We would like to thank the editor and reviewers for giving us insightful suggestions which would help us in depth to improve the quality of the paper. We made a significant revision and the detailed responses are as follows.

Response to Referee comment 1:

Specific comments:

The language is quite poor as it presents some traduction and grammar errors and it is sometimes difficult to follow the logic of the text. Some parts are rather obscure (e.g. lines 123-124 or 249-253).

Response: We have tried our best to improve the grammatical errors and also consulted an English speaker. We hope it will meet with approval. A revision by a mother-tongue has been uploaded. The lines 123-124 have been revised as the lines 149-157 in a new revision of this manuscript. The lines 249-253 have been revised as the lines 286-294 in a new revision of this manuscript.

1. The histogram of Cu % (Fig. 5) seems to be log-normal. If this is the case, the statistical results (mean value and semivariogram parameters) can be biased. The authors are invited to check data distribution and, in case, to make a logarithmic transformation.

Response 1: Thank you. Accept this point. We have checked the Cu data distribution of Pulang deposit. We made a logarithmic transformation for the original data. The histogram and Q-Q plot of the log-transformed Cu data indicate that the distribution of Cu data is log-normal (Fig. 5). We revised the statistical results. The experimental semivariogram of Cu data of Pulang deposit indicates a range and nugget effect of 320.0 m and 0.25, respectively (Fig. 6).

