REPLY to RC1

Dear reviewer,

Thanks for the comments, we modify the manuscript according to the comments and reply them one by one as follows. More details are included in the supplement for the plain text can not display the entire reply especially the symbols.

General comments

- *"The paper needs major changes to address all issues. The developments in Subsection 2.1 from page 4 to page 7 line 5 already appear in the first part of section 2 of ref."Yan PC, Feng GL, Hou W. A novel method for analyzing the process of abrupt climate change. Nonlinear Processes in Geophysics 2015; 22:249-258, doi: 10.5194/npg-22-249-2015" pages 250-251 and do not introduce any new information. They should be omitted and cited or resumed."*

REPLY: In the previous paper *of ref."Yan PC, Feng GL, Hou W. A novel method for analyzing the process of abrupt climate change. Nonlinear Processes in Geophysics 2015; 22:249-258, doi: 10.5194/npg-22-249-2015"*, we introduced the detection method for transition process of climate change in detail. In this manuscript, we develop this method to predict the uncompleted transition process. Thus, the introduction about the method is necessary. Now, we omitted the unnecessary part about the method, and introduce briefly.

- *"The method to determine the values of location parameters α and β, or the position of points A and B is not clearly specified."*

REPLY: We mark point *A* and point *B* in figure 2d, and add two lines in figure 2d to explain how to define parameters α and β. Parameters α and β are defined to introduce x_a and x_b.

- *"The numerical tests of section 2.1 are not fully specified neither its purpose."*

REPLY: In section 2.1, we define a new parameter χ to simplify Eq.(7) for now, and the relationship among χ, α, β is shown in figure 3b. we find that the changing of χ is limited even the change of parameter α is 25%. Besides, there is little influence on the detection of parameters if the position of the points(the values of parameters α and β) are indefinite. We add more explanation in the manuscript.

- *The results of simulated prediction method of section 2.2 are drawn in figure 5 but not quantified in the text, so the quality of the prediction method can not be appreciated.*

REPLY: We add more description in manuscript as follows:

- “For the entire time sequence, there are 500 moments as shown in figure 5a. In figure 5b, only 240 moments are given, and the other moments are unknown. Then, we obtain parameters v and h by regression method. Then, Parameter u is calculated with Eq(8) since parameter k is given. The blue line represent the prediction result. The transition process would be ended in moment 342 with the end state value 2.92. In figure 5c, the end moment and end state are predicted 356 and 2.65 respectively when the time sequence is given 250 moments. In figure 5d, the time sequence is given 260 moments, and the end moment and end state are predicted 359, 2.58 respectively. The ideal experiments predict the end moment and end state of transition process successfully. The results also show that the longer the transition process experience, the more accurate the prediction.”

- *The method described in the paper is based on the use of continuous functions: piecewise linear functions or logistic model; but it is applied to discontinuous functions: see figures 8 and 11. The lines have jumps and the application of the previous equations to discontinuous functions must be justified.*

REPLY: We use the continuous function to express the transition process approximately rather than a piecewise function. And, the real climate time sequence is also continuous indeedly. In the manuscript, we rewrite the paragraph in page 4 line 2-8 as follows:

- "Where t represents time, and x_t represent the system states, which is obtained by the linear regression method. It is noted that the climate system is continuous even the sampling sequence that makes it is discontinuous. We used a continuous function to express this transition period approximately, and we also created a novel method to detect the transition period(Yan et al, 2015). The continuous evolution of Logistic model is consistent with the transition process(May, 1976), which is shown in figure 1d. The modified logistic model is expressed as follows: "

The table with the results of analysis of past 10 years in Section 3.2 is missing.

REPLY: Table 2 was missing, and it is added now.

Specific comments

- *2.1 The detection method of transition process Page 3, lines 21-22. ". . . the period around point C is expanded to a longer period, or the period around point C is observed on a more short time scale ..." It is not the same. Figure 1b corresponds to the second option: "observe on a more short time scale" or better "observe on shorter time scale". The idea is that with a more detailed view, the transition process can be observed.*

REPLY: We did some modification about the description as follows:

"If the period around point C is observed on a more short time scale (as shown in figure 1b), a transition period is obtained, and it is a part of the original time sequence. In fact, many abrupt change could be considered to be a transition period with a more detailed view. "

- *In page 6 line 13 and eq. (12), the amplitude of change is denoted by w, but in eq. (13) the notation is changed to ω.*

REPLY: This mistake is corrected

- *Page 7, line 6 "According to the numerical experiment...". Figure 3 is not a numerical experiment; part (b) is a contour map of χ for $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$*

and part (a) is a section of that contour map along the line α + β = 1 (probably).

REPLY: This mistake is corrected

- *Page 7, line 8. The assertion "the sum of α and β is 1", does it mean that figure 3(a) is the profile of figure 3(b) along the diagonal α + β = 1? Please, clarify. See the remark for the caption of Fig. 3. Page 7, lines 13-15. "Let the sum of α and β be 1, then then the change of parameter χ is only related to parameter α . . . (also in figure 3a)". This is obvious, χ depends on the two parameters (has two degrees of freedom), by imposing a relationship between the two parameters, you reduce the degrees of freedom to one. This sentence does not add any information and should be suppressed.*

REPLY: In fact, due to the symmetry of the transition process to the middle point, we assume that point A and point B are symmetrical about the middle point, and the sum of α and β is 1. We rewrite this part in the manuscript.

- *Page 7, line 16. "In figure 3c, three ideal experiments were carried out...". The experiments were not carried out in the figure 3c. Figure 3c describes the parameters of the three experiments. As noted before, the experiments deserve their own subsection.*

REPLY: These are mistakes. We change "experiment" to be "situation".

- *The setup of the tests is not clearly described. From the figure 3c, we know the parameters u, v and k, from figure 3c. Other parameters are in table 1. The test setup will be clearer if table 1 would include all parameters for each test. Parameter h0 in table 1 is not defined. Nothing is said about the time span of tests; if the points are randomly perturbed and how. See the previous remark.*

REPLY: In figure3c, we test three different situations with different values of parameters u, v and k. When the values of parameter α are different, the values of parameter χ are almost constant. Parameter h_0 and h are defined and obtained as follows in manuscript:

"In addition, linear trends of these three ideal models are calculated according to the points and by regression method which are marked as h_0 in table 1. The linear trends are also calculated by the values of point *A* and point *B* with Eq(5) which are marked as *h* in table 1."
Besides, in table 1, the value of parameter χ is not right. We correct the result in this edition.

- *2.2 The prediction method of transition process Page 8 line 7, ". . . there is the quartic function relationship between linear trend and amplitude of change." Eq. (13) reads $h=k\,\omega^2\,\chi$. This equation is quadratic in the amplitude ω, not quartic.*

REPLY: This mistake is corrected

- *Page 8, line 18, We are supposing the repetition of events, assuming all events have the same k. We obtain v and h. Which is the value of α? We are also assuming that $\alpha+\beta=1$, so we can calculate χ.*

REPLY: This is a mistake. In section 2.1, an explanation is added. Parameter α is set as 0.2, and parameter χ is 0.2164.

- *Page 8, line 23, An ideal time sequence is constructed with a logistic model with parameters $v=-1$, $u=2$ and $k=0.1$. But in figure 5, the steady part of the curve is well above -1 in the left part and above 2 in the right part. From that graphics, the limits seem to be $v=-1.5$, $u=2.5$.*

REPLY: The ideal time sequence is constructed by using the logistic model and random numbers. The random numbers are limited in (0, 1). We correct this mistake in the manuscript.

3.1 Threshold of stability parameter k

- *Page 9, line 15. The origin of the values for the parameter k (green dots) that appear in figure 6a is missing.*

REPLY: It is true that some green dots in figure 6s are missing. These missing dots represent the value of parameter *k* are large. For example, during 1960~1970, the threshold of *k* value is about 200-600 as shown in figure 6a, which means these abrupt changes are unstable. Fortunately, most abrupt changes are stable as statistical results in figure 7.

