A method to predict the uncompleted climate transition process

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Abstract

Climate change is expressed as a climate system transiting from the initial state to a new state in a short time. The period between the initial state and the new state is defined as transition process, which is the key part to connect the two states. By using a piece-wise function, the transition process is stated approximately (Mudelsee, 2000). However, the dynamic processes are not included in the piece-wise function. Thus, we had proposed a method (Yan et al, 2015, 2016) to study the transition process by using a continuous function. In this manuscript, this method is developed to predict the uncompleted transition process based on the dynamic characteristics of the continuous function. We introduce this prediction method in details and apply it to three ideal time sequences and the Pacific Decadal Oscillation (PDO). The PDO is a long-lived El Niño-like pattern of Pacific climate variability (Barnett et al, 1999). This method reveals a new quantitative relationship during the transition process, which explores a nonlinear relationship between the linear trend and the amplitude (difference) between the initial state and the end state. ~~Since~~As the transition process begins, the initial state and the linear trend are estimated. Then, according to the relationship, the end state and end moment of the uncompleted transition process is predicted.

Keywords

Prediction method; Transition process of abrupt change; System stability; Pacific

Decadal Oscillation

1. Introduction

A system transiting from one stable state to another in a short period is called abrupt change (Charney and DeVore, 1979; Lorenz, 1963, 1979). The abrupt change system has two or more states (Goldblatt et al, 2006; Alexander et al, 2012), the system swings between these states that are also called attractors in physics. This phenomena is verified in many fields including biology (Nozaki, 2001), ecology (Osterkamp et al, 2001), climatology (Thom, 1972; Overpeck and Cole, 2006; Yang et al, 2013a, 2013b), brain science (Sherman et al, 1981), etc. The latest observed climate change event is global warning hiatus, which has been studied deeply by many researchers (Amaya et al, 2018; Kosaka and Xie, 2013; Yang et al, 2017). Seven different kinds of abrupt changes are mentioned in Thom's research(1972). Over the last several decades, many methods have been proposed to identify different kinds of abrupt change (Li et al, 1996), ~~like~~such as Moving T-Test, Cramer's (Wei, 1999), Mann-Kendall (MK, Goossens and Berger, 1986), Fisher (Cabezas and Fath, 2002), etc. It is noticed that most abrupt change detection methods suggest~~s~~ that the abrupt change is around a turning point. The significant difference between the average values of the two sequences on both two sides of the turning point is defined as the index to measure the abrupt change. This kind of detection method has a drawback. It is difficult to detect the abrupt change that occurs at the end of sequence.

Mudelsee (2000) studied the abrupt change of a time sequence and illustrated that abrupt change has a duration, which can be quantitatively described with a piece-wise (ramp) function. We developed the detection method by using a continuous function to replace the ramp function(Yan et al, 2014, 2015). The new method can confine the beginning and ending points of abrupt change and quantitatively describes the process of abrupt climate change, and three parameters are introduced. A quantitative relationship among the parameters is revealed (Yan et al, 2015). The relationship could be used to predict the end moment (state) if the system had left the original state but not yet reached to the new state, which is defined as an uncompleted transition process.

In this manuscript, three ideal time sequences are tested to study the prediction method. The prediction method is also applied to study the climate transition process of the PDO, which is an important signal that reveals climatic variability on the decadal timescale (Mantua et al, 1997; Barnett et al, 1999; Zhang et al, 1997; Yang et al, 2004). Previous studies (Lu et al, 2013; Trenberth and Hurrell, 1994) have indicated that there are many ~~climate~~abrupt changes in the PDO over the past 100 years. Most researches mentioned the climate changes happened in the 1940s and 1970s. During the 1940s, the PDO transited from a high state to a low state, while during the 1970s, it did the opposite. All ~~this~~of these changes and their processes had been studied in our previous research~~es~~ (Yan et al, 2015 2016). The climate transition processes were explored clearly. However, we still can not know when the transition processes finish ~~its~~their increasing or decreasing to a stable state if the transition process has begun. We develop a new method to predict the end state and the end moment of a transition process based on the quantitative relationship.

2. Methods

It is necessary to describe the transition process quantitatively before the prediction of the uncompleted climate transition process. We had proposed a detection method by using the logistic model to obtain a transition process. In section 2.1, the method is introduced briefly. On the basis of the detection method, the prediction method for studying the uncompleted transition process is further developed in section 2.2.

2.1 The detection method of transition process

The real time sequence changes abruptly as shown in figure 1a, and the system jumps to a high state in point C. If the period around point C is observed on a shorter time scale (as shown figure 1b), a transition period is obtained, and it is a part of the original time sequence. In fact, many abrupt changes could be considered to be a transition period with a more detailed view. The transition period was expressed with a~~n~~ ramp function in Mudelsee's research (2000) as shown in figure 1c, and the time sequence is divided into three segments, including two equilibrium states and one increasing state. The ramp function is as follows:

$$x_t = \begin{cases} x_1 & t \le t_1 \\ x_1 + (t - t_1)(x_2 - x_1)/(t_2 - t_1) & t_1 < t \le t_2 \\ x_2 & t > t_2 \end{cases} \quad (1)$$
,

w~~W~~here t represents time, and x_t represent the system states, which is obtained by the linear regression method. It is noted that the climate system is continuous; it is even the sampling sequence that makes it is discontinuous. We used a continuous function to express this transition period approximately, and we also created a novel method to detect the transition period (Yan et al, 2015). Here, the detection method is introduced briefly. The continuous evolution of the l~~L~~ogistic model is consistent with the transition process (May, 1976), which is shown in figure 1d. The modified logistic model is expressed as follows:

$$\dot{x} = k(x - u)(v - x) \quad (2)$$
;

Parameters u and v represent the two equilibrium states respectively. Parameter k represents the switching between different states, and it is defined as the instability parameter. As shown in figure 2a, parameters u and v being fixed, and setting k as 0.5, the system transiting to the new state costs a shorter time than that setting k as 0.4. If parameter k is set large enough, the system collapses and becomes chaotic (as shown in figure 2b). When parameter k is set to different values, more situations have been discussed in detail in the previous research (Yan et al, 2016). The result shows that parameter k characterizes the stability of the system (the larger the absolute value, the more unstable the system). According to Thom's theory (1972), the system described by a quar~~dr~~t~~at~~ic function would exhibit tipping-point abrupt change~~,~~ in which the system jumps from one state to a new state abruptly. Thus, we did some mathematical derivation to Eq. (2), and the general potential energy is obtained as follows:

$$V_{(x)} = -\int_0^x \ddot{x}dx = -\int_0^x 2k^2\left[x-(u+v)/2\right](x-u)(x-v)dx$$
$$= \frac{k^2}{2}\left[x^4 - 2(u+v)x^3 + \left(u^2+v^2+4uv\right)x^2 - 2(u+v)uvx\right] \quad (3)$$

It means that Eq. (2) describes a system with tipping-point abrupt change. In figure 2c, the potential energy of Eq. (3) is verified to have two states with the lowest energy, and both of them are stable. This bistable structure is common in the climate system (Goldblatt et al, 2006). Therefore, Eq. (2) can be used to describe the abrupt change system, and the parameters represent different key factors of the transition period during abrupt change. Then, the parameters *u, v* and *h* are obtained by the regression method (Huang, 1990; Yang et al, 2013a) by using Eq. (4), where *i*, x_i denote the time and the state of the system at this time, and $\bar{i}, \bar{x}_i$ are their averages respectively. Variable n_2 is the length of the second segment.

$$\begin{cases} v = \sum_{i=1}^{n_1} x_i / n_1 \\ u = \sum_{i=n_1+n_2+1}^{n} x_i / n_3 \\ h = \sum_{i=n_1+1}^{n_1+n_2} \bar{i} \cdot \bar{x}_i \Big/ \sum_{i=n_1+1}^{n_1+n_2} \bar{i}^2 \end{cases} \quad (4)$$

The linear trend *h* represents the ratio of system state change to time, and it can be expressed by two points on the curve approximately as Eq. (5), where the two points are *A* (x_a, t_a) and *B* (x_b, t_b).

$$h = \frac{x_a - x_b}{t_a - t_b} \quad (5)$$

As shown in figure 2d, the transition period during point *A* (x_a, t_a) and point *B* (x_b, t_b) is approximately linear. Then, we can use the location parameters *α, β* to express system states x_a and x_b. By solving Eq. (2), the relationship between *x* and *t* is determined.

$$t = \frac{1}{k(u-v)} \ln\left(\frac{x_0 - u}{x_0 - v} \cdot \frac{x - v}{x - u}\right) + t_0 \quad (6)$$

Then, parameter h is rewritten as Eq. (7). It is noted that the rightmost part is only related to the location parameters α and β, then let it be χ. Then, the relationship of Eq. (7) is rewritten as Eq. (8):

$$
\begin{aligned}
h &= \frac{x_b - x_a}{\dfrac{1}{(\mu-\nu)k}\ln\dfrac{x_0-\mu}{x_0-\nu}\left(\dfrac{x_b-\nu}{x_b-u}-\dfrac{x_a-\nu}{x_a-u}\right)} \\
&= k(\mu-\nu)^2\frac{(\beta-\alpha)}{\ln\dfrac{\beta(\alpha-1)}{\alpha(\beta-1)}}
\end{aligned} \qquad (7)
$$

,

$$
h = k\omega^2\chi \qquad (8)
$$

.

