

1 General response

I want to thank the authors for the work that they put into the revision, refining various aspects of the derivation of the forecast error covariance, more clearly distinguishing the predictability error and its connection to the state-dependent model error source. I think these additional derivations have addressed the earlier questions and have added strength to the overall analysis in the work. Likewise, I thank the authors for addressing the typos and minor errors found in the last version. Generally I am quite positive about the work and I would only like to make the following minor and technical suggestions. Following these minor points I recommend publication.

2 Minor Points

1. Page 1, lines 39 - 41: I think there are a few other studies worth mentioning that explicitly link numerical discretization error (or lack of numerical precision in a general sense) to the data assimilation cycle and model bias. Though these do not explicitly discuss estimation the model error covariance as in the present study, these works are worth mentioning in this general context: Dubinkina [2018], Hatfield et al. [2018], Grudzien et al. [2020].
2. Page 3, lines 40 - 41: “For instance, assuming that the model error is unbiased, leads to model the bias”. I believe this should be rephrased as “leads to modeling the bias”.
3. Page 3, lines 47 - 50: “Again, assuming the decorrelation between the analysis and the model errors leads to over-estimate the true effect of the model-error with an over-estimation of the true forecast-error uncertainty.” I believe this should be re-phrased as “leads to over-estimating the true effect”.
4. Page 3, lines 91 - 92 : “in that case, a model-error is needed”. I believe this should should read “in that case, a model-error estimate is needed”.
5. Page 3, line 95: “larger enough” should read “large enough”.
6. Page 4, line 41: “not necessary” should read “not necessarily”.
7. Page 5, lines 16 - 18: grammar should be checked on this sentence and clarified.
8. Page 5, line 77: “(assume)” should read “(assumed)”.
9. Page 6, Eq. (23): bracket is misaligned in the \mathbf{g} norm.
10. Page 12, second column, first line: see typo “ad”.
11. Page 18, appendix A2: The equation between (A4) and (A5) has a misplaced transpose in the equation,

$$(\mathbf{NP}^a\mathbf{M}^T)^T \tag{1}$$

References

- S. Dubinkina. Relevance of conservative numerical schemes for an ensemble kalman filter. *Quarterly Journal of the Royal Meteorological Society*, 144(711):468–477, 2018.
- C. Grudzien, M. Bocquet, and A. Carrassi. On the numerical integration of the lorenz-96 model, with scalar additive noise, for benchmark twin experiments. *Geoscientific Model Development*, 13(4):1903–1924, 2020.
- S. Hatfield, P. Düben, M. Chantry, K. Kondo, T. Miyoshi, and T. N. Palmer. Choosing the optimal numerical precision for data assimilation in the presence of model error. *Journal of Advances in Modeling Earth Systems*, 10(9):2177–2191, 2018.