_b} \quad . \quad (5)$$

In equation (6), the values of parameters v and u are calculated. The value of parameter h can be calculated by Then, the parameters u, v and h are obtained by the regression method (Huang, 1990; Yang et al, 2013a) based on the time sequence of the second segment by using Eq. (45),, where i, x_i denote the time and the system state of the system at this time, and $\bar{i}, \bar{x}_i$ are their averages respectively. Variables n_1, n_2 n_2, n_3 isrepresent the lengths of the secondfirst, the second and the third segment respectively.

$$\begin{cases} v = \sum_{i=1}^{n_1} x_i / n_1 \\ u = \sum_{i=n_1+n_2+1}^{n} x_i / n_3 \\ h = \sum_{i=n_1+1}^{n_1+n_2} \bar{i} \cdot \bar{x}_i / \sum_{i=n_1+1}^{n_1+n_2} \bar{i}^2 . \end{cases} \quad (456)$$

The points A (x_a, t_a) and B (x_b, t_b) are alsoThe linear trend h represents the ratio of system state change to time, and it can be expressed by two points on the curve, thus we are going to calculate the solution of equation (2)approximately as Eq. (56), where the two points are A (x_a, t_a) and B (x_b, t_b). The equation 2 is rewritten to be equation 7 and both sides of the equation 7 are integrated as equation (8),

$$dt = \frac{dx}{k(x-u)(v-x)} \quad (7)$$

$$h = \frac{x_a - x_b}{t_a - t_b} \quad (56)$$

$$\int_{t_0}^{t} dt = \int_{x_0}^{x} \frac{1}{k(u-v)} \left(\frac{1}{x-u} - \frac{1}{x-v}\right) dx$$

$$\Rightarrow t\Big|_{t_0}^{t} = \frac{1}{k(u-v)} \ln\left(\frac{x-v}{x-u}\right)\Bigg|_{x_0}^{x} \quad (8)$$

$$\Rightarrow t = \frac{1}{k(u-v)} \ln\left(\frac{x-v}{x-u} \cdot \frac{x_0-u}{x_0-v}\right) + t_0$$

As shown in figure 2d, the transition period during point A (x_a, t_a) and point B (x_b, t_b) is approximately linear. Then, we can use the location parameters α, β to express system states x_a and x_b. By solving Eq. (2), the relationship between x and t is determined.

$$t = \frac{1}{k(u-v)} \ln\left(\frac{x_0 - u}{x_0 - v} \cdot \frac{x - v}{x - u}\right) + t_0 \qquad (67)$$

Where the variables t_0 and x_0 represent the initial time and the initial state respectively. The transition process has been assuming to be linear, thus we define location parameters α and β, $\alpha = \frac{x_a - v}{u - v}, \beta = \frac{x_b - v}{u - v}$, and x_a, x_b are expressed as follows,

$$\begin{cases} x_a = \alpha(u-v) + v \\ x_b = \beta(u-v) + v \end{cases} \qquad (9)$$

Substituting equation (8) and equation (9) into equation (7), we have:

$$h = \frac{x_b - x_a}{t_b - t_a} = \frac{(\beta - \alpha)(u - v)}{\frac{1}{k(u-v)}\left(\ln\left(\frac{x_b - v}{x_b - u}\right) - \ln\left(\frac{x_a - v}{x_a - u}\right)\right)} = k(u-v)^2 \frac{(\beta - \alpha)}{\ln \frac{\beta(\alpha - 1)}{\alpha(\beta - 1)}} \qquad (10)$$

Then, parameter h is rewritten as Eq. (78). It is noted that the rightmost part is only related to parameters α and β,the location parameters α and β, then let it be χ. Then, the relationship of Eq.equation (7810) is rewritten as Eq.equation (8911),:

$$h = \frac{x_b - x_a}{\frac{1}{(u-v)k} \ln \frac{x_0 - u}{x_0 - v}\left(\frac{x_b - v}{x_b - u} - \frac{x_a - v}{x_a - u}\right)} = k(u-v)^2 \frac{(\beta - \alpha)}{\ln \frac{\beta(\alpha - 1)}{\alpha(\beta - 1)}} \qquad (78)$$

$$h = k(u-v)^2 \chi .$$

(8911)

The difference between the initial state v and the end state u, u-v, is called as the amplitude of change. In order to determine the value of parameter χ, the relationship among χ, α, β is displayed in figure 3b. The dash line in figure 3a is the profile of the diagonal in figure 3b, which represents that the sum of α and β is 1. Parameter χ changes little when the location parameter varies in a certain range as marked with warm color in figure 3b. It means that the closer the points (A and B) are to the middle point, the more significant the linear feature is. Then, the process between point A and point B can represent the whole transition process as shown in figure 3c. It is noted that the transition process is symmetrical about the middle point approximately. Thus, we assume that point A and point B are symmetrical about the middle point, and the sum of α and β is 1. The change of parameter χ is only related to parameter α (or parameter β), as shown in the diagonals in figure 3b (also in figure 3a). Parameter χ changes little when parameter α is about 0.2 or larger. In figure 3c, three different situations are carried out to study the influence of parameter α on parameter χ. In each situation, points (A and B) are set to be different positions, and their parameters were calculated respectively in table 1. The parameters α are set as 0.20, 0.25, 0.15 respectively in three different situations marked with S1, S2 and S3. For S2 and S3, both of the percentages of α changing to S1 are 25%, while the percentages of χ changing are only 5.15% and 6.76% respectively, which means the percentage change of χ is much less than α. In addition, linear trends of these three ideal models are calculated according to the points and by regression method which are marked as h_0 in table 1. The linear trends are also calculated by the values of point A and point B with Eq(5) which are marked as h in table 1. It is noted that although the positions of points are different, the trend obtained according to the points is almost the same as that obtained by regression method. The error percentages are 2.36%, 2.25%, 1.38% respectively, which means that we don't have to know the exactly positions of point A and B (the values of parameters α and β). We can approximate the value of χ. Thus, in the following sections parameter α is set as 0.2, and parameter χ is 0.2164.