In order to determine the value of parameter χ, the relationship among χ, α, β is displayed in figure 3b. The dash line in figure 3a is the profile of the diagonal in figure 3b, which represents that the sum of α and β is 1. Parameter χ changes little when the location parameter varies in a certain range as marked with warm color in figure 3b. It means that the closer the points (A and B) are to the middle point, the more significant the linear feature is. Then, the process between point A and point B can represent the whole transition process as shown in figure 3c. It is noted that the transition process is symmetrical about the middle point approximately. Thus, we assume that point A and point B are symmetrical about the middle point, and the sum of α and β is 1. The change of parameter χ is only related to parameter α (or parameter β), as shown in the diagonals in figure 3b (also in figure 3a). Parameter χ changes little when parameter α is about 0.2 or larger. In figure 3c, three different situations are carried out to study the influence of parameter α on parameter χ. In each situation, points (A and B) are set to be different positions, and their parameters were calculated respectively in table 1. The parameters α are set as 0.20, 0.25, 0.15 respectively in three different situations marked with S1, S2 and S3. For S2 and S3, both of the percentages of α changing to S1 are 25%, while the percentages of χ changing are only 5.15% and 6.76% respectively, which means the percentage change of χ is much less than α. In addition, linear trends of these three ideal models are calculated according to the points and by regression method which are marked as h_0 in table 1.

The linear trends are also calculated by the values of point A and point B with Eq(5) which are marked as h in table 1. It is noted that although the positions of points are different, the trend obtained according to the points is almost the same as that obtained by regression method. The error percentages are 2.36%, 2.25%, 1.38% respectively, which means that we don't have to know the exactly positions of point A and B~~when the position of the points~~ (the values of parameters α and β). We can approximate the value of χ ~~are indefinite, there is little influence on the detection of parameter h~~. Thus, in the following sections parameter α is set as 0.2, and parameter χ is 0.2164

2.2 The prediction method of transition process

Eq. (8) shows the quantitative relationship among linear trend, instability parameter, and amplitude of change. There is a linear relationship between linear trend and instability parameter; and there is the quadratic function relationship between linear trend and amplitude of change. We did reveal this quantitative relationship much more than in theory but in real time series (Yan et al, 2016). Based on this relationship, we are going to create a new method to deal with the problem that the transition process has not finished. During the real time sequence, the system transits away from the original state, but it has not reached to a new state as shown in figure 4. The red line represents the period which has been experienced, while the gray line represents the period which has not been experienced. Based on the system states which ~~is~~are far away from the original state, a quasi linear extension of the transition process is established (dash line). Then the parameters v and h are obtained by Eq. (4). Assuming that the parameter k satisfies the statistics in the history of the system, the parameter u can be predicted by Eq. (8), and the end moment is also predicted~~ apparently.~~.

$$\begin{cases} x_t = x_{t-1} + kt(x_t - u)(v - x_t) \\ x'_t = x_t + \eta_t \end{cases} \tag{9}$$

As shown in figure 5, four ideal time sequences are constructed by using the logistic model and random numbers ~~—~~as Eq. (9), where η_t represents the random number. An entire time sequence with 500 moments is shown in figure 5a and three other lengths of time sequences are shown in figures 5b, 5c and 5d respectively. The parameters *v*, *u* and *k* of the logistic model are set as -1.0, 2.0, 0.1, for the ideal time sequence, and the random number is limited in 0-1. The parameters *v*, *h* are obtained by regression method before making prediction. It has to be not~~ic~~ed that in this ideal time sequence there is just one abrupt change, which means that we have no way to obtain the value of the parameter *k* by counting many changes. Thus parameter *k* is given directly, and the prediction of the end state (moment) is drawn in figure 5b, 5c and 5d. For the entire time sequence, there are 500 moments as shown in figure 5a. In figure 5b, only 240 moments are given, and the other moments are unknown. Then, we obtain parameters *v* and *h* by regression method. The parameter *u* is calculated with Eq. (8). The blue line represents the prediction result. The transition process would be ended in moment 342 with the end state value 2.92. In figure 5c, the end moment and end state are predicted to be 356 and 2.65 respectively when the time sequence is given 250 moments. In figure 5d, the time sequence is given 260 moments. The end moment and end state are predicted to be 359 and 2.58 respectively. The end moment and the end state of the prediction result match the presetting lines. The results also show that the longer the transition process experience, the more accurate the prediction.

3. Results

In order to test the validity of this prediction method in a real climate system, we apply this method to predict the uncompleted transition process of the PDO. The PDO index data used is from website of the University of Washington (http://research.jisao.washington.edu/pdo/). The time period from January of 1900 to November of 2015 is studied as the training data, and the time period from December of 2015 to April of 2017 is used as the test data. During the following research, a transition process starting from 2011 is studied. According to the prediction method, several parameters have to be determined in advance. We first determine parameter k

~~firstly~~.

3.1 Threshold of parameter *k*

Parameter k characterizes the stability of the system during climate change, which means that we can get the value of parameter k by counting all abrupt changes of the PDO index. The histogram in Figure 6a shows the PDO time sequence from January of 1900 to November of 2015, and it shows that the PDO went through several changes. The green dots in Figure 6a are parameter k when the sub-sequence length takes 20 years. In the early 1940s and late 1970s, there are two main transitions~~-changes~~ of the PDO~~ mainly~~. The absolute value of the parameter k is large, which means that the system is much more unstable during this two transition ~~change~~processes. In the 1940s, the PDO transits from a positive phase to a negative phase, and ~~the~~ $k < 0$, whereas the situation in the 1970s is the opposite. Figure 6b shows more k values corresponding to the different sub-sequence lengths (as indicated by X-axis, the variation range of the sub-sequence is 20-60 years, with an interval of 1 year). The Y-axis is the start moment, and the locations of the dots indicate the start moments for the corresponding sub-sequence lengths. In particular, the blue dots represent that parameter k is negative, and the red dots represent that it is positive. ~~The~~ There are more dots in the left side region than in the right side region in figure 6. This is because when the length of sub-sequence is short, the amplitude is also often small. ~~Therefore, for the entire sub-sequence, there are many~~ More transition ~~change~~processes are detected. When the length of the sub-sequence reaches or exceeds 50 years, the transition change mainly begin~~s~~ in the 1940s and 1970s, which are also investigated in other research (Shi et al, 2014). The transition ~~change~~processes in these two periods correspond to large k values, which means that these two transition ~~chang~~processes are more unstable than others. More statistical results indicate that the threshold distribution of parameter k values in historical ~~abrupt change~~transition processes exhibit multiple peaks (Figure 7). Specifically, the highest peak with the largest probability is located near to 0. The k value ~~of the largest peak in the distribution~~ is small, which indicates that the abrupt changes~~ that~~

~~correspond to these *k* values~~ are stable. The~~ *k* values also have~~re are some peaks on the left side and right side of ~~the origin~~zero. When k<0, the PDO time sequence transits from the positive phase to the negative phase, wh~~en~~ich the threshold of the k peak is wide and the probability is small; when k>0, the PDO time sequence transits from the negative phase to the positive phase, wh~~en~~ich the threshold of the k value is narrow and the probability is large. This indicates that the~~re are~~ two ~~kind of~~ transitions, which one of them is that the system changes from the positive phase to the negative phase, and the other is that the system changes from the negative phase to the positive phase, are not symmetric, and the latter is more stable. Because there is a difference in parameter k when the selected sub-sequence length is different, ~~the gray region in the upper right corner of~~ Figure 7 also shows the statistical properties of parameter k when the sub-sequence length is 20, 30, 40, 50, or 60 years. When the length of the sub-sequence is 20 years and 30 years, there is only one main peak in the distribution of k values, and the parameter k value of the peak is about 0, which means that the transition change is more stable than the other situations. When the length of the sub-sequence is 40, 50, or 60 years, there are two main peaks. ~~, t~~The peak value on the side of k>0 is not considerably different, which indicates that the stability degree of the transition change from negative to positive is consistent; the location of the peak value on the side of k<0 moves to the left as the sub-sequence length increases, which means that the sub-sequence is longer, the amplitude of detected transition change is larger, and it is more unstable. From the perspective of the value, a k value in the range of (-10, 10) accounts for 80.2% of all k values, a k value in the range of (-5, 5) accounts for 74.2%, and a k value in the range of (-2, 2) accounts for 58.6%. In the following studies, the k value is mainly set in the range of (-2, 2).