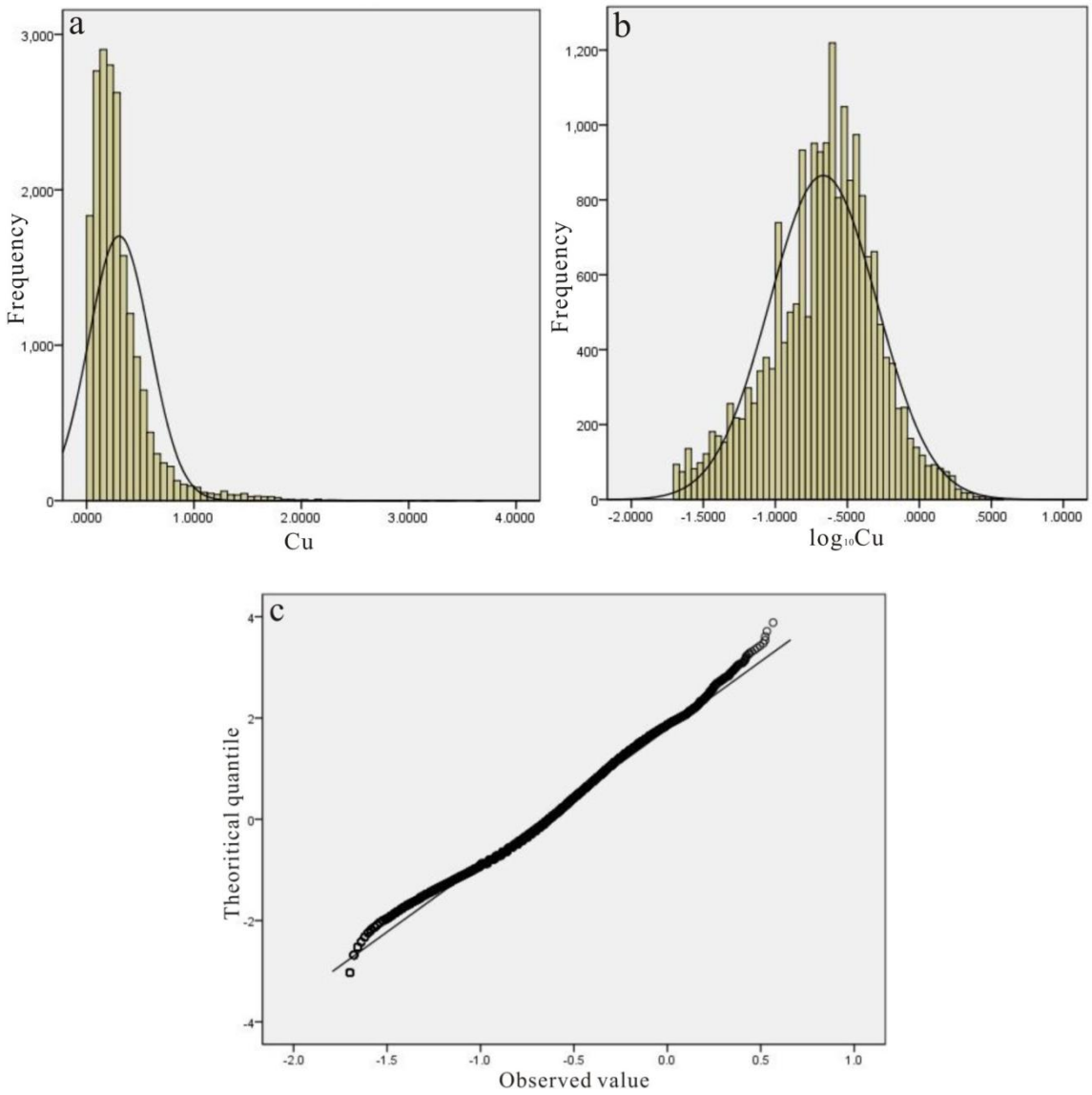


Fig. 5. Histograms of (a) the Cu raw and (b) logarithmic transformation data and (c) Q-Q plot of the log-transformed Cu data in the Pulang deposit.

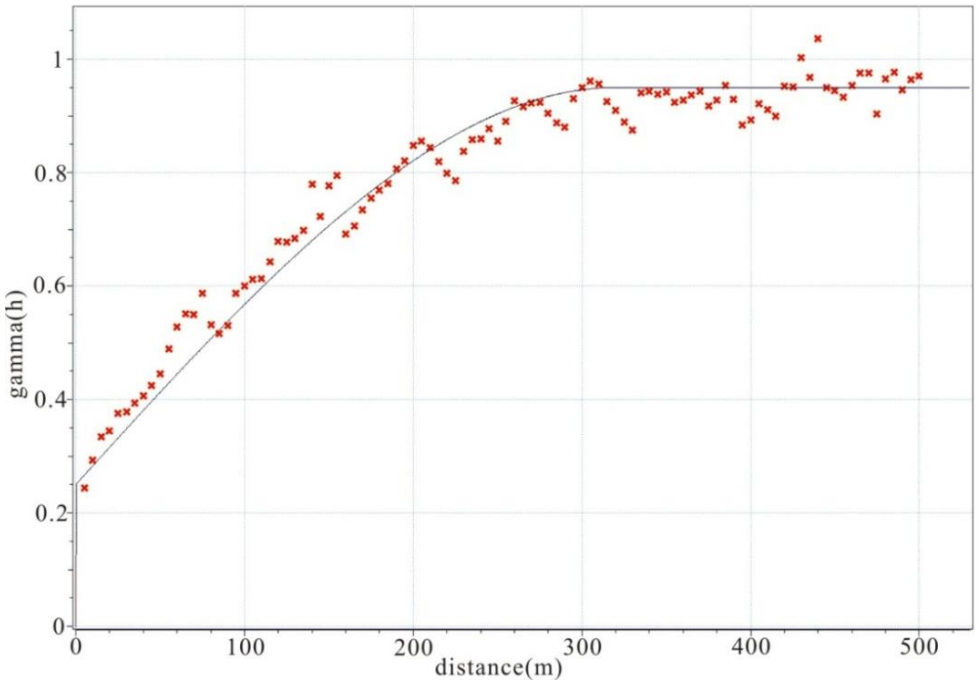


Fig. 6. The experimental semivariogram of Cu data in Pulang deposit.

2. The authors, following Afzal et al. (2011), apply kriging in order to make a 3D interpolation of Cu content. It is not clear if authors use kriging or block kriging. The last procedure in particular (but even the first one) introduces a bias because the fractal behaviour refers to interpolated concentration and not to original data and this aspect may influence fractal analysis. I suggest adding comments on the consequences of the application of an interpolation method on the found fractal ranges.