- *3.2 Determination of abrupt change and the threshold for initial state v and linear trend h*

Page 11, line 8, "We use the method to analyze..." But the method is not specified.
REPLY: "The method" means the method proposed in section 2.2 in this manuscript. We correct this mistake.

- *Page 11, line 10, "Parameters v and h are obtained . . . abrupt changes (Table 1)." But the title of table 1 is "The parameters for ideal models" and also written in page 7, line 18. Is there another missing table?*

REPLY: It should be table 2. Table 2 is added in the manuscript.

- *Page 11, line 21. ". . .the abrupt change determined through the percentile threshold method ...". This method must be described or referenced.*

REPLY: The percentile threshold method is a statistical method for studying extreme events. The reference here is added now.

- *Page 11, line 23. Along the paper, time series were approximated by piece-wise continuous functions: the system was in a steady state, and from that value changed up or down. But in figure 8 time series are approximated by functions with jumps from the value of the steady states to the beginning of the slope lines that approximate the changes. These profiles are different from those used in figure 5 to simulate the process of recovering the parameters and those of the logistic functions.*

REPLY: The continuous functions is used to determine the quantitative relationship among parameters. When calculate the values of parameters, we used the optimal linear regression method in different segments. Thus, these profiles are linear lines which are not like the curve in figure 5.

- *Page 13, line 4, "...the parameter h=1.054/a..." The units of h are not clear, what does a mean? Year? The same problem appears in line 9 in the same page.*

REPLY: The unit of the PDO index with time(month) is 1, so the linear trend of PDO should be month^{-1}. We correct this mistake.

- *Figure 3 Caption. (a) part: it is not stated which diagonal of (b) refers to, $\alpha = \beta$ or $\alpha + \beta = 1$. It, also, would be interesting to mark points 1, 2 and 3 from (b) in the part (a) of figure.*

REPLY: The diagonal refers to $\alpha + \beta = 1$. A gray line is added in figure 3(b). Points 1, 2 and 3 represent three different situations, and we mark them with S1, S2 and S3. We also change the description in table 1.

Technical corrections

- *Cites should be separated from text by a blank, e.g. p. 2 line 1 ". . . change (Charney ...)" and many more. Page 6 line 4 "Then, we do integration...", I consider better "Then, we integrate..."*

REPLY: We correct these mistakes.

REPLY to RC2

Dear reviewer,

Thanks for the comments, we modify the manuscript according to the comments and reply them one by one as follows. More details are included in the supplement for the plain text can not display the entire reply especially the symbols.

General comments

- *1. The abstract needs to be made more clear. The phrase "more details of climate change" is too broad and does not explain what exactly is being addressed by the methods presented in the paper. The PDO is also not explained, nor its relation to climate change.*

REPLY: We rewrite the abstract as follows, and we add more explanation about the PDO.

"Climate change is expressed as a climate system transiting from the initial state to a new state in a short time. The period between the initial state and the new state is defined as transition process, which is the key to connect the two states. By using a piecewise function, the transition process is expressed approximately (Mudelsee, 2000). However, the dynamic processes are not included in the piecewise function. Thus, we had proposed a method to study the transition process by using a continuous function. In this manuscript, the method is developed to predict the unfinished transition process based on the dynamic characteristics of the continuous function. We introduce this method in details and apply it to predict end moment and end state of one unfinished transition process of the Pacific Decadal Oscillation (PDO) time sequence, which is a long-lived El Niño-like pattern of Pacific climate variability (Barnett et al, 1999). This method reveals a new relationship during the transition process, which explores a nonlinear relationship between the linear trend and the amplitude (difference) between the initial state and the end state. Since the transition process begins, the initial state and the linear trend are estimated. Then, according to the relationship, the end states and end moment of the unfinished transition process is predicted. The results of either the ideal experiments or the PDO index show good prediction."

- *2. There is not enough introduction to the methods section before discussing the details of time series analysis.*

REPLY: In this manuscript, we develop a new method to predict the end state and end moment of a uncompleted transition process based on the detection method of transition process. The detection method had been published in our previous papers as follows. Thus, we rewrite this part about the method in this edition of the manuscript. More details about the prediction method are added. Also, we introduce more about the ideal experiments in section 2.2.

Yan PC, Feng GL, Hou W, Wu H Statistical characteristics on decadal abrupt change process of time sequence in 500 hPa temperature field. Chinese Journal of Atmospheric Sciences 2014; 38 (5): 861−873

Yan PC, Feng GL, Hou W. A novel method for analyzing the process of abrupt climate change. Nonlinear Processes in Geophysics 2015; 22:249-258, doi: 10.5194/npg-22-249-2015

Yan PC, Hou W, Feng GL Transition process of abrupt climate change based on global sea surface temperature over the past century, Nonlinear Processes in Geophysics 2016; 23:115–126, doi:10.5194/npg-23-115-2016

- *3. The mathematical notation is inconsistent and unclear.*

Variable k appears to be often interchanged with κ

Parameter k is referred to as both a stability (Fig. 10 and Section 3.3) and instability parameter (Fig. 10).

μ is never formally introduced and is potentially being exchanged with u.

REPLY: Stability parameter *k* should be instability parameter. We corrected this mistake. The other two mistakes are also corrected.

- *4. There is terminology that is used but not defined.*

continued process (pg 1, line 12)

the filtering process (pg 2, line 16)
ramp function (pg 2, line 24)
crush (pg 4, line 21)
percentile threshold (pg 11, line 21)
augmented abrupt change (pg 11, line 28)

REPLY: We correct all above mistakes in manuscript one by one. More details are as follows:

continued process (pg 1, line 12)

The wrong description is moved.

the filtering process (pg 2, line 16)

”filtering” is replaced by “transition”

ramp function (pg 2, line 24)

The ramp function means piecewise function according to Mudelsee’s work. This mistake is corrected.

crush (pg 4, line 21)

It means that the system will be crushed. A reference is added.

percentile threshold (pg 11, line 21)

A reference is added.

augmented abrupt change (pg 11, line 28)

We rewrite this paragraph, and the mistake is corrected.

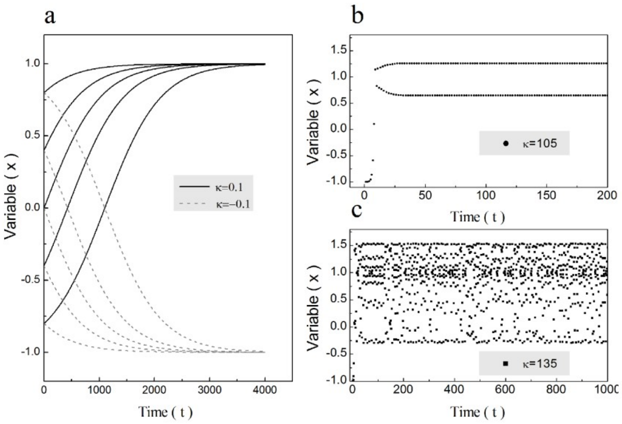
- *5. There is inconsistency between Section 2.2 and Section 3. In Section 2.2 it is stated that the parameter k cannot be obtained from the data (with no explanation as to why), so it is fixed a priori. In Section 3 the parameter k has been estimated from a time series, but again with no explanation as to how one would estimate this.*

REPLY: In section 2.2, the ideal time sequence only have one abrupt change, which means that we have no way to obtain the parameter k because that there is no more other climate changes. While in section 3, the PDO index have several abrupt changes, and parameter k is obtained by counting these changes. We rewrite this paragraph in the manuscript as follows.

" It has to be noticed that in this ideal time sequence there is just one abrupt change, which means that we have no way to obtain the parameter k by counting many changes. Thus parameter k is given directly, and the prediction of the end state (moment) is drawn in figure 5b, 5c and 5d. For the entire ime sequence, there are 500 moments as shown in figure 5a. In figure 5b, only 240 moments are given, and the other moments are unknown. Then, we obtain parameters v and h by regression method. Then, Parameter u is calculated with Eq(8) since parameter k is given. The blue line represent the prediction result. The transition process would be ended in moment 342 with the end state value 2.92. In figure 5c, the end moment and end state are predicted 356 and 2.65 respectively when the time sequence is given 250 moments. In figure 5d, the time sequence is given 260 moments, and the end moment and end state are predicted 359, 2.58 respectively. The ideal experiments predict the end moment and end state of transition process successfully. The results also show that the longer the transition process experience, the more accurate the prediction."