3.2 Values of the initial state *v* and linear trend *h*

We use the method proposed in section 2.2 to analyze the transition changes of the PDO. With different lengths of sub-sequences, three climate changes are detected to start from 1976, 2007 and 2011 respectively. In figure 8, the transition changes starting from 2007 and 2011 are ~~stated~~shown, while the transition ~~change~~process starting from 1976 has not been shown. In table 2, parameters v and h are obtained by regression method ~~when~~for the transition ~~change~~processes starting from 2007 and 2011. When the length of sub-sequence is ~~2~~10 years or ~~3~~20 years, only the transition ~~change~~process starting from 2011 is detected as shown in figure 8a and figure 8b. The parameter v is calculated with the sequence before 2011~~ of the entire sub-sequence~~. Then, the linear trend parameter h is calculated with the segment after 2011~~ of the entire sub-sequence~~. For the transition ~~change~~process starting from 2011, the values of initial state were detected to be -0.45 and -0.03 when the length of sub-sequence is 10 years or 20 years~~,~~ respectively, and both the linear trends are 1.054/month. When the lengths of sub-sequences are set as 30 and 40 years, the transition ~~chang~~process~~e~~ began in 2007 as shown in figure 8c and figure 8d, and the values of initial state are 0.36 and 0.41, respectively, with an linear trend of 0.227/month. ~~Why does the length of the sub-sequence change and the start moment of the transition process change?~~ When we detect the transition ~~change~~process in a sub-sequence, the percentile threshold method (Huang, 1990) is used. Then, a transition ~~change~~process in the sub-sequence is detected~~ anyway~~ (Yan et al, 2015, 2016). The change with the largest amplitude will be detected. ~~When the sub-sequence is set to be 10 years, t~~The start moment of the transition change is identified to be 2011 as shown in table 2.

In figure 8, it is noted that the PDO time sequence is leaving the stable state from the start moment. The transition change occurs over a period of time~~experiences a period~~, which is called ~~as~~the transition process. When the transition process has not finished, it ~~looks like~~appears to be~~ the~~ increasing part. In order to detect whether there are other transition ~~change~~processes, we change the length of the sub-sequences to yearly intervals~~ one year by one year~~. That is, the sub-sequence length is set as 10, 11, 12, ⋯, ~~and~~up to 60 years. Then, the initial state v and the linear trend h of these transition ~~change~~processes are obtained~~. I~~ as shown in figure 9~~,~~. When the sub-sequence length is set less than ~~about~~approximately 40 years, the transition ~~chang~~processes are detected only twice. One began in 2007, and the other began in 2011. The value of parameter h is unchangeable nearly for each transition

~~change~~process, while the value of parameter v is changing when the length of sub-sequence is different. In particular, the ~~abrupt change~~transition process starting from 2007 is detected for the sub-sequences of about 30-40 years, and the value of parameter v is in the range of (0.28, 0.45). The transition ~~change~~process starting from 2011 is detected for the sub-sequences of about 10-30 years, and the value of parameter v increases as the length of the sub-sequence increases, whereas the variation range of parameter v~~threshold~~ is (-0.48, 0.12), which is significantly different from the situation of the transition ~~change~~process starting from 2007.

3.3 Prediction of the uncompleted transition process beginning in 2011

After the threshold ranges for parameters k, v, and h are determined, according to the quantitative relationship, we can calculate the end state and the end moment of the transition process. Using the transition process~~change~~ in 2011 as an example, we study the ending state and end moment for the PDO index transition process~~change~~. According to the research results that are presented in Sections 3.1 and 3.2, the parameter is h=1.054/month in this transition process~~change~~, and the threshold range of parameter k is determined to be (0, 2). The range of parameter v is determined to be (-0.48, 0.12), and the variation situation of parameter u and end moment with parameters k and v are shown in Figure 10. The results indicate that the threshold range of parameter u for the ending state is (1, 7), and the time range of the ending moment is (2013, 2017). According to the probability of parameter k, the end moment of this transition process is about 2015, and after that time, the sequence stops to increase, approaching to a stable state with value of 1.6.

In figure 11, a sketch map is displayed to briefly explain how the prediction method works~~ briefly~~. The PDO time sequence is displayed as a black line. The period during 2006~2011 is detected as the initial state, and a transition process is increasing from this initial state. It is not able to be known whether the increasing process has been completed or not. Based on the linear regression method, the initial state and the linear trend are obtained and shown as purple dash lines. Then by the method proposed in section 2.2, all possible end states of this transition process are obtained with Eq. (8) as shown in figure 10, and the most likely end state is marked as a green dash line.. Unlike the uncompleted transition process of ideal experiment, the transition process has completed in about 2015 since we detected the PDO change in 2016. This transition process started from 2011~~,~~ and ends in 2015. The initial moment and the end moment are marked as black dash lines. However, we are still not sure whether the PDO finish this transition process completely or not for it it appears at the end of the sequence. ~~As we all know, m~~Many statistical methods are not accurate for the detecting both ends of the sequence. Thus, the real PDO sequence during 2016~2017 is added to the end of the PDO time sequence. The PDO value from 2015 to 2017 is almost unchanged, which is consistent with the predicted result.

4. Conclusion and discussion

A novel method had been proposed to identify the transition process of climate change in our previous research. By defining initial state parameter v, linear trend parameter h, end state parameter u, and instability parameter k, a quantitative relationship among th~~is~~ese parameters was revealed. Based on the relationship, we develop a method to study uncompleted transition processes. The method is applied to predict ideal time sequences and the PDO time sequence. In the ideal experiments, three different time sequences with different length are constructed. Based on the initial state and the linear trend which the system had experienced, and the given parameter, the end state and end moment of the transition process are predicted. The prediction result does match the ideal time sequence well. For the PDO time sequence, a transition change beg~~an~~inning in 2011 was taken to test the prediction method. The end moment of this transition process is predicted to be 2015. which is consistent with the real time sequence.

In this prediction method, the quantitative relationship among the parameters characterizing the transition process is vital. According to the segment of the transition process which has been ~~happen~~occurred, we determine the parameters~~. Then, we~~ and predict the end moment and the end state. In fact, this is also a extrapolation method. ~~However, if the transition process has not begun, we can not predict this climate change. There is no other statistical method that can predict the climate change which has not occurred only by time sequence.~~ It is noted that the uncompleted climate change we studied is closed to the end of the sequence. Due to the la~~k~~ck~~e~~ of enough data, it is difficult to study the end of time sequence by using other statistical methods.

