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Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "A methodology to obtain model-error covariances due to the discretization scheme from the parametric Kalman filter perspective" *by* Olivier Pannekoucke et al.

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1 General comments

This paper is a new and very interesting piece of work for the geophysical data assimilation community.

The authors propose a way to include spatial-discretization (model) error in the parametric Kalman filter (PKF) framework. In this aim, the model error covariance is approximated by the difference between the predictability error covariance and the forecast error covariance. Predictability and forecast errors are associated to the erroneous and perfect models respectively, the erroneous model being a spatial discretization of the

perfect model. Using an almost equivalent partial derivative equation for the erroneous model and the Gaussian closure of Pannekoucke et al. (2018), the authors derive evolution laws for predictability and forecast errors, their PKF representations (variance and diffusion tensor) and finally the model error PKF representation.

Representing spatial-discretization error, the use of modified equations (32) for this purpose and the combination with PKF are very good ideas and seems to be promising paths. Nevertheless, I was confused by the predictability and forecast errors definitions and by the ensuing model error proxy.

2 Specific comments

- Some parts of section 2.1 are unclear to me, especially between lines 85 and 90. The forecast χ^f (and thus the forecast error ϵ^f) is first defined by the perfect model $\mathcal{N}$ (eq. (4)). Then, it is re-defined by the erroneous model $\mathcal{M}$ (eq. (11)). I understand that you wanted to introduce the error complexity step by step. But, I think that lines 85 to 90 confuse the reader since in the following of the paper we are not sure of what is your definitions of the forecast χ^f and forecast error ϵ^f.
- I am particularly confused by the equation of the line 88 :

$$\epsilon^f = M\epsilon^a - \epsilon^m, \tag{1}$$

which leads to the central proxy equation (13) (and then leading to (15), (16), (17), ...). If the analysis error is zero, then the forecast error ϵ^f should be equal to 0 according to the definition (2)-(5) and this would imply zero model error ϵ^m. However, model error can exist even with perfectly known initial condition. In my understanding,

$$\epsilon^f = N\epsilon^a, \tag{2}$$

$$
\begin{aligned}
&= M\epsilon^a + (N - M)\epsilon^a, && (3)\\
&= M\epsilon^a + \int_0^t (\mathcal{N} - \mathcal{M})(\chi^t + \epsilon^a) - \int_0^t (\mathcal{N} - \mathcal{M})(\chi^t), && (4)\\
&\neq M\epsilon^a - \int_0^t (\mathcal{N} - \mathcal{M})(\chi^t), && (5)\\
&= M\epsilon^a - \epsilon^m. && (6)
\end{aligned}
$$

The model error source term, $\int_0^t (\mathcal{N} - \mathcal{M})(\chi^t + \epsilon^a) = \int_0^t (\mathcal{N} - \mathcal{M})(\chi^a)$, is missing. I would expect to see this source term, at least later, in the draft, like in the equation (33). For the example 3.2, this source term corresponds to the residual of the spatial discretization:

$$
(\mathcal{N} - \mathcal{M})(\chi^a) = [(v - U) \cdot \nabla - \kappa\Delta]\,\chi^a \qquad (7)
$$

Intuitively, I thought that this term should appear somewhere to increase the model error along time (especially if ones neglects the correlation between analysis error and model error). But, I may miss something. Perhaps, the authors do not consider it because they consider it as a bias and not as a centred random error? or because this source term is negligible? Is it possible to quantify its relative order of magnitude? If the source term is negligible, how can the independent-from-analysis-error model error component can growth? If the source term is not negligible but not considered by the authors because it is a bias, perhaps the source term should be replaced by a centered noise having the same effect (eg the same norm)? The authors should discuss these points.

- From a more general perspective, if ones focuses on the part of the model error which is independent of the analysis error, I guess that the methodology should work with zero analysis error. Am I right? Can your method deal with zero analysis error?

- I am not sure to understand what is the predictability error ϵ^p. I guess it is just defined equation (14). So, in the linear approximation, ϵ^p is the propagation in time of the analysis error ϵ^a with the erroneous model $\mathcal{M}$. Am I right? If not, it may alter my previous comments but then I would not understand the equation (33).

3 Technical corrections and typos

- Several equations appeared separated in two, like in lines 110 and 264. Perhaps, this is just due to the draft template.
- line 227 : trom $\mapsto$ from
- line 255 : a space is missing at the end of the sentence

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