- *6. In Section 3.1 it is stated "When the length of the sub-sequence is 20 years and 30 years, there is only one peak in the distribution of k values: : :" (pg 10, lines 21-24). This seems strange, as there are said to be multiple peaks for a smaller subsequence (10 years), a single peak for 20 and 30, and then multiple peaks for larger subsequences. I would assume there would be a more continuous relationship. This is not discussed why this is not the case. Also, a quantitative measure is not specified of what defines a peak.*

REPLY: The description in this part was not right. We rewrite this part about the values of parameter k. Parameter k characterizes the stability of the system during climate change. If it is detected to be large, the system is not stable. The ideal time sequences are shown in our previous work as follows. The evolution of the system expressed by the logistic model with different stability parameters: (a) the system reaches to the stable states with a different initial variable when parameter $k = \pm 0.01$; (b) the system becomes bifurcated when the parameter $k = 105$; (c) the system becomes chaotic when the parameter k = 135. However, we can not identify the value of parameter directly, but we can find its threshold. Thus, in section 3.1 of this manuscript, we obtain parameter k by counting the climate changes of the PDO index.



By referring: Yan PC, Feng GL, Hou W. A novel method for analyzing the process of abrupt climate change. Nonlinear Processes in Geophysics 2015; 22:249-258, doi: 10.5194/npg-22-249-2015

- *7. The motivation for Section 3.2 is absent and it is not obvious how this section relates to the overall goal of Section 3.*

REPLY: In section 3.1, we obtain the parameter k, and in section 3.2, we obtain the parameters v, h. More explanation as follows in the first paragraph of section 3 is added. We also rewrite the first paragraph of section 3.2.

"During the following research, a transition process starting from 2011 is studied. According to the prediction method, several parameters have to be determined in advance. We determine parameter k firstly."

- *8. "Abrupt change" appears to be used synonymously with "transition process" in Section 3.2 and this does not seem consistent with the rest of the paper. Please maintain the same terminology for clarity.*

REPLY: We check all the manuscript, and change inconsistent description.

- *9. The final paragraph of Section 3.2 (pg 12, lines 10-24) discusses three abrupt changes. The previous paragraph discussed four. There is much confusion as to what abrupt change events are being discussed throughout this paragraph.*

REPLY: We correct this mistake and give more explanation in section 3.2.

- *10. The lengths of the sub-sequences mentioned in Section 3.2 do not match the numbers on the colour bar in Fig 9. It is therefore not clear what Fig 9 is showing.*

REPLY: We add more explanation about figure 9 in section 3.2. In figure 9, the transition process starting from 1976 should not be shown. It is corrected now. Only the transition process starting from 2007 and 2011 are stated.

- *11. There is no discussion as to which abrupt change detection (year 2007 or 2011) is correct, which leads to a lack of motivation for studying only the 2011 event. It needs to be more clearly explained why the 2011 event is chosen for the prediction experiment.*

REPLY: Both of the climate changes starting from 2007 and 2011 are right. When the sub-sequence are set as different lengths, which means we test the climate change in different time scale, the start moments of climate changes might be different. In this manuscript, only the climate change starting from 2011 is studied for testing the prediction method. More explanation is added in section 3.

- *12. The "variation situation of parameter μ" (pg 13, lines 6-7) was never introduced nor explained.*

REPLY: This is a mistake. It is corrected to be "*u*".

- *13. The "prediction result" (pg 13, lines 13-14) was not specified. Additionally, it is not clear which prediction is being shown in Fig. 11.*

REPLY: We rewrite this paragraph as follows in the manuscript. More explanation about the prediction in figure 11 is included.

"In figure 11, the PDO time sequence is displayed as black line. The period during 2006~2011 is detected as the initial state, and a transition process is increasing from this initial state. It is not able to be known whether the increasing process has been completed or not. Based on the linear regression method, the initial state and the linear trend are obtained and shown as purple dash lines. Then by the method proposed in section 2.2, the end state of transition process are obtained with Eq(8), and it is marked as green dash line.. Unlike the uncompleted transition process of ideal experiment, the transition process has completed in 2015 since we detected the PDO change in 2016. This transition process started from 2011, and end in 2015. The initial moment and the end moment are marked as black dash lines. However, we are still not sure whether the PDO complete this transition process or not for it it appears at the end of the sequence. As we all know, many statistical methods are not accurate for the detecting both ends of the sequence. Thus, the real PDO sequence during 2016~2017 is added to the end of the PDO time sequence. The PDO value from 2015 to 2017 is almost unchanged, which is consistent with the predicted result."

- *14. Conclusion needs to be expanded upon much more.*

REPLY: We rewrite the conclusion and discussion, and all three following mistakes are corrected now.

The sentence "The abrupt change with smaller time scales has a continuous process, and the abrupt change with larger time scales becomes abrupt change point." (pg 14, lines 8-10) is not easily understandable and alludes to material that does not appear to have been discussed in the paper.

The phrase "a detected abrupt change beginning in 2011 appears relatively close to the end of the 115-year sequence, and it is difficult to identify by using other methods" (pg 14, line 11-13) was not previously discussed in the manuscript. Please expand on why the abrupt change is difficult to identify through other methods.

There is not enough evidence in the manuscript to support the claim "The findings increases the possibility of resolving the problem associated with difficult processing at the end of a time sequence" (pg 14, lines 14-16). Please add discussion of the *problem of difficult processing at the end of a time sequence.*

Specific comments

1. pg 1, line 14 - Change "self" to "itself"

REPLY: This mistake is corrected.

2. pg 1, line 15 - Add full reference to paper on PDO

REPLY: The reference is added.

3. pg 1, line 16 - Remove "And" from beginning of sentence

REPLY: This mistake is corrected.

4. pg 2, lines 4-6 - Add references for each of the fields mentioned

REPLY: Two references are added.

5. pg 2, line 6 - Change "famous" to "observed"

REPLY: This mistake is corrected.

6. pg 2, line 8 - Add reference for "Thom's research"

REPLY: The reference is added.

7. pg 2, line 8 - Remove "And" from beginning of sentence

REPLY: This mistake is corrected.

8. pg 2, line 24 - "Ramp Function" does not need to be capitalised

REPLY: This mistake is corrected.

9. pg 2, line 26 - "Non-linear Function" and "Ramp Function" do not need to be capitalised

REPLY: This mistake is corrected.

10. pg 2, line 29 - Remove "Besides" from beginning of sentence

REPLY: This mistake is corrected.

11. pg 3, line 1 - Change "got reach to" to "reached"

REPLY: This mistake is corrected.

12. pg 3, line 3-4 - Capitalise "decadal oscillation"

REPLY: This mistake is corrected.

13. pg 3, line 4 - Move reference to end of sentence

REPLY: The reference is moved..

14. pg 3, line 5 - Remove "has"

REPLY: This mistake is corrected.

15. pg 3, line 20 - Change "change" to "changes"

REPLY: This mistake is corrected.

16. pg 3, line 22 - Change "more short" to "shorter"

REPLY: This mistake is corrected.

17. pg 3, line 24 - Change "change" to "changes"

REPLY: This mistake is corrected.

18. pg 4, eq 1 - Add punctuation to equations (including all subsequent equations in paper)

REPLY: This mistake is corrected.

19. pg 4, lines 7-8 - The sentence starting "The population changed: : :" is not clear.

REPLY: This mistake is corrected.

20. pg 5, line 5 - Change "would" to "could"

REPLY: This mistake is corrected.

21. pg 8, line 11 - Change "which" to "that"

REPLY: This mistake is corrected.

22. pg 8, lines 20-21 - Please write out the equation for the logistic model with noise

REPLY: The equation is added.

23. pg 8, lines 21-22 - Please specify the difference between the "three uncompleted changes". Is the same noise realisation used but for different lengths of trajectories?