2.2 The prediction method of transition process

Equation. (8911) shows the quantitative relationship among linear trend, instability parameter, and amplitude of change. There is a linear relationship between linear trend and instability parameter; and there is thea quadratic function relationship between linear trend and amplitude of change. We ~~did~~ revealed this quantitative relationship ~~much more than in theory but in real time series~~ based on sea surface temperature(Yan et al, 2016). ~~Based on~~According to this relationship, we are going to ~~create~~develop a new method to predict the transition process which has not been completed. A brief description about the new method is stated with the schematic diagram in ~~During the real time sequence~~figure 4~~,~~. ~~the system transits away from the original state, but it has not reached to a new state as shown in figure 4.~~ The red line represents the period which has been experienced, while the gray line represents the period which has not been experienced. Based on the system states which are far away from the original state, a quasi linear extension of the transition process is established ~~(~~as dash line~~)~~. Then the parameters v and h are obtained by ~~Eq.~~equation (4~~5~~6). The instability parameter k represents the stability of abrupt changes of the system, which means that it's related to the system. Its threshold can be estimated by history data. ~~Assuming that the parameter k satisfies the statistics in the history of the system, t~~The parameter u ~~can be~~is predicted by ~~Eq.~~equation (~~8~~9~~11~~), and the end moment can be determined according to the definition of linear trend h. ~~, and the end moment is also predicted.~~

In order to test the prediction method

$$\begin{cases} x_t = x_{t-1} + kt(x_t - u)(v - x_t) \\ x'_t = x_t + \eta_t \end{cases}$$

~~(9~~10~~)~~

~~As shown in figure 5, ,four~~an ideal time sequence~~s~~ ~~are~~is constructed by using ~~the logistic model and random numbers as Eq.~~equation (~~9~~1~~0~~2), which is the sum of the logistic model and random numbers, ~~,~~where η_t represents the random number,

$$\begin{cases} x_t = x_{t-1} + kt(x_t - u)(v - x_t) \\ x'_t = x_t + \eta_t \end{cases} \quad (12)$$

As shown in figure 5, An entire time sequence with 500 moments is shown in figure 5a and three other ~~lengths of~~ time sequences with different lengths are shown in figures 5b, 5c and 5d respectively. The parameters v, u and k of the logistic model are set as -1.0, 2.0, 0.1, for the ideal time sequence, and the random number is limited in

0-1. The parameters *v*, *h* are obtained by regression method before making prediction. It has to be noted that in this ideal time sequence there is just one abrupt change, which means that we have no way to obtain the value of the parameter *k* by counting ~~many~~other abrupt changes. Thus parameter *k* is given directly, and the prediction of the end state (moment) is drawn in figure 5b, 5c and 5d. For the entire time sequence, there are 500 moments as shown in figure 5a. In figure 5b, only 240 moments are given, and the other moments are unknown. Then, we obtain parameters *v* and *h* by regression method. The parameter *u* is calculated with ~~Eq.~~equation (~~8~~96). The blue line represents the prediction result. The transition process would be ended in moment 342 with the end state value 2.92. In figure 5c, the end moment and end state are predicted to be 356 and 2.65 respectively when the time sequence is given 250 moments. In figure 5d, the time sequence is given 260 moments. The end moment and end state are predicted to be 359 and 2.58 respectively. The prediction end moment and the prediction end state~~ of the prediction result~~ are basically consistent with the original time sequences~~match the presetting lines~~. However, the average absolute prediction errors of the three time sequences are 0.37, 0.27 and 0.26 respectively. When the length of the sequence is 240, the prediction state is overestimated, and the average absolute prediction error is 0.37. With the length of the system experienced expanding, the predic~~i~~tion error decreases. The prediction states are very close to the original states when the length is 260. Therefore, in the actual prediction, we hope that the transition process has been experienced for a long enough time, which will help to predict accurately.~~ The results also show that the longer the transition process experience, the more accurate the prediction.~~

3. Results

In order to test the validity of this prediction method in a real climate system, we apply this method to predict the uncompleted transition process of the PDO. The PDO index data used is from website of the University of Washington (http://research.jisao.washington.edu/pdo/). The time period from January of 1900 to November of 2015 is studied as the training data, and the time period from December of 2015 to April of 2017 is used as the test data. During the following research, a transition process starting from 2011 is studied. According to the prediction method, several parameters have to be determined in advance. We first determine parameter k .