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Table1. The parameters of ideal models

Situations	α	χ	$h0$	h	$\lvert h0-h\rvert/h$
S1	0.20	21.64E-2	12.99E-4	12.69E-4	2.36%
S2	0.25	22.76E-2	9.10E-4	8.90E-4	2.25%
S3	0.15	20.18E-2	32.27E-4	32.72E-4	1.38%

Table2. Parameters v and h obtained with different sub-sequences

Length of sub-sequence	Start moment (year.month)	v	h ($month^{-1}$)
10a	2011.06	-0.45	1.054
20a	2011.06	-0.03	1.054
30a	2007.11	0.36	0.227
40a	2007.11	0.41	0.227

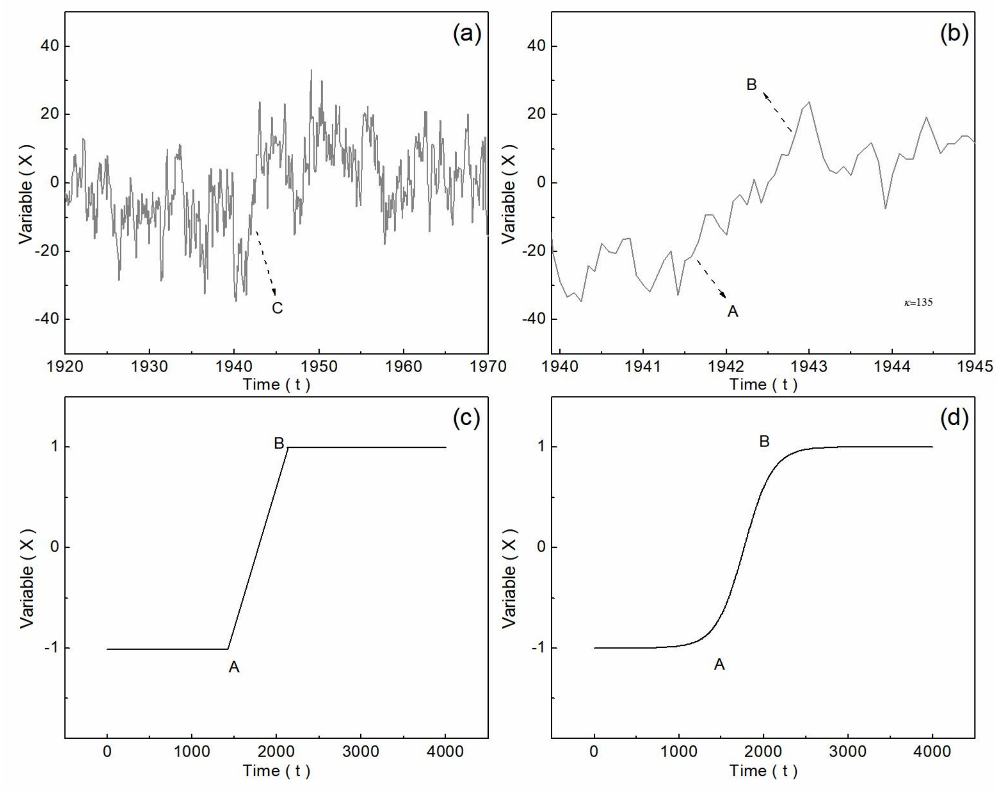


Figure 1. Transition process of abrupt change in real time sequence and ideal time sequence. (a) The PDO time sequence during 1920 to 1970; (b) The PDO time sequence during 1940 to 1945; (c) The transition process presented by piece-wise function; (d) The transition process presented by continuous function

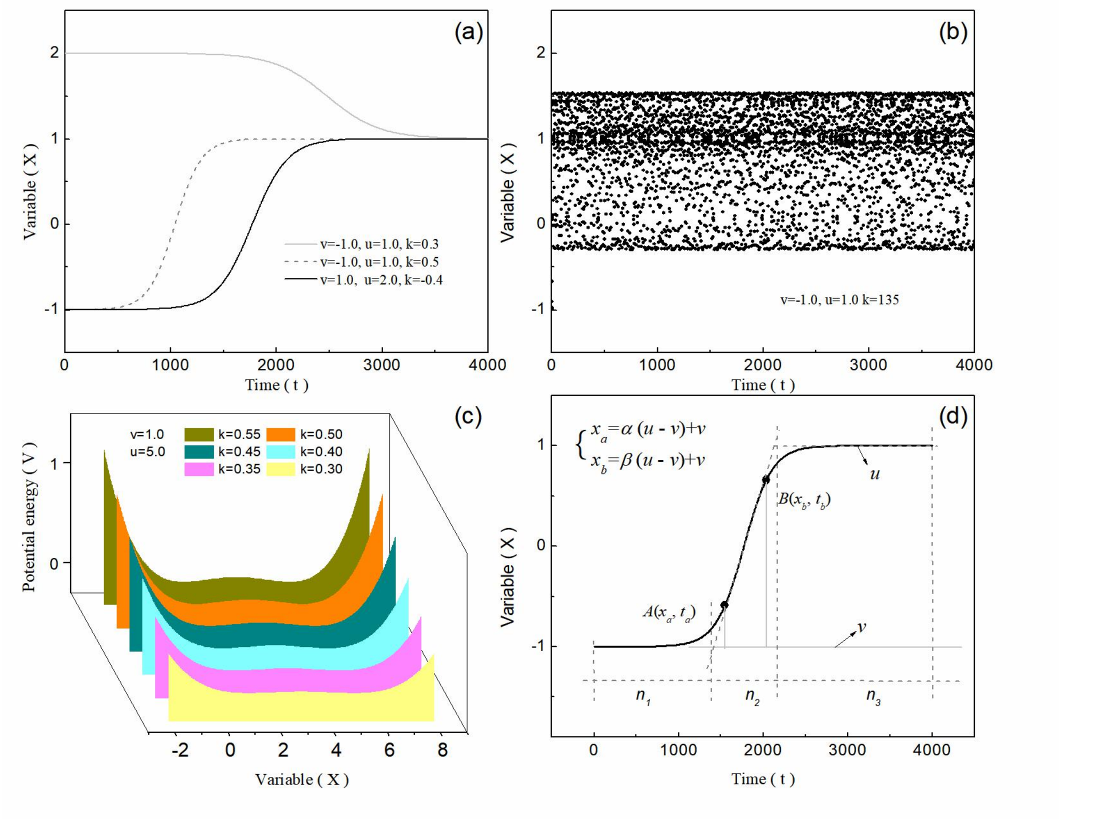


Figure 2. The system presented by Eq. (2). (a)The transition processes of system swinging between different stable states since the parameters are different; (b)The system stays in unstable states; (c)The generalized potential energy function of system performs differently since the parameters are different; (d)Different segments of the transition process in the ideal time sequence and the system states x expressed with location parameters.

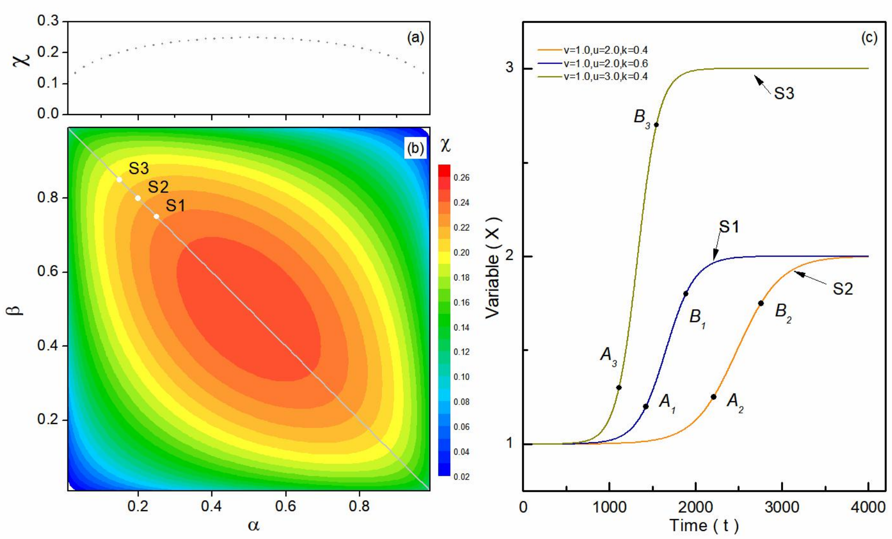


Figure 3. The influence of different value of parameters α and β on parameter χ and parameter h. (a) Diagonal section of parameter χ in figure b (gray line); (b) Parameter χ with location parameters α and β; (c)Points A and B stay in different positions in three situations marked as S1, S2, S3.