REPLY: An entire time sequence with 500 moments is shown in figure 5a and three other lengths of time sequences are shown in figures 5b, 5c and 5d respectively. More explanation is added in title of figure 5.

24. pg 8, lines 27-29 - The sentence starting with "The results show: : :" is not clear.

REPLY: More explanation is added in manuscript.

25. pg 10, line 9 - Add "of the largest peak" after "The k value"

REPLY: It is added.

26. pg 10, line 10 - Change "are distributed in the" to "also have"

REPLY: This mistake is corrected.

27. pg 10, line 26 - Please quantify what is meant by "tiny"

REPLY: We rewrite this sentence as:

It is difficult to detect an abrupt change with huge amplitude if the abrupt change takes almost no time.

28. pg 11, lines 1-5 - Percentages of what?

REPLY: Percentages of all *k* values.We correct this mistake.

29. pg 11, line 8 - Add "in the PDO" after "abrupt changes"

REPLY: This mistake is corrected.

30. pg 11, line 10 - Add "the" before "two"

REPLY: This mistake is corrected.

31. pg 11, line 13 - Write out the set of sub-sequence lengths in words

REPLY: We rewrite this sentences in the manuscript.

32. pg 13, line 1 - Remove "after the abrupt change"

REPLY: This mistake is corrected.

33. pg 13, line 9 - Remove the "a" after each year in the brackets

REPLY: This mistake is corrected.

34. pg 13, line 23 - Remove "the" before "uncompleted" and change "process" to "processes"

REPLY: These mistakes are corrected.

35. pg 13, line 24 - Remove "the" before "ideal"

REPLY: This mistake is corrected.

36. pg 13, line 26 - Remove "started

REPLY: This mistake is corrected.

A method to predict the uncompleted climate transition process

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Abstract

Climate change is expressed as a climate system transiting from the initial state to a new state in a short time. The period between the initial state and the new state is defined as transition process, which is the key part to connect the two states. By using a piece-wise function, the transition process is stated approximately (Mudelsee, 2000). However, the dynamic processes are not included in the piece-wise function. Thus, we had proposed a method to study the transition process by using a continuous function. In this manuscript, this method is developed to predict the uncompleted transition process based on the dynamic characteristics of the continuous function. We introduce this prediction method in details and apply it to three ideal time sequences and the Pacific Decadal Oscillation (PDO). The PDO is a long-lived El Niño-like pattern of Pacific climate variability (Barnett et al, 1999). This method reveals a new quantitative relationship during the transition process, which explores a nonlinear relationship between the linear trend and the amplitude (difference) between the initial state and the end state. Since the transition process begins, the initial state and the linear trend are estimated. Then, according to the relationship, the end state and end moment of the uncompleted transition process is predicted.

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删除: predict end moment and end state of one uncompleted transition process of

删除: By considering the short period as a continued process, which is called transition process, more details of climate change would be described according to analysis the time sequence self. We had proposed a method to quantify the transition process of

删除: time sequence, which

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删除: The results of either the ideal experiments or the PDO index show good prediction.

Keywords

Prediction method; Transition process of abrupt change; System stability; Pacific Decadal Oscillation

1. Introduction

A system transiting from one stable state to another in a short period is called abrupt change (Charney and DeVore, 1979; Lorenz, 1963, 1979). The abrupt change system has two or more states (Goldblatt et al, 2006; Alexander et al, 2012), the system swings between these states that are also called attractors in physics. This phenomena is verified in many fields including biology (Nozaki, 2001), ecology (Osterkamp et al, 2001), climatology (Thom, 1972; Overpeck and Cole, 2006; Yang et al, 2013a, 2013b), brain science (Sherman et al, 1981), etc. The latest observed climate change event is global warning hiatus, which has been studied deeply by many researchers (Amaya et al, 2018; Kosaka and Xie, 2013; Yang et al, 2017). Seven different kind of abrupt changes are mentioned in Thom's research(1972). Over the last several decades, many methods have been proposed to identify different kinds of abrupt change (Li et al, 1996), like Moving T-Test, Cramer's (Wei, 1999), Mann-Kendall (MK, Goossens and Berger, 1986), Fisher (Cabezas and Fath, 2002), etc. It is noticed that most abrupt change detection methods suggests that the abrupt change is around a turning point. The significant difference between the average values of the two sequences on both two sides of the turning point is defined as the index to measure the abrupt change. This kind of detection method has a drawback. It is difficult to detect the abrupt change occurs at the end of sequence.

Mudelsee (2000) studied the abrupt change of a time sequence and illustrated that abrupt change has a duration, which can be quantitatively described with a piece-wise (ramp) function. We developed the detection method by using a continuous function to replace the ramp function(Yan et al, 2014, 2015). The new method can confine the beginning and ending points of abrupt change and

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quantitatively describes the process of abrupt climate change, and three parameters are introduced. A quantitative relationship among the parameters is revealed (Yan et al, 2015). The relationship could be used to predict the end moment (state) if the system had left the original state but not yet reached to the new state, which is defined as uncompleted transition process.

In this manuscript, three ideal time sequences are tested to study the prediction method. The prediction method is also applied to study the climate transition process of the PDO, which is an important signal that reveals climatic variability on the decadal timescale (Mantua et al, 1997; Barnett et al, 1999; Zhang et al, 1997; Yang et al, 2004). Previous studies (Lu et al, 2013; Trenberth and Hurrell, 1994) have indicated that there are many climate changes in the PDO over the past 100 years. Most researches mentioned the climate changes happened in the 1940s and 1970s. During the 1940s, the PDO transited from a high state to a low state, while during the 1970s, it did the opposite. All this changes and their processes had been studied in our previous researches (Yan et al, 2015, 2016). The climate transition processes were explored clearly. However, we still can not know when the transition processes finish its increasing or decreasing to a stable state if the transition process has begun. We develop a new method to predict the end state and the end moment of a transition process based on the quantitative relationship.

2. Methods

2.1 The detection method of transition process

The real time sequence changes abruptly as shown in figure 1a, and the system jumps to a high state in point C. If the period around point C is observed on a shorter time scale (as shown figure 1b), a transition period is obtained, and it is a part of the original time sequence. In fact, many abrupt changes could be considered to be a transition period with a more detailed view. The transition period was expressed with an ramp function in Mudelsee's research (2000), as shown in figure 1c, and the time

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sequence is divided into three segments, including two equilibrium states and one increasing state. The ramp function is as follows:

$$x_t = \begin{cases} x_1 & t \le t_1 \\ x_1 + (t - t_1)(x_2 - x_1)/(t_2 - t_1) & t_1 < t \le t_2 \\ x_2 & t > t_2 \end{cases} \qquad (1)$$

Where t represents time, and x_t represent the system states, which is obtained by the linear regression method. It is noted that the climate system is continuous even the sampling sequence that makes it is discontinuous. We used a continuous function to express this transition period approximately, and we also created a novel method to detect the transition period (Yan et al, 2015). Here, the detection method is troduced briefly. The continuous evolution of Logistic model is consistent with the transition process (May, 1976), which is shown in figure 1d. The modified logistic model is expressed as follows:

$$\dot{x} = k(x-u)(v-x) \qquad (2)$$

Parameters u and v represent the two equilibrium states respectively. Parameter k represents the switching between different states, and it is defined as instability parameter. As shown in figure 2a, parameters u and v being fixed, and setting k as 0.5, the system transiting to the new state costs a shorter time than that setting k as 0.4. If parameter k is set large enough, the system collapses and becomes chaotic (as shown in figure 2b). When parameter k is set to different values, more situations have been discussed in detail in the previous research (Yan et al, 2016). The result shows that parameter k characterizes the stability of the system (the larger the absolute value, the more unstable the system). According to Thom's theory (1972), the system described by a quadratic function would exhibit tipping-point abrupt change, which the system jumps from one state to a new state abruptly. Thus, we did some mathematical derivation to Eq. (2), and the general potential energy is obtained as follows:

$$\begin{aligned} V_{(x)} &= -\int_0^x \ddot{x}dx = -\int_0^x 2k^2\left[x-(u+v)/2\right](x-u)(x-v)dx \\ &= \frac{k^2}{2}\left[x^4 - 2(u+v)x^3 + \left(u^2+v^2+4uv\right)x^2 - 2(u+v)uvx\right] \end{aligned} \qquad (3)$$