3.1 Threshold of parameter *k*

Parameter *k* characterizes the stability of the system during climate change, which means that we can ~~get~~estimate the value of parameter *k* by counting all abrupt changes of the PDO index. The histogram in Figure 6a shows the PDO time sequence from January of 1900 to November of 2015, and it shows that the PDO went through several transition processe~~change~~s. The green dots in Figure 6a are parameter *k* when the sub-sequence length takes 20 years. In the early 1940s and late 1970s, there are two main transitions of the PDO. The absolute value of the parameter *k* is large, which means that the system is much more unstable during this two transition processes. In the 1940s, the PDO transits from a positive phase to a negative phase, and $k < 0$, whereas the situation in the 1970s is the opposite. Figure 6b shows more *k* values corresponding to the different sub-sequence lengths (as indicated by X-axis, the variation range of the sub-sequence is ~~20~~15-60 years, with an interval of 1 year). The Y-axis is the start moment, and the locations of the dots indicate the start moments for the corresponding sub-sequence lengths. In particular, the blue dots represent that parameter *k* is negative, and the red dots represent that it is positive. There are more dots in the left side region than in the right side region in figure 6. This is because when the length of sub-sequence is short, the amplitude is also often small~~.~~, which leads that ~~M~~more transition processes are detected. When the length of the sub-sequence reaches or exceeds 50 years, the transition ~~change~~processes mainly begin in the 1940s and 1970s~~,~~. Such climate changes~~which~~ are also investigated in ~~other~~the previous research (Shi et al, 2014). The transition processes in these two periods correspond to large *k* values, which means that these two transition processes are more unstable than others. The small figure in figure 7 (a) shows that the *k* values (marked with green dots) are more that 100 during 1960~1970 when the length of sub-sequence is 20 years. The frequencies of parameter *k* values when the length of sub-sequence are 10, 20, 20, 40, 50, and 60 years respectively are displayed in figure 7. It is noted that some of the values of parameter *k* are so large that their furequencies are almost zero which are not necessary to be counted. The frequencies of *k* values which belongs to -10 to 10 are shown in figure 7. By considering the frequency which is bigger than 5% as a peak, there is only one peak when the length of sub-sequence is 10 years. Most of the *k* values are concentrated around zero, which means that the transition processes detected are stable. For the situations which the lengths of sub-sequences are 20 and 30 years, there is one mainly peak, and it is near zero. When the lengths of sub-sequences are more than 30 years, there are two mainly peaks. One is near zero, and another is much less than zero. It states that the much more unstable transition processes are detected when the lengths of sub-sequences are large. More statistical results indicate that the threshold distribution of parameter *k* values in historical transition processes exhibit multiple peaks (Figure 7). Specifically, the highest peak with the largest probability is located near to 0. The *k* value is small, which indicates that the abrupt changes are stable. There are some peaks on the left side and right side of zero. When *k*<0, the PDO time sequence transits from the positive phase to the negative phase, which the threshold of the *k* peak is wide and the probability is small; when *k*>0, the PDO time sequence transits from the negative phase to the positive phase, which the threshold of the *k* value is narrow and the probability is large. This indicates that the two transitions, which one of them is that the system changes from the positive phase to the negative phase, and the other is that the system changes from the negative phase to the positive phase, are not symmetric, and the latter is more stable. Because there is a difference in parameter *k* when the selected sub-sequence length is different, Figure 7 also shows the statistical properties of parameter *k* when the sub-sequence length is 20, 30, 40, 50, or 60 years. When the length of the sub-sequence is 20 years and 30 years, there is only one main peak in the distribution of *k* values, and the parameter *k* value of the peak is about 0, which means that the transition change is more stable than the other situations. When the length of

~~the sub-sequence is 40, 50, or 60 years, there are two main peaks. The peak value on the side of *k*>0 is not considerably different, which indicates that the stability degree of the transition change from negative to positive is consistent; the location of the peak value on the side of *k*<0 moves to the left as the sub-sequence length increases, which means that the sub-sequence is longer, the amplitude of detected transition change is larger, and it is more unstable.~~ From the perspective of the *k* threshold values, the~~a~~ *k* values in the range of (-10, 10) accounts for 63.90%, 70.64%, 77.00%, 90.05%, 93.69%, and 89.90%~~80.2%~~ of all *k* values for that the lengths of sub-sequences are 10, 20, 30, 40, 50, 60 years respectively. They are 55.64%, 67.52%, 73.99%, 83.45%, 85.46%, 84.82% for the range of (-5, 5) and 35.64%, 62.22%, 59.36%, 68.28%, 66.25%, 47.55% for the range of (-2, 2).~~, a *k* value in the range of (-5, 5) accounts for 74.2%, and a *k* value in the range of (-2, 2) accounts for 58.6%.~~ In the following studies, the *k* values ~~is~~are mainly ~~set~~considered to be in the range of (-2, 2).

3.2 Values of the initial state *v* and linear trend *h*

We use the method proposed in section 2.2 to analyze the ~~transition change~~transition processes of the PDO. With different lengths of sub-sequences, three climate changes are detected to start from 1976, 2007 and 2011 respectively. In figure 8, the ~~transition change~~transition processes starting from 2007 and 2011 are shown, while the transition process starting from 1976 has not been shown. In table 2, parameters *v* and *h* are obtained by regression method for the transition processes starting from 2007 and 2011. When the length of sub-sequence is 10 years or 20 years, only the transition process starting from 2011 is detected as shown in figure 8a and figure 8b. The parameter *v* is calculated with the sequence before 2011. Then, the linear trend parameter *h* is calculated with the segment after 2011. For the transition process starting from 2011, the values of initial state were detected to be -0.45 and -0.03 when the length of sub-sequence is 10 years or 20 years respectively, and both the linear trends are 1.054/month. When the lengths of sub-sequences are set as 30 and 40 years, the transition process began in 2007 as shown in figure 8c and figure 8d, and the values of initial state are 0.36 and 0.41, respectively, with an linear trend of 0.227/month. When we detect the transition process in a sub-sequence, the percentile threshold method (Huang, 1990) is used. Then, a transition process in the sub-sequence is detected (Yan et al, 2015, 2016). The change with the largest amplitude will be detected. The start moment of the ~~transition change~~transition process is identified to be 2011 as shown in table 2.

In figure 8, it is noted that the PDO time sequence is leaving the stable state from the start moment. The ~~transition change~~transition process occurs over a period of time, which is called the transition process. When the transition process has not finished, it appears to be increasing part. In order to detect whether there are other transition processes, we change the length of the sub-sequences to yearly intervals. That is, the sub-sequence length is set as 10, 11, 12, ⋯, up to 60 years. Then, the initial state *v* and the linear trend *h* of these transition processes are obtained as shown in figure 9. When the sub-sequence length is set less than approximately 40 years, the transition processes are detected only twice. One began in 2007, and the other began in 2011. The value of parameter *h* is unchangeable nearly for each transition process, while the value of parameter *v* is changing when the length of sub-sequence is different. In particular, the transition process starting from 2007 is detected for the sub-sequences of about 30-40 years, and the value of parameter *v* is in the range of (0.28, 0.45). The transition process starting from 2011 is detected for the sub-sequences of about 10-30 years, and the value of parameter *v* increases as the length of the sub-sequence increases, whereas the variation range of parameter *v* is (-0.48, 0.12), which is significantly different from the situation of the transition process starting from 2007.