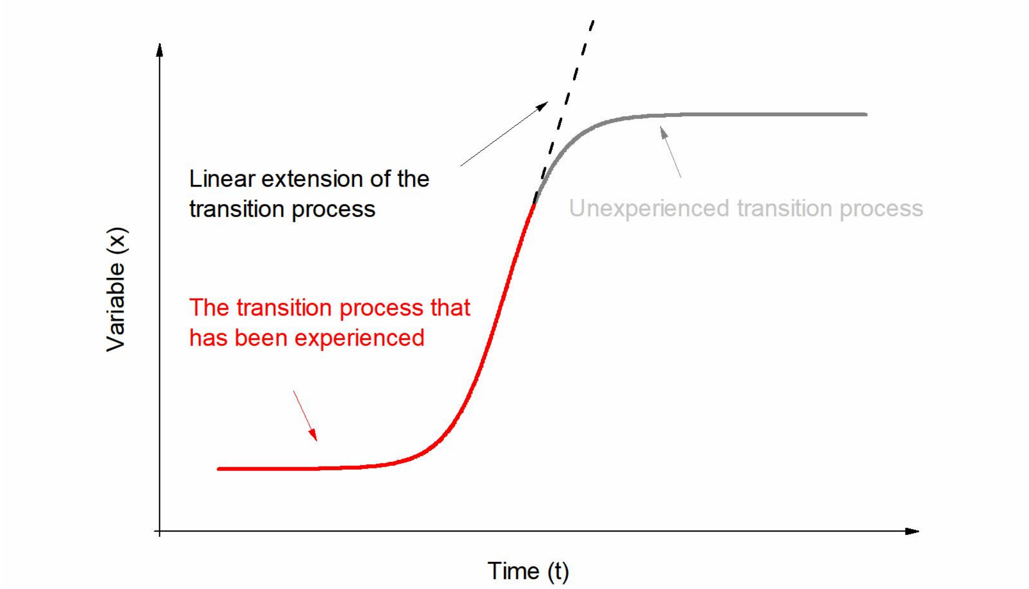


Figure 4. The schematic diagram of prediction method.

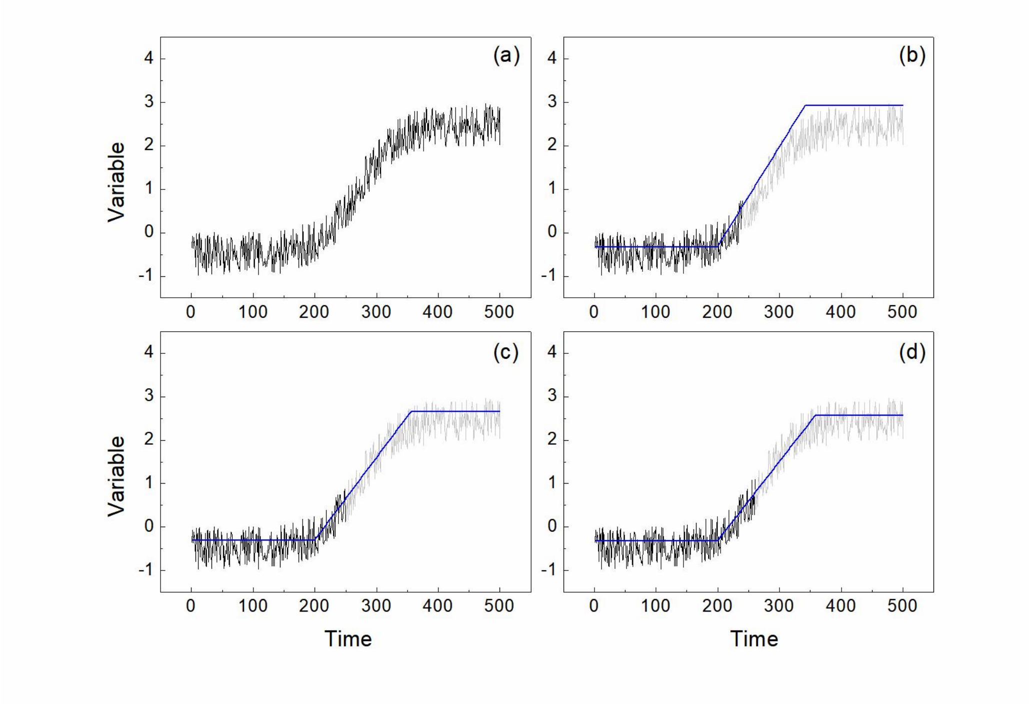


Figure5. The ideal time sequence constructed by the logistic model and random numbers. (a) Completed transition process with 500 moments, Uncompleted transition processes (the gray lines) and their prediction result (the blue lines) with (b) 240

moments, (c) 250 moments, and (d) 260 moments, the light gray lines are the original entire ideal time sequences.

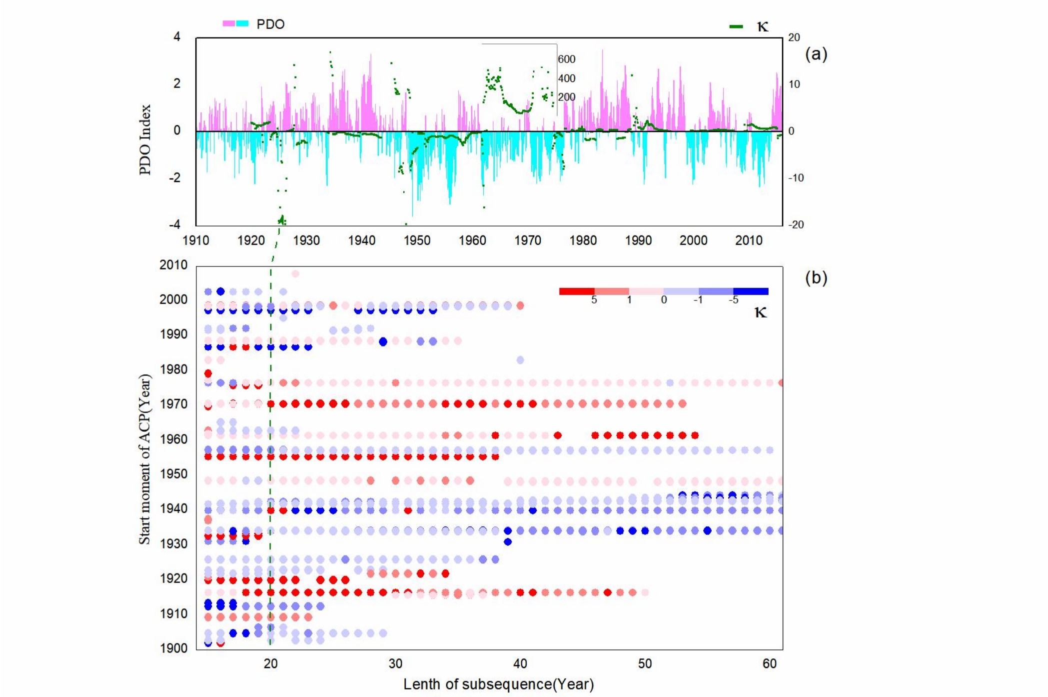


Figure 6. Identification of the PDO time sequence and instability parameter k with different sub-sequence lengths. (a) The X-axis is the year, the histogram in the figure shows the PDO time sequence (left ~~panel~~Y-axis), and the green dots indicate the value of parameter k when the sub-sequence is 20 years (right Y-axis~~panel~~); (b) the start moments of transition ~~chang~~processes with different sub-sequence lengths (the red color dots represent increasing ~~chang~~processes, and blue color dots represent decreasing changes, with deeper colors representing higher values). The X-axis is the sub-sequence length (month), and the Y-axis is the start moment of abrupt change (year).