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In figure 2c, the potential energy of Eq. (3) is verified to have two states with the lowest energy, and both of them are stable. This bistable structure is common in the climate system (Goldblatt et al, 2006). Therefore, Eq. (2) can be used to describe the abrupt change system, and the parameters represent different key factors of the transition period during abrupt change. Then, the parameters *u*, *v* and *h* are obtained by regression method (Huang, 1990; Yang et al, 2013a) by using Eq. (4), where i, x_i denote the time and the state of the system at this time, and $\bar{i}, \bar{x}_i$ are their averages respectively. Variable n_2 is the length of second segment. The linear trend *h* represents the ratio of system state change to time, and it can be expressed by two points on the curve approximately as Eq. (5), where the two points are *A* (x_a, t_a) and *B* (x_b, t_b).

$$\begin{cases} v = \sum_{i=1}^{n_1} x_i / n_1 \\ u = \sum_{i=n_1+n_2+1}^{n} x_i / n_3 \\ h = \sum_{i=n_1+1}^{n_1+n_2} \bar{i} \cdot \bar{x}_i \Big/ \sum_{i=n_1+1}^{n_1+n_2} \bar{i}^2 \end{cases} \qquad (4)$$

$$h = \frac{x_a - x_b}{t_a - t_b} \qquad (5)$$

As shown in figure 2d, the transition period during point *A* (x_a, t_a) and point *B* (x_b, t_b) is approximately linear. Then, we can use the location parameters *α*, *β* to express system states x_a and x_b. By solving Eq. (2), the relationship between *x* and *t* is determined.

$$t = \frac{1}{k(u-v)} \ln\left(\frac{x_0 - u}{x_0 - v} \cdot \frac{x - v}{x - u}\right) + t_0 \qquad (6)$$

Then, parameter *h* is rewritten as Eq. (7). It is noted that the rightmost part is only related to the location parameters *α* and *β*, then let it be *χ*. Then, the relationship of Eq. (7) is rewritten as Eq. (8).

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$$h = \frac{x_b - x_a}{\frac{1}{(\mu-\nu)\kappa}\ln\frac{x_0-\mu}{x_0-\nu}\left(\frac{x_b-\nu}{x_b-u}-\frac{x_a-\nu}{x_a-u}\right)} = \kappa(\mu-\nu)^2\frac{(\beta-\alpha)}{\ln\frac{\beta(\alpha-1)}{\alpha(\beta-1)}} \qquad (7)$$

$$h = \kappa\omega^2\chi \qquad (8)$$

In order to determine the value of parameter χ, the relationship among χ, α, β is displayed in figure 3b. The dash line in figure 3a is the profile of the diagonal in figure 3b, which represents that the sum of α and β is 1. Parameter χ changes little when the location parameter varies in a certain range as marked with warm color in figure 3b. It means that the closer the points (A and B) are to the middle point, the more significant the linear feature is. Then, the process between point A and point B can represent the whole transition process as shown in figure 3c. It is noted that the transition process is symmetrical about the middle point approximately. Thus, we assume that point A and point B are symmetrical about the middle point, and the sum of α and β is 1. The change of parameter χ is only related to parameter α (or parameter β), as shown in the diagonals in figure 3b (also in figure 3a). Parameter χ changes little when parameter α is about 0.2 or larger. In figure 3c, three different situations are carried out to study the influence of parameter α on parameter χ. In each situation, points (A and B) are set to be different positions, and their parameters were calculated respectively in table 1. The parameters α are set as 0.20, 0.25, 0.15 respectively in three different situations marked with S1, S2 and S3. For S2 and S3, both of the percentages of α changing to S1 are 25%, while the percentages of χ changing are only 5.15% and 6.76% respectively, which means the percentage change of χ is much less than α. In addition, linear trends of these three ideal models are calculated according to the points and by regression method which are marked as h_0 in table 1. The linear trends are also calculated by the values of point A and point B with Eq(5) which are marked as h in table 1. It is noted that although the positions of points are

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different, the trend obtained according to the points is almost the same as that obtained by regression method. The error percentages are 2.36%, 2.25%, 1.38% respectively, which means that when the position of the points (the values of parameters α and β) are indefinite, there is little influence on the detection of parameter h. Thus, in the following sections parameter α is set as 0.2, and parameter χ is 0.2164

2.2 The prediction method of transition process

Eq. (8) shows the quantitative relationship among linear trend, instability parameter, and amplitude of change. There is a linear relationship between linear trend and instability parameter; and there is the quadratic function relationship between linear trend and amplitude of change. We did reveal this quantitative relationship much more than in theory but in real time series (Yan et al, 2016). Based on this relationship, we are going to create a new method to deal with the problem that the transition process has not finished. During the real time sequence, the system transits away from the original state, but it has not reached to a new state as shown in figure 4. The red line represents the period which has been experienced, while the gray line represents the period which hasn't been experienced. Based on the system states which is far away from the original state, a quasi linear extension of the transition process is established (dash line). Then the parameters v and h are obtained by Eq. (4). Assuming that the parameter k satisfies the statistics in the history of the system, the parameter u can be predicted by Eq. (8), and the end moment is also predicted apparently.

$$\begin{cases} x_t = x_{t-1} + kt(x_t - u)(v - x_t) \\ x'_t = x_t + random_t \end{cases} \tag{9}$$

As shown in figure 5, four ideal time sequences are constructed by using the logistic model and random numbers as Eq. (9). An entire time sequence with 500 moments is shown in figure 5a and three other lengths of time sequences are shown in figures 5b, 5c and 5d respectively. The parameters v, u and k of the logistic model are

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set as -1.0, 2.0, 0.1, for the ideal time sequence, and the random number is limited in 0-1. The parameters v, h are obtained by regression method before making prediction. It has to be noticed that in this ideal time sequence there is just one abrupt change, which means that we have no way to obtain the value of the parameter k by counting many changes. Thus parameter k is given directly, and the prediction of the end state (moment) is drawn in figure 5b, 5c and 5d. For the entire time sequence, there are 500 moments as shown in figure 5a. In figure 5b, only 240 moments are given, and the other moments are unknown. Then, we obtain parameters v and h by regression method. The parameter μ is calculated with Eq. (8). The blue line represent the prediction result. The transition process would be ended in moment 342 with the end state value 2.92. In figure 5c, the end moment and end state are predicted to be 356 and 2.65 respectively when the time sequence is given 250 moments. In figure 5d, the time sequence is given 260 moments. The end moment and end state are predicted to be 359 and 2.58 respectively. The end moment and the end state of prediction result match the presetting lines. The results also show that the longer the transition process experience, the more accurate the prediction.

3. Results

In order to test the validity of this prediction method in a real climate system, we apply this method to predict the uncompleted transition process of the PDO. The PDO index data used is from website of the University of Washington (http://research.jisao.washington.edu/pdo/). The time period from January of 1900 to November of 2015 is studied as the training data, and the time period from December of 2015 to April of 2017 is used as the test data. During the following research, a transition process starting from 2011 is studied. According to the prediction method, several parameters have to be determined in advance. We determine parameter k firstly.