3.3 Prediction of the uncompleted transition process beginning in 2011

After the threshold ranges for parameters *k, v,* and *h* are determined, according to the quantitative relationship, we can calculate the end state and the end moment of the transition process. Using the transition process in 2011 as an example, we study the ending state and end moment for the PDO index transition process. According to the research results that are presented in Sections 3.1 and 3.2, the parameter is h=1.054/month in this transition process, and the threshold range of parameter k is determined to be (0, 2). The range of parameter v is determined to be (-0.48, 0.12), and the variation situation of parameter u and end moment with parameters k and v are shown in Figure 10. The results indicate that the threshold range of parameter u for the ending state is (1, 7), and the time range of the ending moment is (2013, 2017). According to the probability of parameter k, the end moment of this transition process is about 2015, and after that time, the sequence stops to increase, approaching to a stable state with value of 1.6.

In figure 11, a sketch map is displayed to briefly explain how the prediction method works. The PDO time sequence is displayed as a black line. The period during 2006~2011 is detected as the initial state, and a transition process is increasing from this initial state. It is not able to be known whether the increasing process has been completed or not. Based on the linear regression method, the initial state and the linear trend are obtained and shown as purple dash lines. Then by the method proposed in section 2.2, all possible end states of this transition process are obtained with ~~Eq.~~equation (~~8~~9) as shown in figure 10, and the most likely end state is marked as a green dash line.. Unlike the uncompleted transition process of ideal experiment, the transition process has completed in about 2015 since we detected the PDO change in 2016. This transition process started from 2011 and ends in 2015. The initial moment and the end moment are marked as black dash lines. However, we are still not sure whether the PDO finish this transition process completely or not for it it appears at the end of the sequence. Many statistical methods are not accurate for the detecting both ends of the sequence. Thus, the real PDO sequence during 2016~2017 is added to the end of the PDO time sequence. The PDO value from 2015 to 2017 is almost unchanged, which is consistent with the predicted result.

4. Conclusion and discussion

A novel method had been proposed to identify the transition process of climate change in our previous research. By defining initial state parameter *v*, linear trend parameter *h*, end state parameter *u*, and instability parameter *k*, a quantitative relationship among these parameters was revealed. Based on the relationship, we develop a method to study uncompleted transition processes. The method is applied to predict ideal time sequences and the PDO time sequence. In the ideal experiments, three different time sequences with different length are constructed. Based on the initial state and the linear trend which the system had experienced, and the given parameter, the end state and end moment of the transition process are predicted. The prediction result does match the ideal time sequence well. For the PDO time sequence, a ~~transition change~~transition process beginning in 2011 was taken to test the prediction method. The end moment of this transition process is predicted to be 2015. which is consistent with the real time sequence.

In this prediction method, the quantitative relationship among the parameters characterizing the transition process is vital. According to the segment of the transition process which has been occurred, we determine the parameters and predict the end moment and the end state. In fact, this is also a extrapolation method. It is noted that the uncompleted climate change we studied is closed to the end of the sequence. Due to the lack of enough data, it is difficult to study the end of time sequence by using other statistical methods.

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Table1. The parameters of ideal models

Situations	α	χ	h_0	h	$\|h_0-h\|/h$
S1	0.20	21.64E-2	12.99E-4	12.69E-4	2.36%
S2	0.25	22.76E-2	9.10E-4	8.90E-4	2.25%
S3	0.15	20.18E-2	32.27E-4	32.72E-4	1.38%

Table2. Parameters v and h obtained with different sub-sequences

Length of sub-sequence	Start moment (year.month)	v	h (month^{-1})
10a	2011.06	-0.45	1.054
20a	2011.06	-0.03	1.054
30a	2007.11	0.36	0.227
40a	2007.11	0.41	0.227