Response 2: Thank you. Accept this point. The 3D model of the Cu concentration distribution of the Pulang deposit was produced with the ordinary kriging method using Geovia Surpac software on the basis of the semivariogram and anisotropic ellipsoid. Fundamentally, the accuracy of the interpolation results mainly depends on whether the interpolation model accurately fits the spatial distribution characteristics of the deposit. The original drillhole data of ore element concentrations were interpolated by using the ordinary kriging method to calculate the $V(\leq \upsilon)$ and $V(\geq \upsilon)$ enclosed by a concentration contour in a 3D model in this study. The method estimates values in unsampled locations based on the moving average of the interest variables, satisfying various distribution forms of data. Ordinary kriging is a spatial estimation method that provides a minimum error-variance estimate of any unsampled value. The

correct variogram in kriging interpolation can guarantee the accuracy of the interpolation results. The accuracy of the spatial interpolation analysis is verified by comparing the difference between the measured values and the predicted values to select the best variogram model. In order to test the variogram model, the cross-validation method is used to determine whether the parameters of the variogram model are correct (Fig. 7). The distribution of the residual is normal and the mean of error between the actual and estimated Cu grade values is equal to 0 (Table 1). This result indicates that this model is reasonable and that the variogram parameters used for estimating the Cu grade are unbiased.

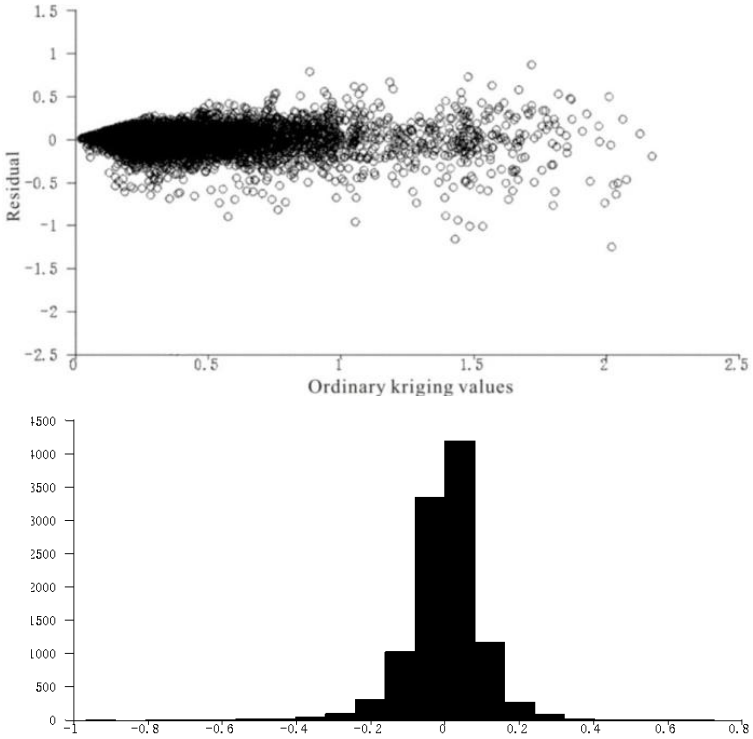


Fig. 7. The cross-validation results: (a) residual VS Cu grade; (b) the residual distribution histogram.

Table 1 The results of statistical characteristics of the residual.

Variables	Residual
Mean	0.000
Variance	0.016
Standard Deviation	0.127

3. The paper basically presents a comparison between two methods of analysis, for this reason, more comments should be added in the conclusions instead of simply describing the results.

Response 3: Thank you. Accept this point. We have revised the conclusions. Given the referee comment 2, we have tried to add the N-S fractal model to improve the structure of the paper and revolutionize the style of the paper. We have added more comments and rewritten the conclusion as follows.

In many cases, drillhole logging is dealing with the lack of proper diagnosis of geological phenomena, which can undermine the delineation of mineralized zones because it depends on the subjective interpretation of individual loggers, and no two loggers provide the same interpretations. However, conventional geological modeling based on drillhole data is fundamentally important for understanding the orebody spatial structure. Grades of ore elements are not determined by conventional methods of geological ore modeling, while the variation in ore grades in a mineral deposit is an obvious and salient feature. Given the problems mentioned above, using a series of newly established methods based on mathematical analyses such as fractal modeling seems to be inevitable.

In this paper, the number-size (N-S), concentration-volume (C-V) and power spectrum-volume (S-V) fractal models were used to delineate and recognize various Cu mineralized zones of the Pulang porphyry copper deposit in the southern end of the Yidun continental arc, Southwest China. All these fractal models reveal that high-grade Cu mineralized zones are situated in the central and southern parts of the deposit. The Cu threshold values of highly mineralized zones are 1.45% and 1.88% based on the N-S and C-V fractal models. The Cu threshold of supergene enrichment zones is 1.33% based on the S-V fractal model. The models of moderately mineralized zones contain 0.28-1.45% Cu according to the N-S model and 1.48-1.88% Cu according to the C-V model. The hypogene zones contain 0.23-1.33% Cu according to the S-V model. The N-S model reveals weakly mineralized zones and barren host rocks containing <0.28% Cu. In contrast, the C-V model reveals that the barren host rocks contain <0.25% and that the weakly mineralized zones contain

0.25-1.48% Cu. The S-V model reveals that the barren host rock and leached zone contain <0.23% Cu.