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3.1 Threshold of parameter *k*

Parameter *k* characterizes the stability of the system during climate change, which means that we can get the value of parameter *k* by counting all changes of the PDO index. The histogram in Figure 6a shows the PDO time sequence from January of 1900 to November of 2015, and it shows that the PDO went through several changes. The green dots in Figure 6a are parameter *k* when the sub-sequence length takes 20 years. In the early 1940s and late 1970s, there are two transition changes of the PDO mainly. The absolute value of the parameter *k* is large, which means that the system is much more unstable during this two transition changes. In the 1940s, the PDO transits from a positive phase to a negative phase, and the $k < 0$, whereas the situation in the 1970s is the opposite. Figure 6b shows more *k* values corresponding to the different sub-sequence lengths (as indicated by X-axis, the variation range of the sub-sequence is 20-60 years, with an interval of 1 year). The Y-axis is the start moment, and the locations of the dots indicate the start moments for the corresponding sub-sequence lengths. In particular, the blue dots represent that parameter *k* is negative, and the red dots represent that it is positive. The dots in the left side region are more than the dots in the right side region in figure 6. This is because when the length of sub-sequence is short, the amplitude is also often small. Therefore, for the entire sub-sequence, there are many transition changes. When the length of the sub-sequence reaches or exceeds 50 years, the transition change mainly begins in the 1940s and 1970s, which are also investigated in other research (Shi et al, 2014). The transition changes in these two periods correspond to large *k* values, which means that these two transition changes are more unstable than others. More statistical results indicate that the threshold distribution of parameter *k* values in historical abrupt change processes exhibit multiple peaks (Figure 7). Specifically, the peak with the largest probability is located near to 0. The *k* value of the largest peak in the distribution is small, which indicates that the abrupt changes that correspond to these *k* values are stable. The *k* values also have peaks on the left side and right side of the origin. When $k<0$, the PDO time sequence transits from the positive phase to the

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negative phase, when the threshold of the k peak is wide and the probability is small; when k>0, the PDO time sequence transits from the negative phase to the positive phase, when the threshold of the k value is narrow and the probability is large. This indicates that there are two kind of transitions, which one of them is that the system changes from the positive phase to the negative phase, and the other is that the system changes from the negative phase to the positive phase, are not symmetric, and the latter is more stable. Because there is a difference in parameter k when the selected sub-sequence length is different, the gray region in the upper right corner of Figure 7 also shows the statistical properties of parameter k when the sub-sequence length is 20, 30, 40, 50, or 60 years. When the length of the sub-sequence is 20 years and 30 years, there is only one peak in the distribution of k values, and the parameter k value of the peak is about 0, which means that the transition change is more stable than the other situations. When the length of the sub-sequence is 40, 50, or 60 years, the peak value on the side of k>0 is not considerably different, which indicates that the stability degree of the transition change from negative to positive is consistent; the location of the peak value on the side of k<0 moves to the left as the sub-sequence length increases, which means that the sub-sequence is longer, the amplitude of detected transition change is larger, and it is more unstable. From the perspective of the value, a k value in the range of (-10, 10) accounts for 80.2% of all k values, a k value in the range of (-5, 5) accounts for 74.2%, and a k value in the range of (-2, 2) accounts for 58.6%. In the following studies, the k value is mainly set in the range of (-2, 2).

3.2 Values of the initial state *v* and linear trend *h*

We use the method proposed in section 2.2 to analyze the transition changes of the PDO. With different lengths of sub-sequences, three climate changes are detected to start from 1976, 2007 and 2011 respectively. In figure 8, the transition changes starting from 2007 and 2011 are stated, while the transition change starting from 1976 has not been shown. In table 2, parameters v and h are obtained by regression method when the transition change starting from 2007 and 2011. When the length of

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In figure 8, it is noted that the PDO time sequence is leaving the stable state from the start moment. The transition change experiences a period, which is called as transition process. When the transition process has not finished, it looks like the increasing part. In order to detect whether there are other transition change, we change the length of sub-sequence one year by one year. That is, the sub-sequence length is set as 10, 11, 12, ⋯, and 60 years. Then, the initial state v and the linear trend h of these transition changes are obtained. In figure 9, the sub-sequence length is set less than about 40 years, the transition changes are detected only twice. One began in 2007, and the other began in 2011. The value of parameter h is unchangeable nearly for each transition change, while the value of parameter v is changing when the length of sub-sequence is different. In particular, the abrupt change starting from 2007 is detected for the sub-sequence of about 30-40 years, and the value of parameter v is in the range of (0.28, 0.45). The transition change starting from 2011 is detected for the sub-sequence of about 10-30 years, and the value of parameter v increases as the

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length of the sub-sequence increases, whereas the variation range of threshold is (-0.48, 0.12), which is significantly different from the situation of the transition change starting from 2007.

3.3 Prediction of the uncompleted transition change beginning in 2011

After the threshold ranges for parameters *k, v,* and *h* are determined, according to the quantitative relationship, we can calculate the end state and the end moment of the transition process. Using the transition change in 2011 as an example, we study the ending state and end moment for the PDO index transition change. According to the research results that are presented in Sections 3.1 and 3.2, the parameter is *h*=1.054/month in this transition change, and the threshold range of parameter *k* is determined to be (0, 2). The range of parameter *v* is determined to be (-0.48, 0.12), and the variation situation of parameter *u* and end moment with parameters *k* and *v* are shown in Figure 10. The results indicate that the threshold range of parameter *u* for the ending state is (1, 7), and the time range of the ending moment is (2013, 2017). According to the probability of parameter *k*, the end moment of this transition process is about 2015, and after that time, the sequence stops to increase, approaching to a stable state with value of 1.6.

In figure 11, a sketch map is displayed to explain how the prediction method works briefly. The PDO time sequence is displayed as black line. The period during 2006~2011 is detected as the initial state, and a transition process is increasing from this initial state. It is not able to be known whether the increasing process has been completed or not. Based on the linear regression method, the initial state and the linear trend are obtained and shown as purple dash lines. Then by the method proposed in section 2.2, all possible end states of this transition process are obtained with Eq. (8) as shown in figure 10, and the most likely end state is marked as green dash line. Unlike the uncompleted transition process of ideal experiment, the

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transition process has completed in about 2015 since we detected the PDO change in 2016. This transition process started from 2011, and end in 2015. The initial moment and the end moment are marked as black dash lines. However, we are still not sure whether the PDO finish this transition process completely or not for it it appears at the end of the sequence. As we all know, many statistical methods are not accurate for the detecting both ends of the sequence. Thus, the real PDO sequence during 2016~2017 is added to the end of the PDO time sequence. The PDO value from 2015 to 2017 is almost unchanged, which is consistent with the predicted result.

4. Conclusion and discussion

A novel method had been proposed to identify the transition process of climate change in our previous research. By defining initial state parameter y, linear trend parameter h, end state parameter u, and instability parameter k, a quantitative relationship among this parameters was revealed. Based on the relationship, we develop a method to study uncompleted transition processes. The method is applied to predict ideal time sequences and the PDO time sequence. In the ideal experiments, three different time sequences with different length are constructed. Based on the initial state and the linear trend which the system had experienced, and the given parameter, the end state and end moment of the transition process are predicted. The prediction result does match the ideal time sequence well. For the PDO time sequence, a transition change began in 2011 was taken to test the prediction method. The end moment of this transition process is predicted to be 2015, which is consistent with the real time sequence.

In this prediction method, the quantitative relationship among the parameters characterizing the transition process is vital. Accord to the segment of the transition process which has been happened, we determine the parameters. Then, we predict the end moment and the end state. In fact, this is also a extrapolation method. However, if the transition process has not begun, we can not predict this climate change. There is no other statistical method that can predict the climate change which has not occurred

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only by time sequence. It is noted that the uncompleted climate change we studied is closed to the end of the sequence. Due to the lake of enough data, it is difficult to study the end of time sequence by using other statistical methods.

Acknowledgements

We thank two anonymous reviewers for their valuable suggestions. This study was jointly sponsored by National Key Research and Development Program of China (Grant No. 2018YFE0109600), National Natural Science Foundation of China (Grant Nos. 41875096, 41775078, 41675092), Meteorological scientific research project of Gansu Meteorological Bureau (MS201914).