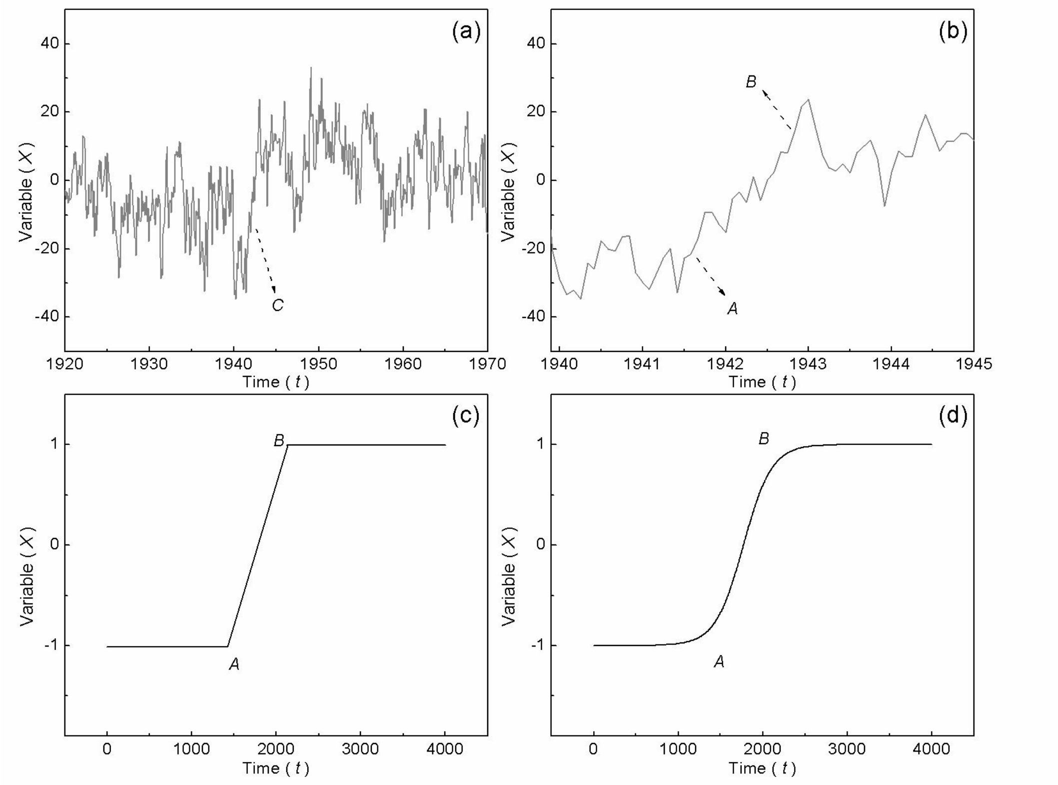

Figure 1. Transition process of abrupt change in real time sequence and ideal time sequence. (a) The PDO time sequence during 1920 to 1970; (b) The PDO time sequence during 1940 to 1945; (c) The transition process presented by piece-wise function; (d) The transition process presented by continuous function.

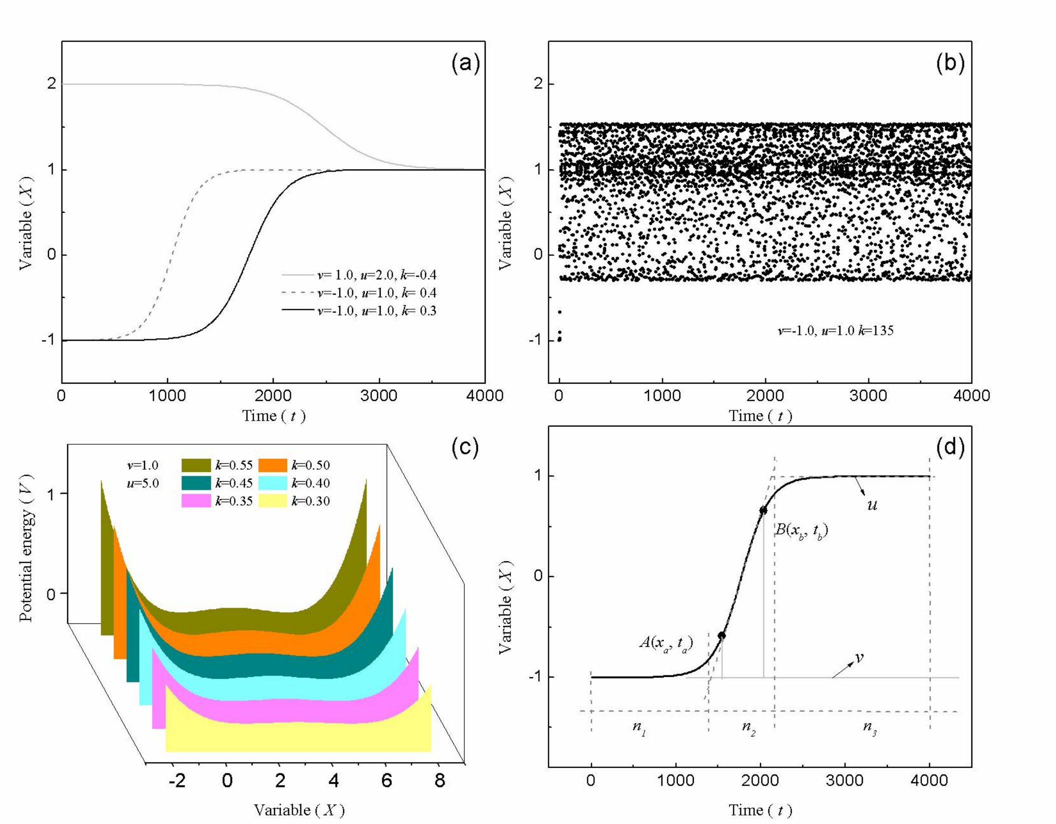

Figure 2. ~~The system presented by Eq. (2).~~ (a)The transition processes of system swinging between different stable states since the parameters are different; (b)The system stays in unstable states; (c)The generalized potential energy function of system performs differently since the parameters are different; (d)Different segments of the transition process in the ideal time sequence~~ and the system states x expressed with location parameters~~.

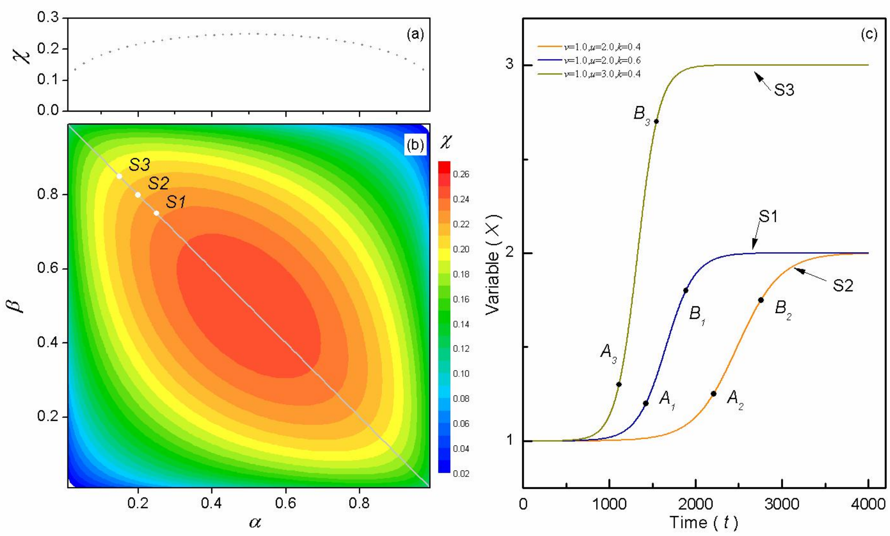

Figure 3. The ~~influence of different value of~~relationship among the parameters α ~~and~~, β ~~on parameter~~, χ and ~~parameter~~ h. (a) Diagonal section of parameter χ in figure b (gray line); (b) Parameter χ with location parameters α and β; (c)Points A and B stay in different positions in three situations marked as S1, S2, S3, and the dash lines connect the two points.