The comparison between highly mineralized zones based on the fractal models and potassic zones resulting from the 3D geological model illustrates that the S-V fractal model is better than the N-S and C-V model because the number of overlapped voxels (A) in the S-V model is higher than those in the N-S and C-V model. The overall accuracies for the C-V, N-S and S-V models are 0.50, 0.51 and 0.52, respectively (Table 6), which indicates that the S-V model gives the best results for identifying highly mineralized zones in the deposit. On the other hand, the correlation (from OA results) between the highly mineralized zones obtained from S-V modeling and the potassic alteration zones is better than those of the N-S and C-V models because of a strong proportional relationship between the extension and positions of the voxels in the S-V model and potassic alteration zones in the 3D geological model.

A comparison between phyllic alteration zones obtained from the 3D geological model and moderate grade mineralization zones obtained from the fractal models indicates that the OA values of the C-V, N-S and S-V fractal methods in reference to the phyllic alteration zones of the geological model are 0.59, 0.54 and 0.56, respectively. The overall accuracy of the moderately and weakly mineralized zones obtained from C-V modeling is higher than the mineralized zones obtained from N-S and S-V modeling (Table 7).

According to the correlation between the results driven by fractal modeling and geological logging from drillholes in the Pulang porphyry copper deposit, high-grade mineralization zones generated by fractal models, especially the S-V model, have a better correlation with potassic alteration zones resulting from the 3D geological model than from the N-S and C-V models. The highly and moderately mineralized zones obtained from the fractal models are both situated in the southern and central parts of the Pulang deposit and coincide with potassic and phyllic alteration zones. There is a better relationship between the moderately and weakly mineralized zones derived by the C-V model and the phyllic alteration zones from the 3D geological model than those derived by the N-S and S-V models.

4. The lines 268-274 refer to particular samples that could validate results, but the outcome is not clear.

Response 4: Thank you. Accept this point. We have revised this part.

It could be considered that there are spatial correlations between different modeled Cu zones and geological features such as alterations and mineralogy. Several samples were collected from different drillholes in different grade mineralization zones of the Pulang deposit to validate the results of the fractal models. These samples were analyzed by microscopic identification and XRF (X-ray fluorescence spectrometry). The PL-B82 sample was collected from the drillhole situated in a high-grade mineralization zone and includes a high chalcopyrite content and some molybdenite (Fig. 16a). The PL-B62 sample was collected from the drillhole situated in a moderate-grade mineralization zone and includes a low chalcopyrite content and some pyrrhotite in the polished section (Fig. 16b). The PL-B74 sample was collected from the drillhole located in a weakly mineralized zone with lower chalcopyrite content and some pyrrhotite (Fig. 16c and Fig. 16d). The results obtained from the mineralogy, microscopic identification and drillhole scanning by XRF of these samples indicate that the Cu concentrations are 1.80%, 1.32% and 0.41% in the PL-B82, PL-B62 and PL-B74 samples, respectively (Table 8).

Table 8

Sample no.	Mineralized zones obtained by fractal models	Cu(%)
PL-B74	Weakly mineralized zones	0.41
PL-B62	Moderately mineralized zones	1.32
PL-B82	Highly mineralized zones	1.80