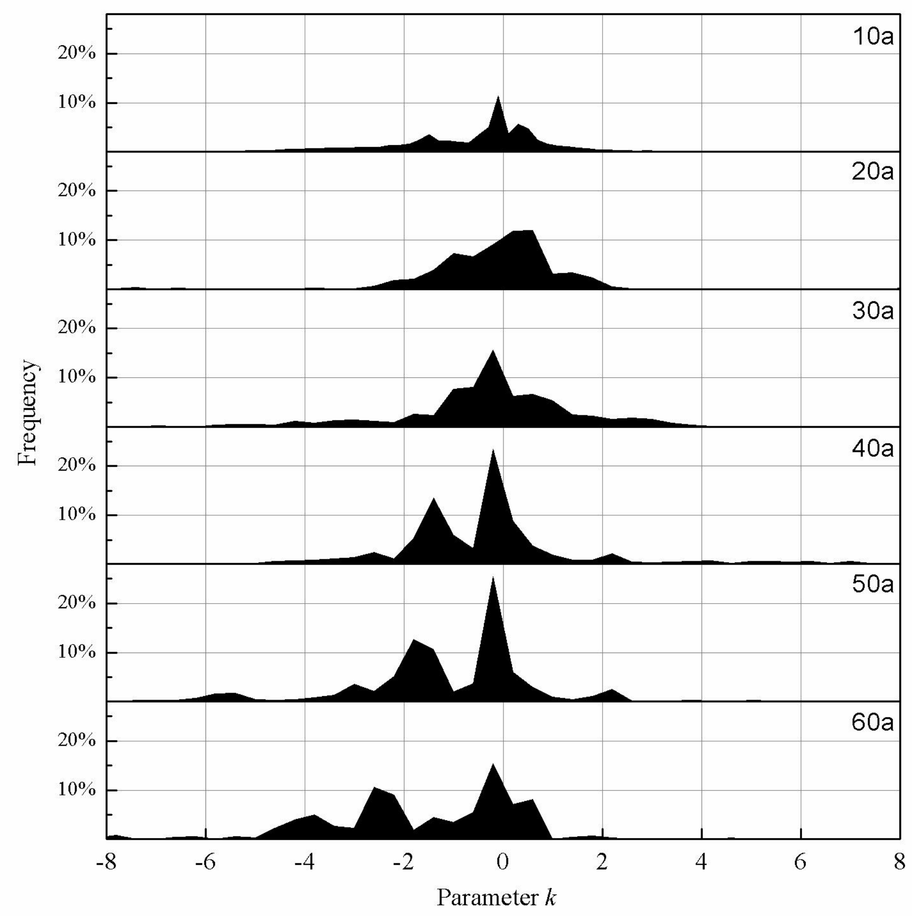


Figure 7. Statistical results of instability parameters for different sub-sequences lengths. The X-axis is the value of the parameter, and the Y-axis is the statistical frequency with a sub-sequence length of 10 years. The gray region in the upper-right corner is for the sub-sequence of 20-60 years.

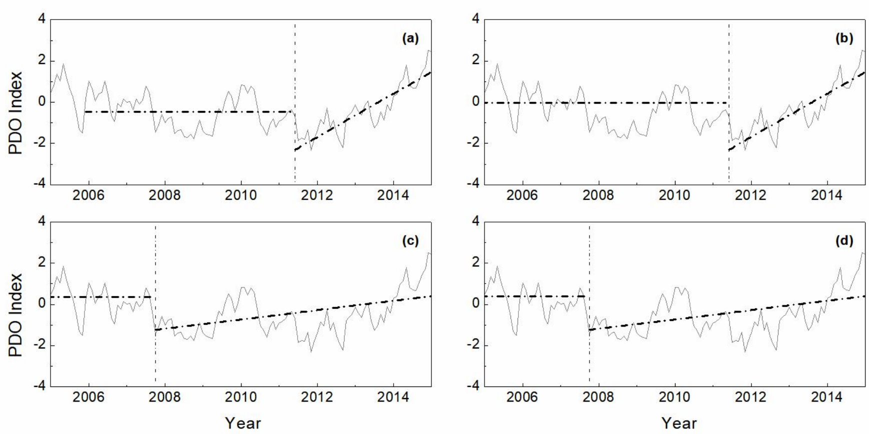


Figure 8. The PDO time sequences and the detection of parameters v and h when the

sub-sequence was set as (a)10 years, (b)20 years, (c)30 years, (d)40 years. The gray lines ~~is~~represent the PDO time sequence~~s~~. The horizontal dash lines represent initial states, the slope dash lines represent linear trend lines of the transition ~~change~~process, and vertical dotted lines represent the start moment.

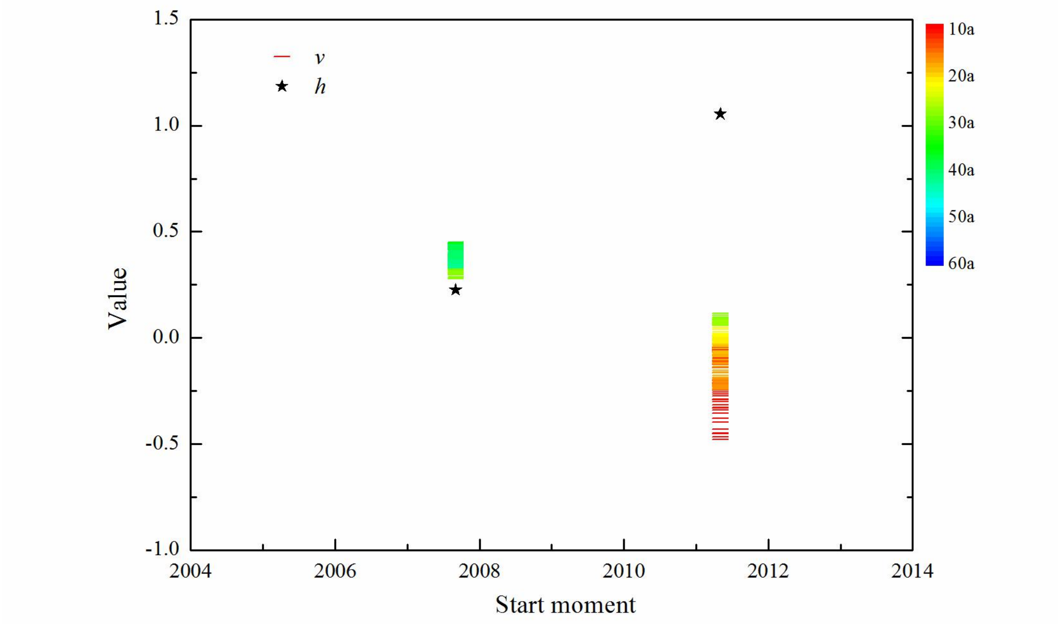


Figure 9. The values of the parameters v and k of two transition ~~chang~~processes with different lengths of sub-sequence. The black stars represent the values of parameter h, and the colourful short bar represent the values of parameter v. The colour bar represents years of the sub-sequence length from 10 to 60 in intervals of 1.~~.~~

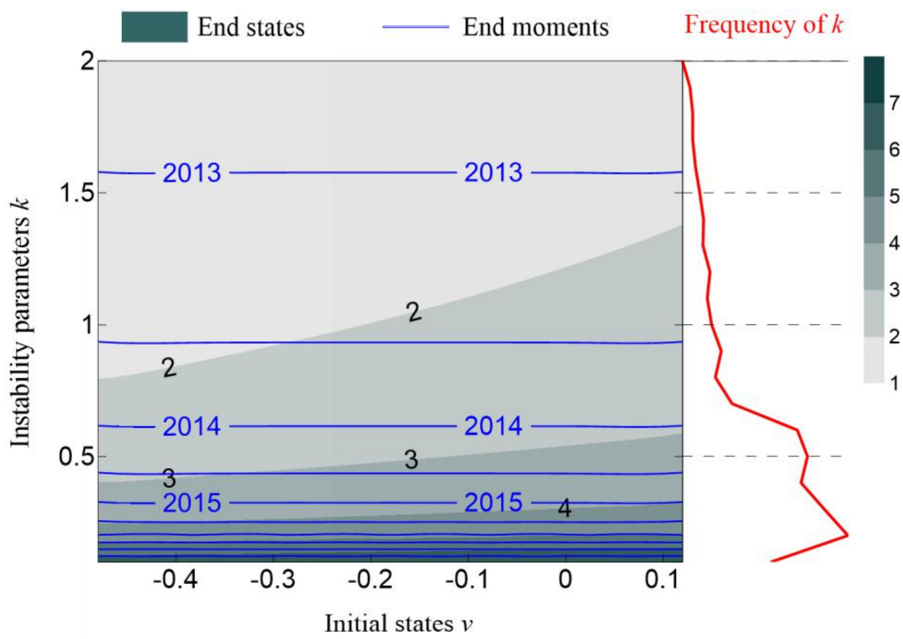


Figure 10. Variation end state and end moment with the initial state parameter *v* (horizontal ordinate) and instability parameter *k* (vertical coordinate). The red line on the right side shows the probability distribution of instability parameter *k*.

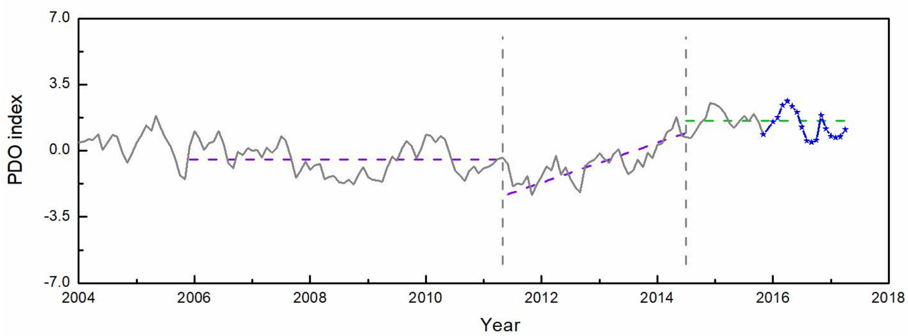


Figure 11. Prediction of the PDO index. The gray line ~~is~~represents the PDO index before 2015; the blue line~~ with starts~~ ~~i~~represents the PDO index after 2015; the gray dash line represent the start moment and end moment; the purple dash lines represent the initial state and the linear trend line, the green line represent the prediction end state.