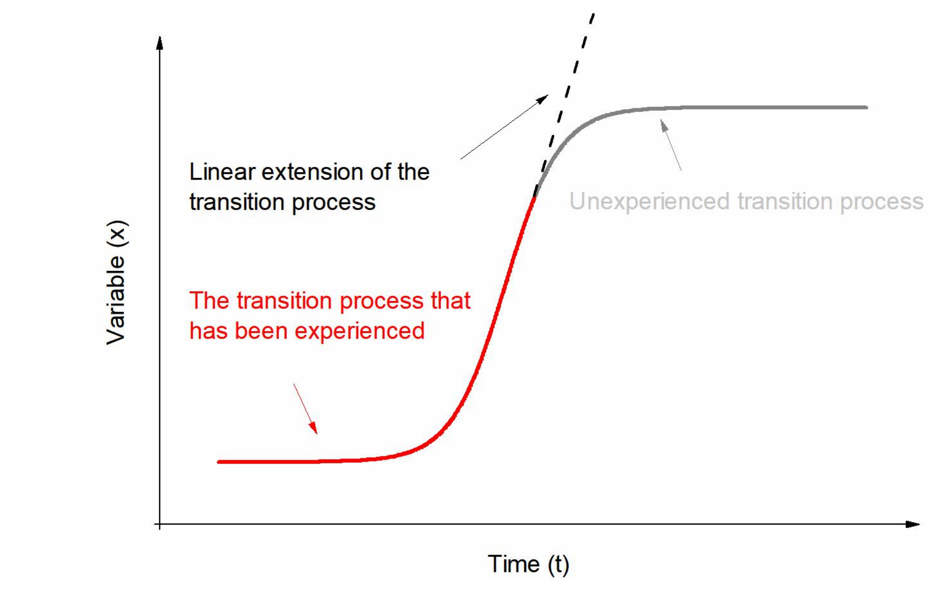

Figure 4. The schematic diagram of prediction method.

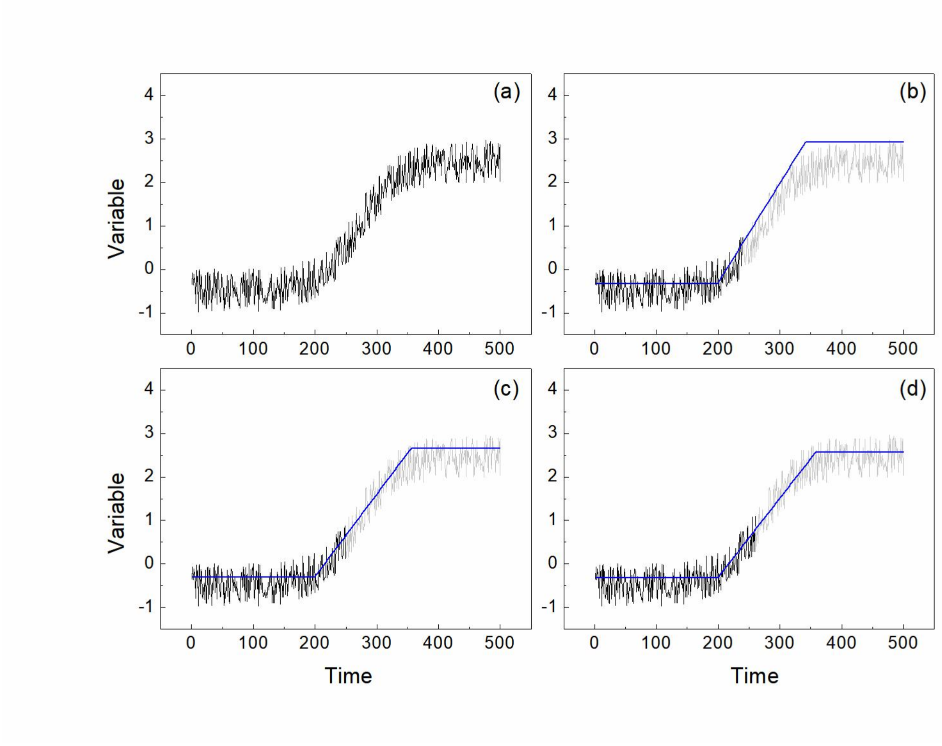

Figure_5. The ideal time sequence constructed by the logistic model and random numbers. The X-axis represents time, and the Y-axis represents variable x_t'. (a)

Completed transition process with 500 moments, Uncompleted transition processes (the gray lines) and their prediction result (the blue lines) with (b) 240 moments, (c) 250 moments, and (d) 260 moments, the light gray lines are the original entire ideal time sequences.