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Table1. The parameters of ideal models

Situations	α	χ	$h0$	h	$\lvert h0-h \rvert/h$
S1	0.20	21.64E-2	12.99E-4	12.69E-4	2.36%
S2	0.25	22.76E-2	9.10E-4	8.90E-4	2.25%
S3	0.15	20.18E-2	32.27E-4	32.72E-4	1.38%

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Table2. Parameters v and h obtained with different sub-sequences

Length of sub-sequence	Start moment (year.month)	v	h (month^{-1})
10a	2011.06	-0.45	1.054
20a	2011.06	-0.03	1.054
30a	2007.11	0.36	0.227
40a	2007.11	0.41	0.227

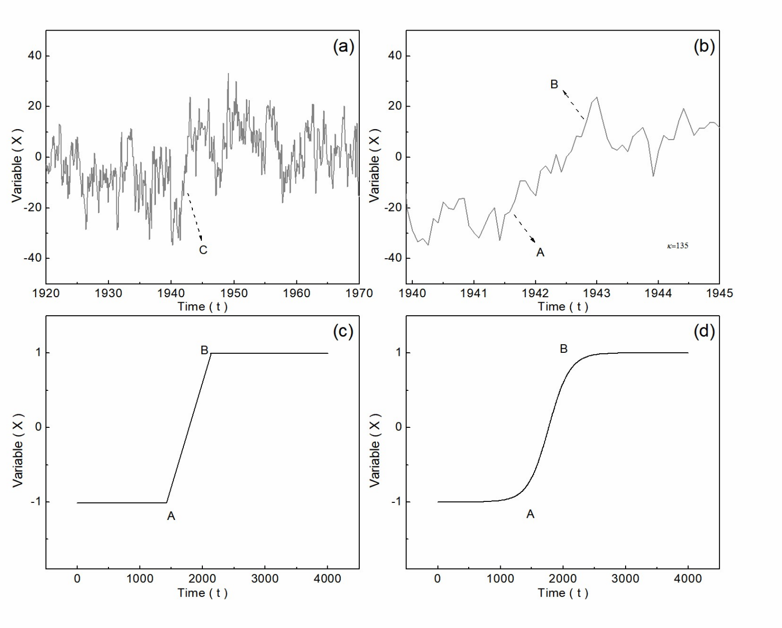

Figure 1. Transition process of abrupt change in real time sequence and ideal time sequence. (a) The PDO time sequence during 1920 to 1970; (b) The PDO time sequence during 1940 to 1945; (c) The transition process presented by piece-wise function; (d) The transition process presented by continuous function

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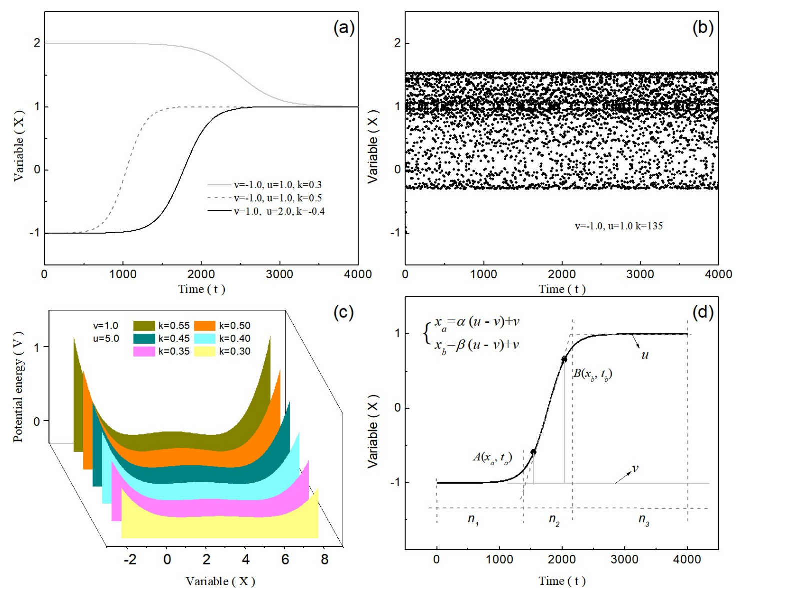

Figure 2. The system presented by Eq. (2). (a)The transition processes of system swinging between different stable states since the parameters are different; (b)The system stays in unstable states; (c)The generalized potential energy function of system performs differently since the parameters are different; (d)Different segments of the transition process in the ideal time sequence and the system states x expressed with location parameters.

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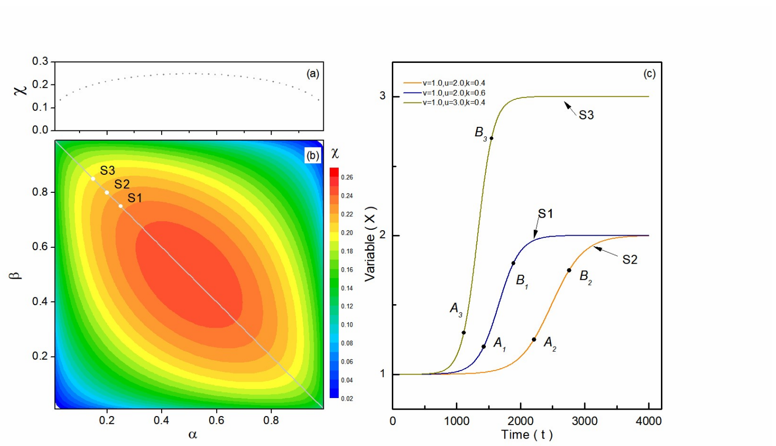

Figure 3. The influence of different value of parameters α and β on parameter χ and parameter h. (a) Diagonal section of parameter χ in figure b (gray line); (b) Parameter χ with location parameters α and β; (c)Points A and B stay in different positions in three situations marked as S1, S2, S3.

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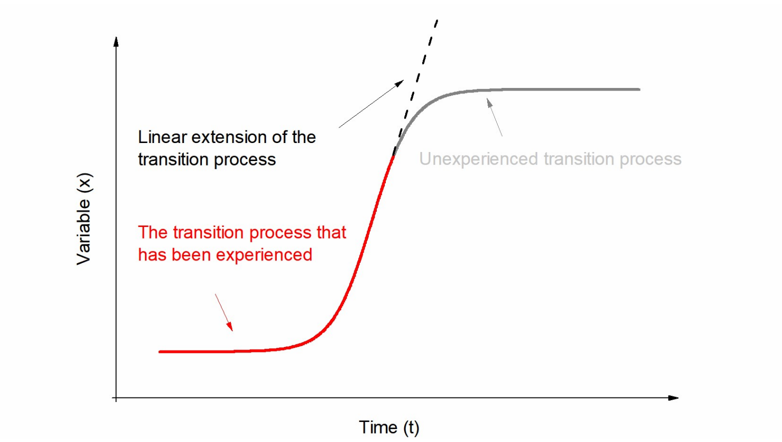

Figure 4. The schematic diagram of prediction method.

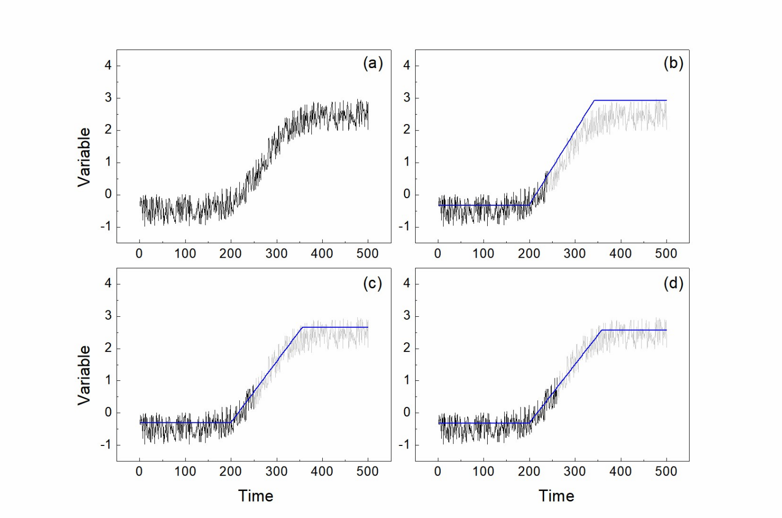

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Figure5. The ideal time sequence constructed by the logistic model and random numbers. (a) Completed transition process with 500 moments, Uncompleted transition

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processes (the gray lines) and their prediction result (the blue lines) with (b) 240 moments, (c) 250 moments, and (d) 260 moments, the light gray lines are the original entire ideal time sequences.