Dear reviewers,

We do thank you very much. we revise this manuscript based on your comments and reply them one by one as follows.

REPLY TO RC1

Original comments not addressed

1. General comments #2 There is not enough introduction to the methods section before discussing the details of time series analysis.

 • The authors responded to this by rewriting some parts of the Subsections 2.1 and 2.2. I still feel as though a few sentences of introduction to this general Section 2 are necessary before there is a jump to Sub-section 2.1.

REPLY: It seems that we misunderstood the original comment. Before Section 2.1, we added some explanations as follows:

It is necessary to describe the transition process quantitatively before the prediction of the uncompleted climate transition process. We had proposed a detection method by using the logistic model to obtain a transition process. In section 2.1, the method is introduced briefly. On the basis of the detection method, the prediction method for studying the uncompleted transition process is further developed in section 2.2.

2. General comments #3 Variable k appears to be often interchanged with κ

 • Figure 7 still uses κ on the x-axis.

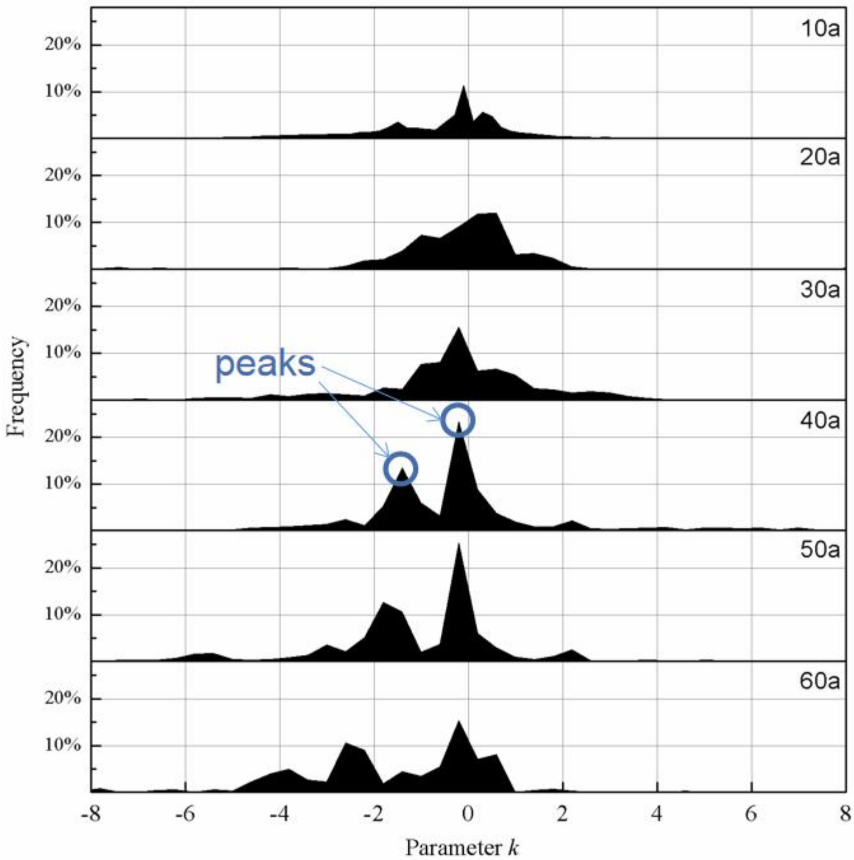
REPLY: Figure 7 is replaced with a new edition, and “κ” is changed to be “k”.

3. General comments #6 In Section 3.1 it is stated “When the length of the sub-sequence is 20 years and 30 years, there is only one peak in the distribution of k values . . . ” This seems strange, as there are said to be multiple peaks for a smaller sub-sequence (10 years), a single peak for 20

and 30, and then multiple peaks for larger sub-sequences. I would assume there would be a more continuous relationship. This is not discussed why this is not the case. Also, a quantitative measure is not specified of what defines a peak.

- The reply from the authors does not address my comments at all. The authors discuss the stability behaviour of k rather than the behaviour of the distribution for different sub-sequence lengths. They mention that the text around this discussion in the manuscript is changed when it has not been. Additionally they still do not specify how they define a peak.

REPLY: In section 3.1, figure 7 is replaced to be a new edition as follows. We consider the extremely high frequency marked by blue circle in the following figure as peaks. It is true that there is a continuous relationship. There is only one main peak for sub-sequence of 10a, 20a, and 30a. There are two main peaks for sub-sequence of 40a, 50a, and 60a. We modify the description in the manuscript as follows:



When the length of the sub-sequence is 20 years and 30 years, there is only one main peak in the distribution of k values, and the parameter k value of the peak is about 0, *which means that the transition change is more stable than the other situations. When the length of the sub-sequence is 40, 50, or 60 years, there are two main peaks.*

4. General comments #8 "Abrupt change" appears to be used synonymously with "transition process" in Section 3.2 and this does not seem consistent with the rest of the paper. Please maintain the same terminology for clarity.

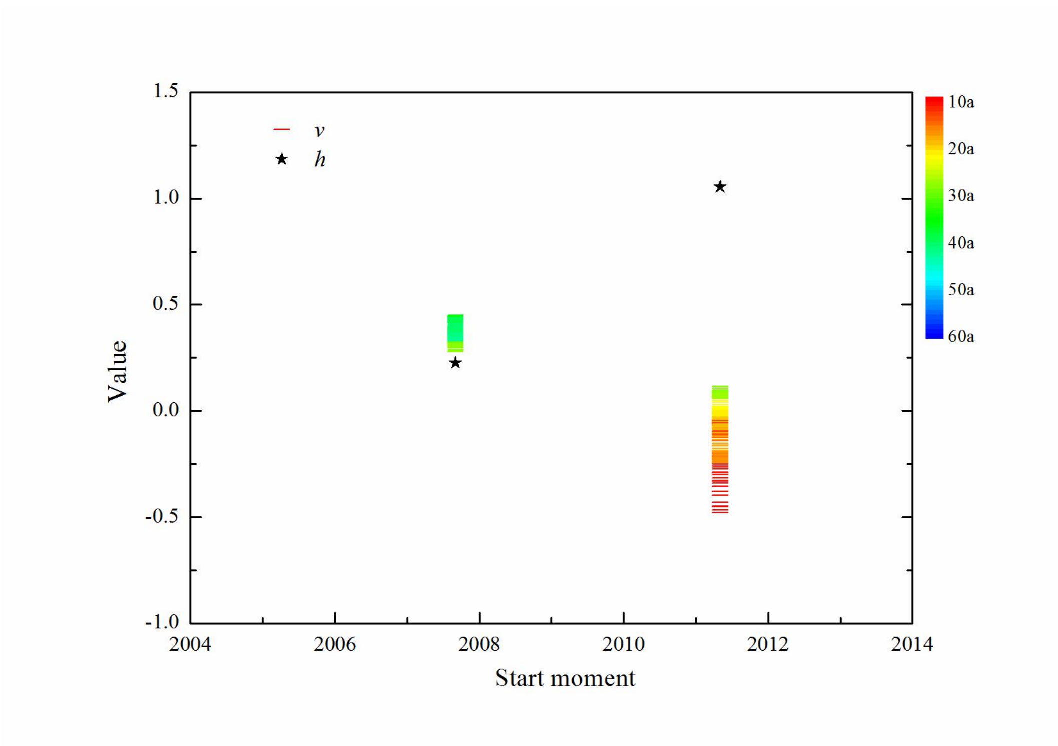
- The phrase was changed in many places to read "transition change". This is redundant. I have noted all the instances below (in specific comments) where it needs to be fixed to "transition process", along with a few instances of "abrupt change" that were missed.

REPLY: Thank you very much for these very detailed comments. We have corrected all the mistakes according to the "*Specific comments/technical corrections*".

5. General comments #10 The lengths of the sub-sequences mentioned in Section 3.2 do not match the numbers on the colour bar in Fig 9. It is therefore not clear what Fig 9 is showing.

- The authors' response does not address the colour bar mismatch at all. The labels on the colour bar still do not clearly represent years from 10 to 60 in intervals of 1.