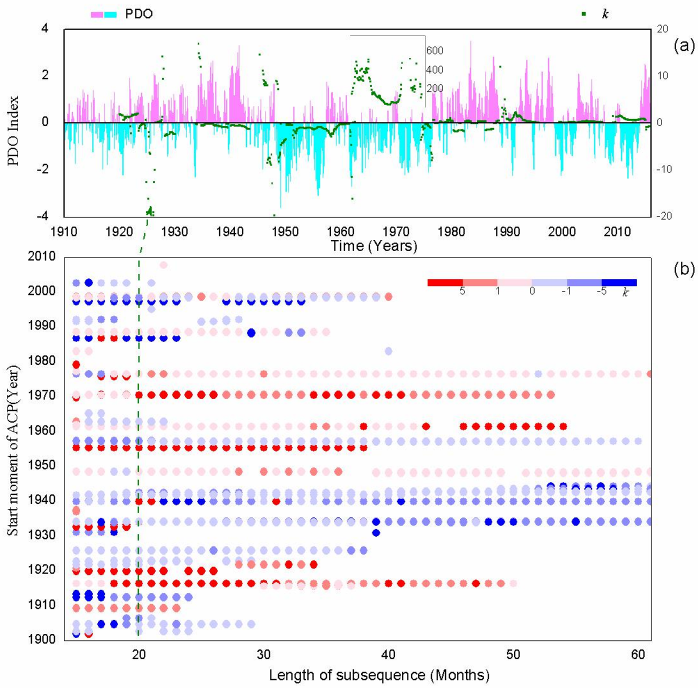

Figure 6. Identification of the PDO time sequence and instability parameter *k* with different sub-sequence lengths. (a) The X-axis represents the sub-sequence length in ~~is the year~~months, the histogram in the figure shows the PDO time sequence (left Y-axis), and the green dots indicate the value of parameter k when the sub-sequence is 20 years (right Y-axis); (b) the start moments of transition processes with different sub-sequence lengths (the red color dots represent increasing processes, and blue color dots represent decreasing changes, with deeper colors representing higher values). The X-axis is the sub-sequence length (month), and the Y-axis is the start moment of abrupt change (year).

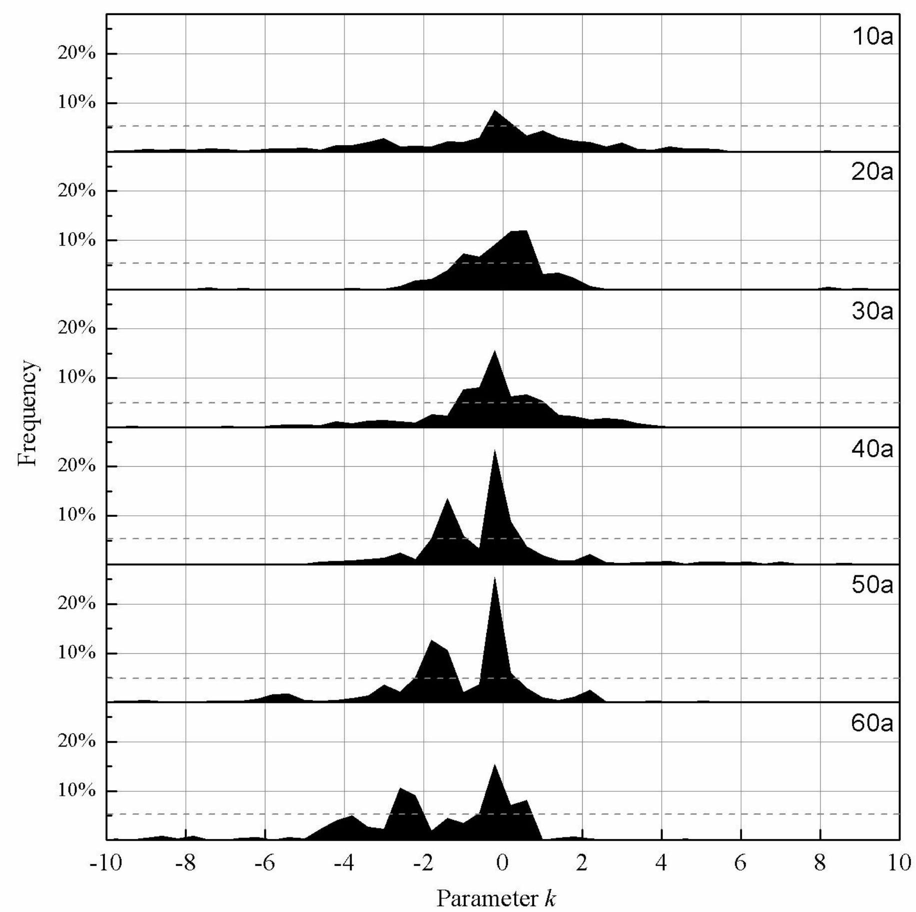

Figure 7. Statistical results of instability parameters for different sub-sequences lengths. The X-axis is the value of ~~the~~ parameter *k*, and the Y-axis is the statistical frequency ~~with a~~for the sub-sequence length of 10~~ years~~a, 20a 30a 40a 50a and 60a~~. The gray region in the upper-right corner is for the sub-sequence of 20-60 years~~.

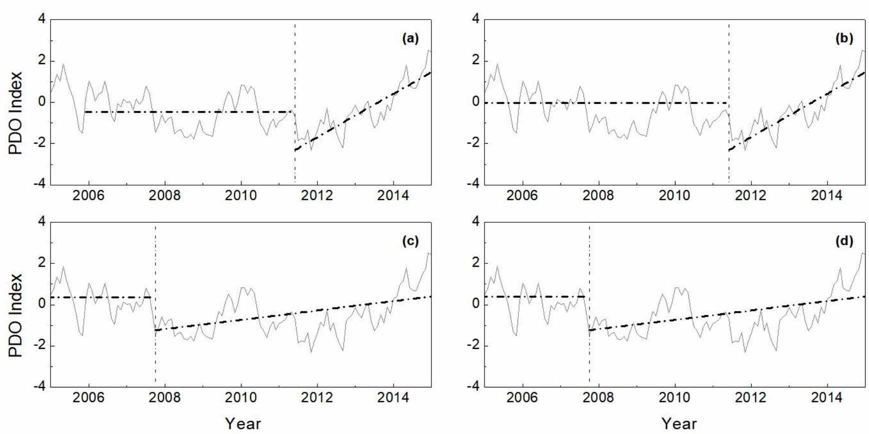

Figure 8. The PDO time sequences and the detection of parameters *v* and *h* when the

sub-sequence was set as (a)10 years, (b)20 years, (c)30 years, (d)40 years. The gray lines represent the PDO time sequence. The horizontal dash lines represent initial states, the slope dash lines represent linear trend lines of the transition process, and vertical dotted lines represent the start moment.