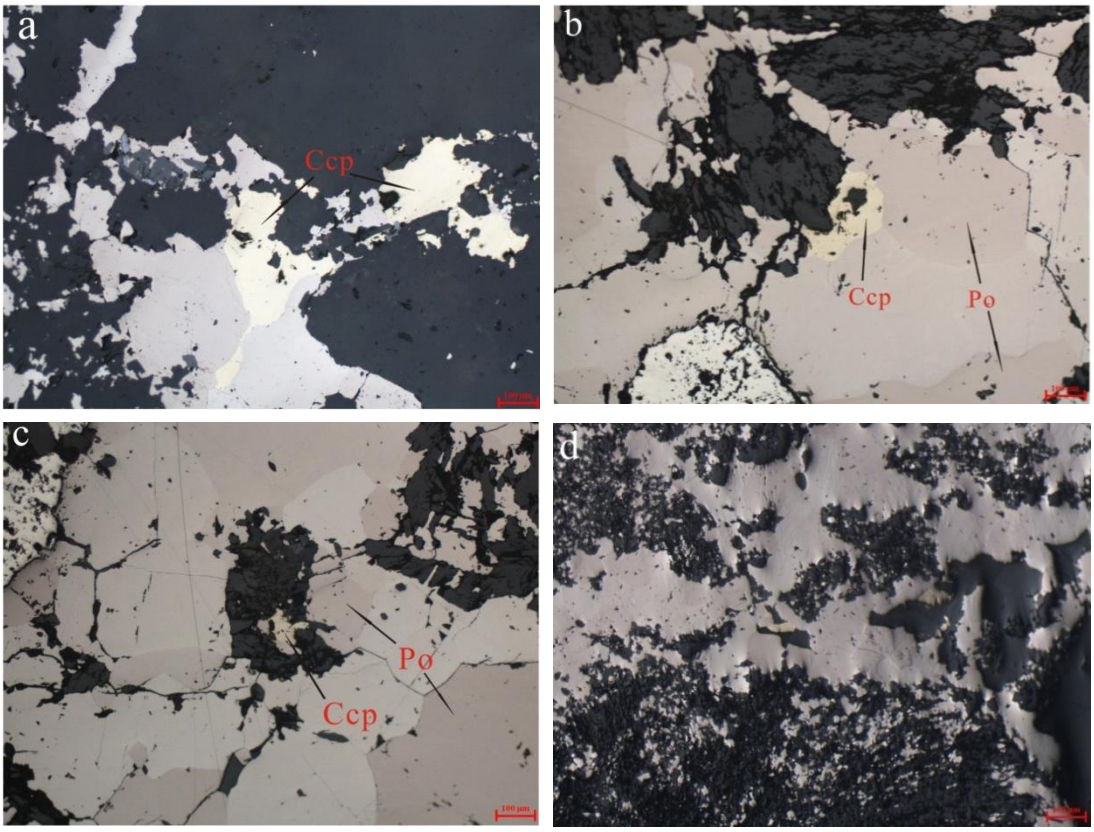


Fig.16. Chalcopyrite content in several samples based on mineralographical study: (a) PL-B82 sample was collected from the drillhole situated in the high grade mineralization zones.; (b) PL-B62 sample was collected from the drillhole situated in the moderately grade mineralization zones.; (c) and (d) PL-B74 sample was collected from the drillhole located at the weakly mineralized zones.

5. Many of the articles listed in References are not cited in the text.

Response 5: Thank you. Accept this point. We have revised the manuscript. Many of the articles listed in references have been added and cited in the text. A new revision by a mother-tongue has been uploaded.

Response to Referee comment 2:

1. Typing/spacing issues

Response 1: Thank you. Accept this point. We have revised it as suggested. A revision by a mother-tongue has been uploaded.

2. Grammatical issues

Response 2: Accept this point. We have tried our best to improve the grammatical issues and also consulted an English speaker, hope it will meet with approval. A revision by a mother-tongue has been uploaded.

3. You can improve the paper with many other, even newer references.

Response 3: Accept this point. We have added many newer references to improve the paper. And a new revision of this manuscript has been uploaded.

4. In some cases, the paper is prolonged by repeating obvious things. For example, about the amounts of the thresholds, some tables could be informative enough and no need to mention them.

Response 4: Accept this point. We have checked this paper and found that the paper is prolonged by repeating obvious things for example the amounts of the thresholds. We have deleted these obvious things. And a new revision of this manuscript has been uploaded.

5. Honestly to me, there was nothing new in this paper and the paper was totally like what Afzal et al have done but on a different case study. This is acceptable, but the readers may need at least a very tiny interesting, innovative or new thing in it. If you make something different, for example from a different point of view, it would make a big bonus for your paper. So, I believe if you revolutionize the style of the paper or even add one more fractal model, like N-S fractal model, to improve and change the structure of the paper, it would be great.

Response 5: Accept this point. Given the referee comments, we have tried to add the N-S fractal model to improve the structure of the paper and revolutionize the style of

the paper. Furthermore, the results of N-S fractal model were compared with the C-V and S-V fractal models.