REPLY: It seems that we misunderstood the mismatch about the color bar. We replace figure 9 with a new edition as follows.



Additional general comments on revised manuscript

1. The statements "According to Thom's theory . . . the general potential energy is obtained as follows" (pg 4, lines 21-24) are not clear. What is meant by "the system described be a quadratic function" and how is it related to the general potential energy? Do you mean that the potential

energy should be described by a quadratic function?

REPLY: It should be "quartic function". In part C, section 5.3 of Thom's book " *Stability Structural and Morphogenesis*", he introduced a general potential energy, $V=\frac{1}{4}x^4+\frac{1}{2}x^2+vx$. The quartic function describes a system with tipping point, which is an abrupt change type. Thus, we study the general potential energy of Eq.2 in the manuscript as:

$$V_{(x)}=-\int_0^x \ddot{x}dx=-\int_0^x 2k^2\left[x-(u+v)/2\right](x-u)(x-v)dx$$
$$=\frac{k^2}{2}\left[x^4-2(u+v)x^3+\left(u^2+v^2+4uv\right)x^2-2(u+v)uvx\right]$$

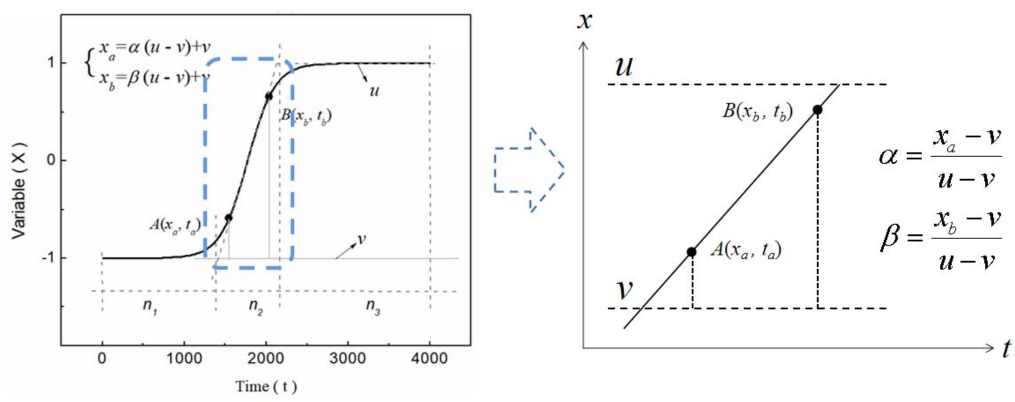
, which is similar to Thom's equation. It also means that the equation, $\dot{x}=k(x-u)(v-x)$, describes a system with tipping point abrupt change. We change "quadratic function" to be "quartic function" in section 2.1.

2. Equations 5 and 6 need to be incorporated into sentences and full punctuation is needed for all equations. Additionally, h is defined twice. If I understand the rest of the section correctly I believe one should be $h0$.

REPLY: The full punctuations are added after equations 5 and 6. In equation 5, we define h with two points (A and B). In equation 7, we calculate h value by using the solution of function (2), and we have the quantitative relationship. For the relationship (Eq. 8), $h = k(u-v)^2\chi$, h, u, v are obtained by equation 4. Then, we have k value.

3. The introduction of location parameters α, β (pg 5, line 14) seems out of place. They are not used in any equations up to that point. Please explain more formally how these are related to the system states x_a and x_b in Eq. 5?

REPLY: The parameters α and β are introduced to describe the positions of points A and B. Figure 2d is shown as follows, and the transition process is extracted and placed on the right. The parameters α and β are defined with u, v and x. In figure 3, a numerical test is stated to study the impact of the positions of points A and B (also parameters α and β) on parameter χ. Finally, it is noted that parameter χ change sightly and it is given as an invariant constant.



4. Equations 7 and 8 should be incorporated into the sentences where they are introduced for improved clarity and understanding for the reader.

REPLY: Punctuation after equation 8 has been modified.

5. What does the term "indefinite" mean here (pg 7, line 4)? It seems to be used synonymously with "unknown".

REPLY: When point A and point B change around their original positions (α=0.2 and β=0.8), the χ value changes very little. This means that we don't have to know the exactly positions of point A and B. we can approximate the value of χ. We rewrite this part in the manuscript.

6. In Eq. 9 the random terms are just labeled as "random_t". It would be more appropriate to label with a variable name (e.g. σ, η, etc.) and state from which distribution the random variable is chosen.

REPLY: We modify Eq. 9 and replace $random_t$ with η_t. More description is added as follows:

As shown in figure 5, four ideal time sequences are constructed by using the logistic model and random numbers as Eq. (9), where η_t represents the random number.

7. The sentence "Therefore, for the entire sub-sequence, there are many transition changes" (pg 9, line 19) is unclear what the message. In particular the phrase "the entire sub-sequence" is used multiple times throughout the paper and I am not sure what is meant by it.

REPLY: Three "entire sub-sequence" were included. They are not necessary, and all of them are removed. In section 3.1, the sentence "Therefore, for the entire sub-sequence, there are many transition changes" has been changed to be:

More transition processes are detected.

Specific comments/technical corrections

All technical mistakes are corrected based on the comments. They are too many to be listed. We sincerely thank these detailed comments.

REPLY TO RC2

General comments

The paper has been improved but some issues remain.

The method of obtaining the value of k remains unclear. Page 8, line 27 "… we can get the value of parameter k by counting all changes …". If it is obtained by counting changes, how can be negative?

REPLY: The "changes" should be "abrupt changes". According the logistic model, there is no abrupt change if k =0. If $k \neq 0$, the abrupt change occurs. We can calculate the k value based on the abrupt change. If k>0, the time sequence transits from the negative phase to the positive phase and vice versa.

The use of functions with jumps (see fig. 8) remains unjustified. All the developments and definitions are done with the logistic model or piecewise continuous functions, but the application to the real system has jumps between the initial state and the transition process and between the transition process and the final state.

REPLY: In order to get optimums fitting effect, we did not use continuous piece-wise function to fit the real system. In most cases, due to the continuity of the real time series itself, the fitting results have a slight jump, which has little impact on the final prediction results. If there is a significant jump in the time series, the prediction results will be significantly affected. In the future, we will carry out more ideal experiments to study the influence of the abnormal jump of the sequence on the prediction results.

Specific comments

Page 7, line 26. It is stated "The parameters v, u and k of the logistic model are set as -1.0, 2.0, 0.1, ..." but in figure 5, the v and u values seem to be -0.5 and 2.5 respectively.

REPLY: The parameters v, u and k of the logistic model are set as -1.0, 2.0, 0.1, for the ideal time sequence, and the random number is limited in 0-1. We built the ideal time sequence by using the sum of the logistic model and the random number. Thus, the *v* and *u* values seem to be -0.5 and 2.5 respectively in figure 5.

In page 8, the recovered values are 2.92, 2.65 and 2.58, converging to 2.5 as deduced from the graphic instead to the value stated in the text (2.0). The recovered values show big differences (2.92 – 46% error, 2.65 – 32.5% error and 2.58 – 29% error) with respect to to the original value (2.0). This lack of agreement contrasts with the good results in the real case showed in fig. 11. May be, this disagreement due to the introduction of random variations (uniformly distributed?) which are always positive (range 0-1).

REPLY: Due to the introduction of random variations (white noise), the value of end state is 2.5 according to the ideal time sequence which is built by using the sum of the logistic model (the value of *u* is 2.0) and the random number (the average value is 0.5). The recovered values are 2.92, 2.65 and 2.58 when lengths of time sequence are set to be 240, 250 and 260 respectively. Then, the deviation rate are ((2.92-2.50)/2.50=) 16.8%, ((2.65-2.50)/2.50=) 6% and ((2.58-2.50)/2.50=) 3.2%. The prediction value of *u* is approaching to be 2.5 when the length of time sequence is given enough to cover the entire transition process.

Technical corrections

Page 2, line 19-20 “It is difficult to detect the abrupt change occurs at the end of sequence.” Is there something missing? For example “… abrupt change [that] occurs …”.

REPLY: This mistake is corrected.

Page 4, line 6 “detect the transition period (Yan et al, 2015). Here, the detection method is

troduced” Should be “introduced”? Misspelling?

REPLY: This mistake is corrected.

Page 13, line 26 “Due to the lake of enough data…” Misspelling? Lack?

REPLY: This mistake is corrected.