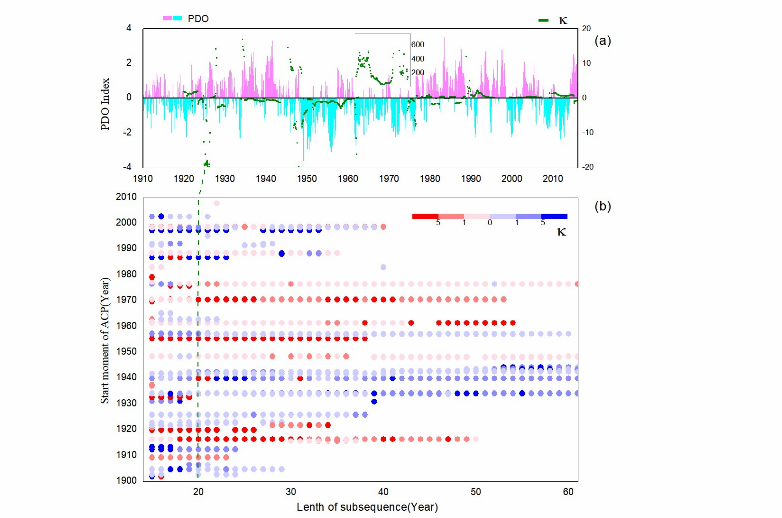

Figure 6. Identification of the PDO time sequence and instability parameter k with different sub-sequence lengths. (a) The X-axis is the year, the histogram in the figure shows the PDO time sequence (left panel), and the green dots indicate the value of parameter k when the sub-sequence is 20 years (right panel); (b) the start moments of transition changes with different sub-sequence lengths (the red color dots represent increasing changes, and blue color dots represent decreasing changes, with deeper colors representing higher values). The X-axis is the sub-sequence length (month), and the Y-axis is the start moment of abrupt change (year).

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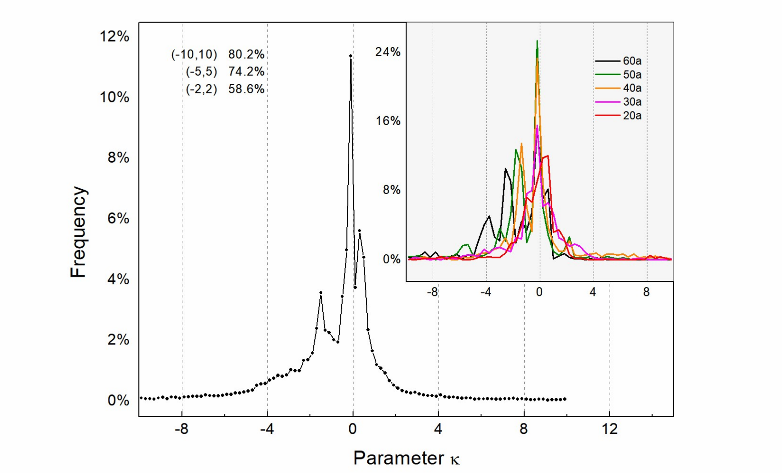

Figure 7. Statistical results of instability parameters for different sub-sequence lengths. The X-axis is the value of the parameter, and the Y-axis is the statistical frequency with a sub-sequence length of 10 years. The gray region in the upper-right corner is for the sub-sequence of 20-60 years.

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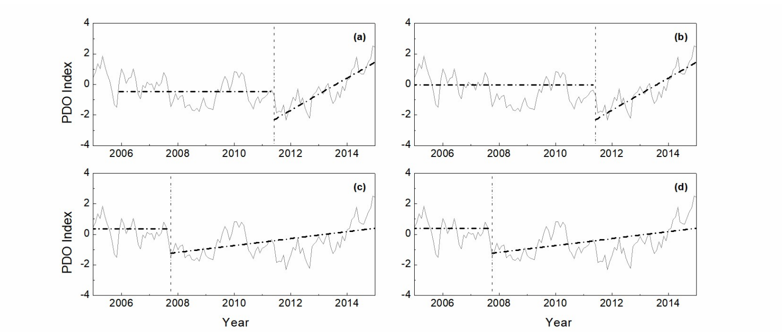

Figure 8. The PDO time sequences and the detection of parameters *v* and *h* when the sub-sequence was set as (a)10 years, (b)20 years, (c)30 years, (d)40 years. The gray lines is PDO time sequences. The horizontal dash lines represent initial states, the slope dash lines represent linear trend lines of transition change, and vertical dotted line represent the start moment.

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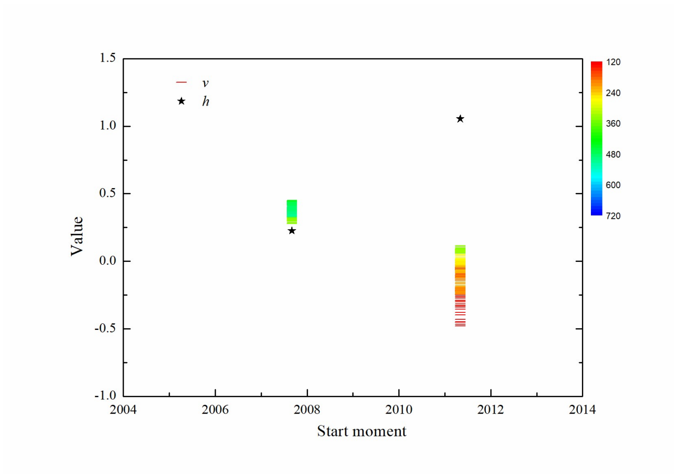

Figure 9. The values of the parameters v and k of two transition changes with different lengths of sub-sequence. The black stars represent the values of parameter h, and the colourful short bar represent the values of parameter v. The colour bar represents the sub-sequence length.

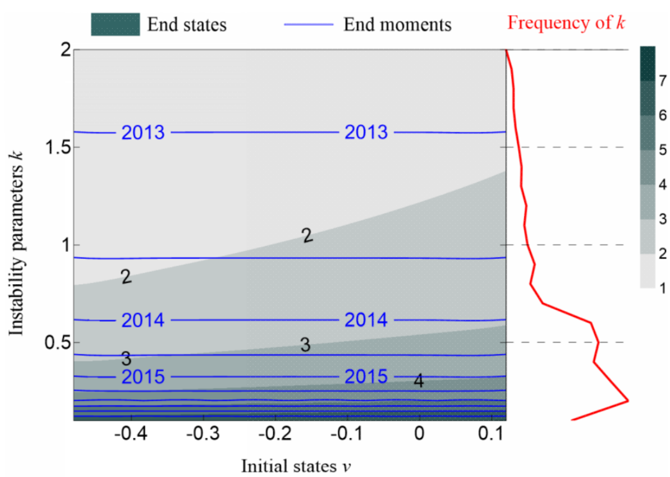

Figure 10. Variation end state and end moment with the initial state parameter v (horizontal ordinate) and instability parameter k (vertical coordinate). The red line on the right side shows the probability distribution of instability parameter k.

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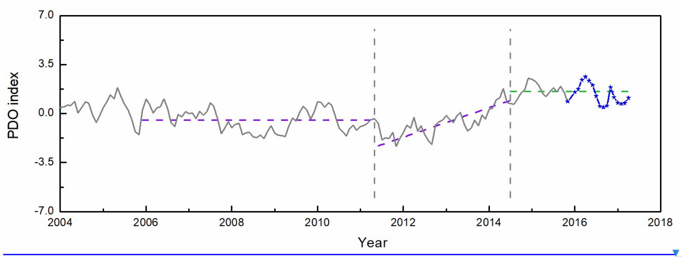
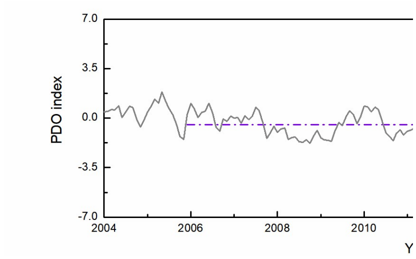


Figure 11. Prediction of the PDO index. The gray line is the PDO index before 2015; the line with starts is the PDO index after 2015; the gray dash line represent the start moment; the purple dash lines represent the initial state and the linear trend line, the green line represent the prediction end state.

删除: 

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