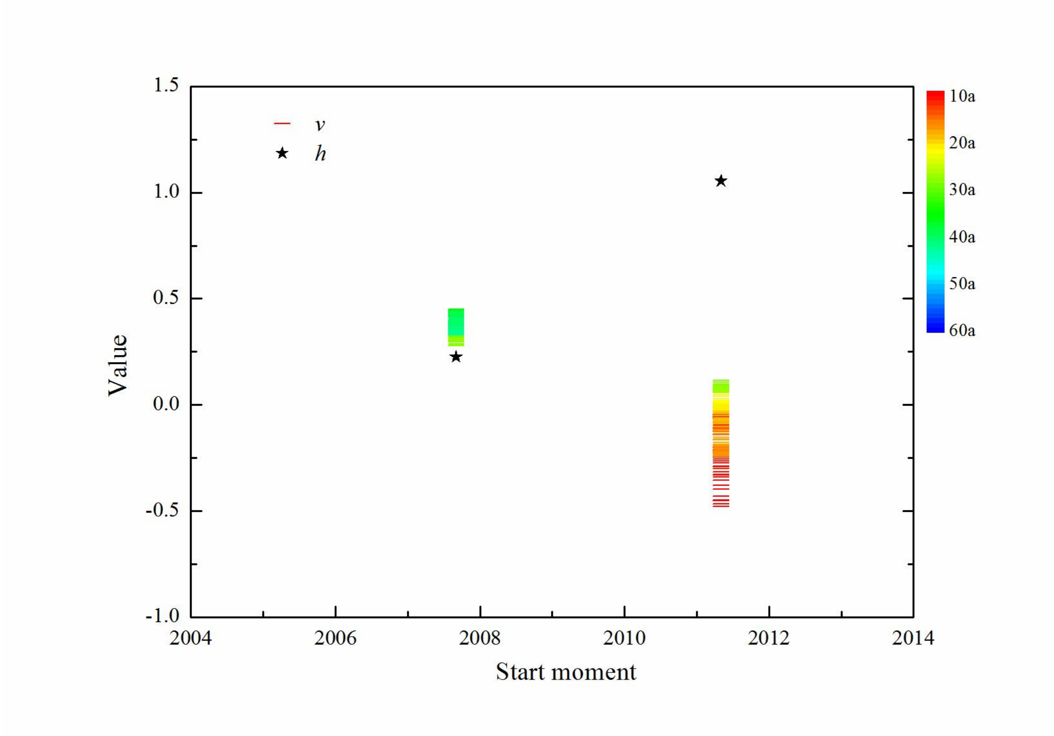

Figure 9. The values of the parameters *v* and *k* of two transition processes with different lengths of sub-sequence. The black stars represent the values of parameter *h*, and the colourful short bar represent the values of parameter *v*. The colour bar represents years of the sub-sequence length from 10 to 60 in intervals of 1.

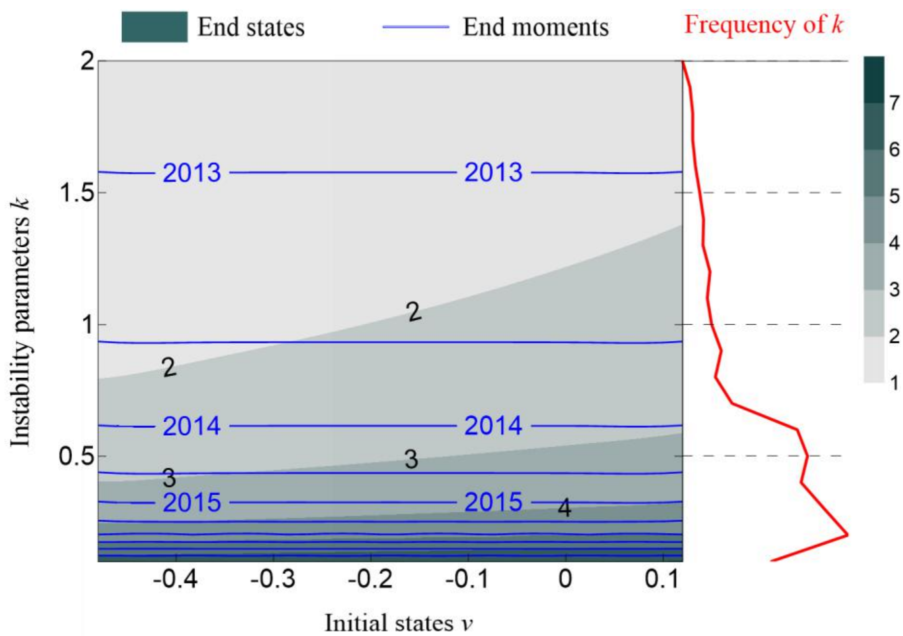

Figure 10. Variation end state and end moment with the initial state parameter *v* (horizontal ordinate) and instability parameter *k* (vertical coordinate). The red line on the right side shows the probability distribution of instability parameter *k*.

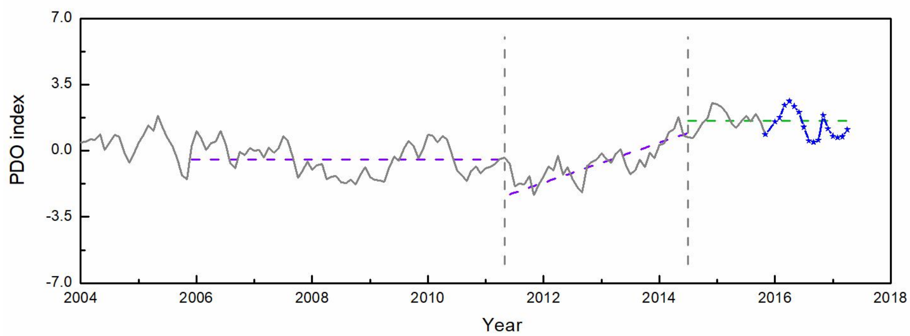

Figure 11. Prediction of the PDO index. The gray line represents the PDO index before 2015; the blue line represents the PDO index after 2015; the gray dash line represent the start moment and end moment; the purple dash lines represent the initial state and the linear trend line, the green line represent the prediction end state.