Answers to the comments of Referee 1

Note: Line numbers refer to the track-changed version

Comment 1
"The method shares many aspects with standard spectral clustering methods (e.g. Fiedler's). One of the known limitations of these methods is that they partition the network in 'balanced' (i.e. not too different sizes) parts. This is usually an advantage in image processing and in computer-load redistribution, but I find this an important limitation in the present application to fluid flows. I ask the authors to state if this is a limitation of the present method and its possible impact on applications."

Answer to comment 1
Thank you for this comment. Indeed, the NCut finds more balanced clusters in terms of their weighted size. This is also why this kind of clustering is expected to fail when looking for very small structures compared to the fluid domain, e.g. for ocean eddies. For our application however, this is not a limitation because we are looking for large scale structures in the North Atlantic Ocean. We will include this in the methods section.

Changes in text, line 164:
Minimizing the NCut leads to a clustering result that tries to balance the different terms in eq. (6) such that the resulting clusters are of approximately equal size in terms of their total relative weight (Shi & Malik 2000). While this poses no serious problem for detecting large scale flow features in the ocean, it is certainly a limitation for detecting even smaller structures such as eddies or jets in a large ocean domain, and we explicitly exclude such examples from the scope of our method.

Comment 2
"Is there any criterion to determine an 'optimum' number K of network parts, or when to stop the iterative hierarchical partitioning?"

Answer to comment 2
There is no general criterion how to truncate the spectral embedding. A heuristic was proposed in different contexts (also fluid dynamics) to look for the largest spectral gap. But this might not work for many applications. We will include a more detailed discussion about this issue in the methods section.

Changes in text, line 228:
Note that there is no general rule to determine the number of clusters K in algorithm 1, or where to stop the hierarchical clustering procedure in algorithm 2. A popular heuristic to determine K is to look for a prominent gap in a spectrum of L_s and choose K as the number of smallest eigenvalues before that gap (Hadjighasem et al. 2016). This is however problematic for systems with no prominent spectral gap, which is the case for the systems considered here (cf. section 4). For algorithm 1, we therefore compute the clustering results for different values of K to see how the results depend on this choice. For algorithm 2, one can set a maximum value on the cost function in

eq. (6) as suggested by Shi & Malik (2000). Yet, the cost function is rather abstract, and there is no general rule what this value should be. Here, we compute the clusters up to a certain (arbitrary) order and compare the results to known structures in oceanography.

Comment 3
In the application to the North-Atlantic drifter data set, the authors declare to look for rigid, stationary features. Nevertheless, the particle position at the beginning of the trajectories and at the end (Figs 6a and 6b) are different. Could you discuss the implications of this on the 'stationarity' of the structures and in relationship with trajectory duration?

Answer to comment 3
Thank you for this comment. Indeed we were not clear on the fact that the trajectories of the different particle groups do have non-zero overlap (no perfect clustering). As a result, the initial positions of particles of one cluster can overlap with the final positions of another cluster, i.e. the spatial extent of the cluster is only approximately stationary. We will include some more clarification on this issue in the results section.

Changes in text, line 362:
Note here that the trajectories in the different clusters do have small but non-zero overlap, such that the spatial extent of the clusters can be different at initial and final time.

Comment 4
Could you comment on the reasons for the change in size of the ellipsoidal structures identified in Fig. 4 with respect to the ones in Fig. 3?

Answer to comment 4
Thanks for this comment. This question is quite difficult to answer, especially as there are many steps involved in the network construction and the clustering that are not obvious or where it is difficult to get an intuition for. Increasing the bin size will lead to more trajectory overlaps, but what that means directly for the clusters resulting from k-Means on the eigenvectors is difficult to understand. For us, having a decrease in the size of these structures indicates that increasing the bin size does not preserve all structures, but only the most dominant ones (i.e. the separation between left and right of the fluid domain). But we were not specific enough about this in the previous version. We will add more explanation about this point in the new version.

Changes in text, line 305:
The corresponding clustering result still resolves the most dominant structure up to very high resolution: the split between the left and right side is preserved, and even the structure of the transport barrier is very similar to the one in fig. 3. The results for K=3 and K=4 are however different from fig. 3, as the gyre centres appear smaller. This indicates that only the most prominent structures, here the separation between the left and right sides, are preserved under coarsening the partition. Nevertheless, figs. 4c-d do still give an impression about the flow structures at higher orders,

though not completely equal to the high resolution case. Note that the singular values (fig. 4a) are strongly suppressed compared to the M=1,250 case in fig. 3.

Comment 5
The set S in the denominator of Eq. (6) is not defined.

Answer to comment 5
Thank you for noting. We will do so.

Changes in text, line 155:
Assume we are given an undirected network defined on a discrete set S containing N vertices, with edges given by the symmetric adjacency matrix $Q \in \mathbb{R}^{N \times N}$.

Comment 6
"The authors use the word 'similar' in lines 172, 341 and 350 in an unclear meaning, especially because in other parts of the manuscript some 'similarity' measures are de-fined and used. Please use 'equivalent' or 'equal' if this is the intended meaning of 'similar' there, of choose a more precise word if it is not."

Answer to comment 6
Thank you for noting. Indeed we were inconsistent in the usage of 'similar' and imprecise. We will change this in the revised version.

Changes in text, line 185:
... it follows that these eigenvectors are equal to the left singular vectors corresponding to...

Changes in text, line 438:
Minimizing the generalized normalized cut of A_P *defined in eq. (6) with spectral relaxation is equal to maximizing the generalized coherence ratio*

Changes in text, line 447:
We now show that the right singular vectors of R defined in eq. (10) are equal to the right eigenvectors of $\hat{P}$ *if the particle measure is invariant and uniform.*