Number-size (N-S) fractal model

The number-size (N-S) method proposed by Mandelbrot (1983) can be utilized to describe the distribution of geochemical populations (Sadeghi et al., 2012). In this method, geochemical data does not undergo any preprocessing (Mao et al., 2004). This model shows a relationship between desirable attributes (e.g., Cu concentration in this study) and their cumulative number of samples (Sadeghi et al.,2012). A power-law frequency model has been proposed to explain the N-S relationship according to the frequency distribution of elemental concentrations and cumulative number of samples with those attributes (e.g., Li et al., 1994; Sadeghi et al., 2012; Sanderson et al.,1994; Shi and Wang, 1998; Turcotte, 1996; Zuo et al., 2009a).

The N-S model proposed by Mandelbrot (1983) can be expressed as follows:

$$N(\geq\rho)=F\rho^{-D} \quad (1)$$

where ρ denotes the element concentration, $N(\geq\rho)$ denotes the cumulative number of samples with concentrations greater than or equal to ρ, F is a constant and D is the scaling exponent or fractal dimension of the distribution of element concentrations. According to Mandelbrot (1983), log-log plots of $N(\geq\rho)$ versus ρ show linear segments with different slopes -D corresponding to different concentration intervals.

Number-size (N-S) fractal modeling

The N-S model was applied to the Cu data (Fig. 8). The selection of breakpoints as threshold values is an objective decision because geochemical populations are defined by different line segments in the N-S log-log plot. The straight fitted lines were obtained based on least-square regression (Agterberg et al., 1996; Spalla et al., 2010). In other words, the intensity of element enrichment is depicted by each slope of the line segments in the N-S log-log plots (Afzal et al., 2010; Bai et al., 2010).

Based on the classification of the 3D model of Cu data and the thresholds obtained from the N-S fractal model (Table 2), highly mineralized zones are situated in the southern and central parts of the Pulang deposit and coincide with the

potassium-silicate alterations. However, small and highly mineralized zones are located in the central parts of the Pulang deposit (Fig. 9). Moderately mineralized zones occur along a northwest-southeast trend and correlate with the phyllic zones. Weakly mineralized zones and barren host rocks are situated in the marginal parts of the area.

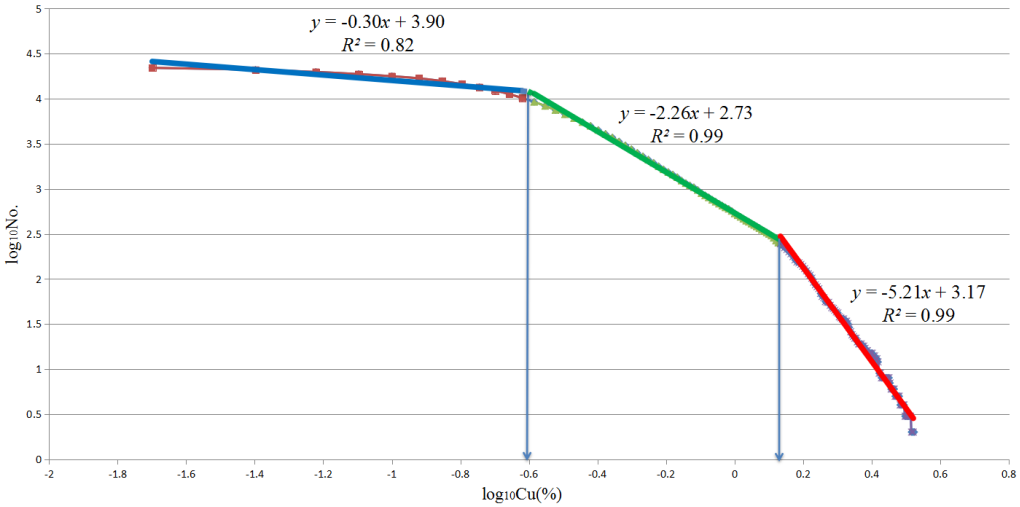


Fig.8. N–S log–log plot for Cu concentrations in the Pulang deposit.

Table 2 Thresholds concentrations obtained by using N-S model based on Cu% in Pulang deposit.

Mineralized zones	Thresholds(Cu%)	Range(Cu%)
Barren host rock and weakly mineralized		<0.28
Moderatelymineralized	0.28	0.28-1.45
Highly mineralized	1.45	>1.45

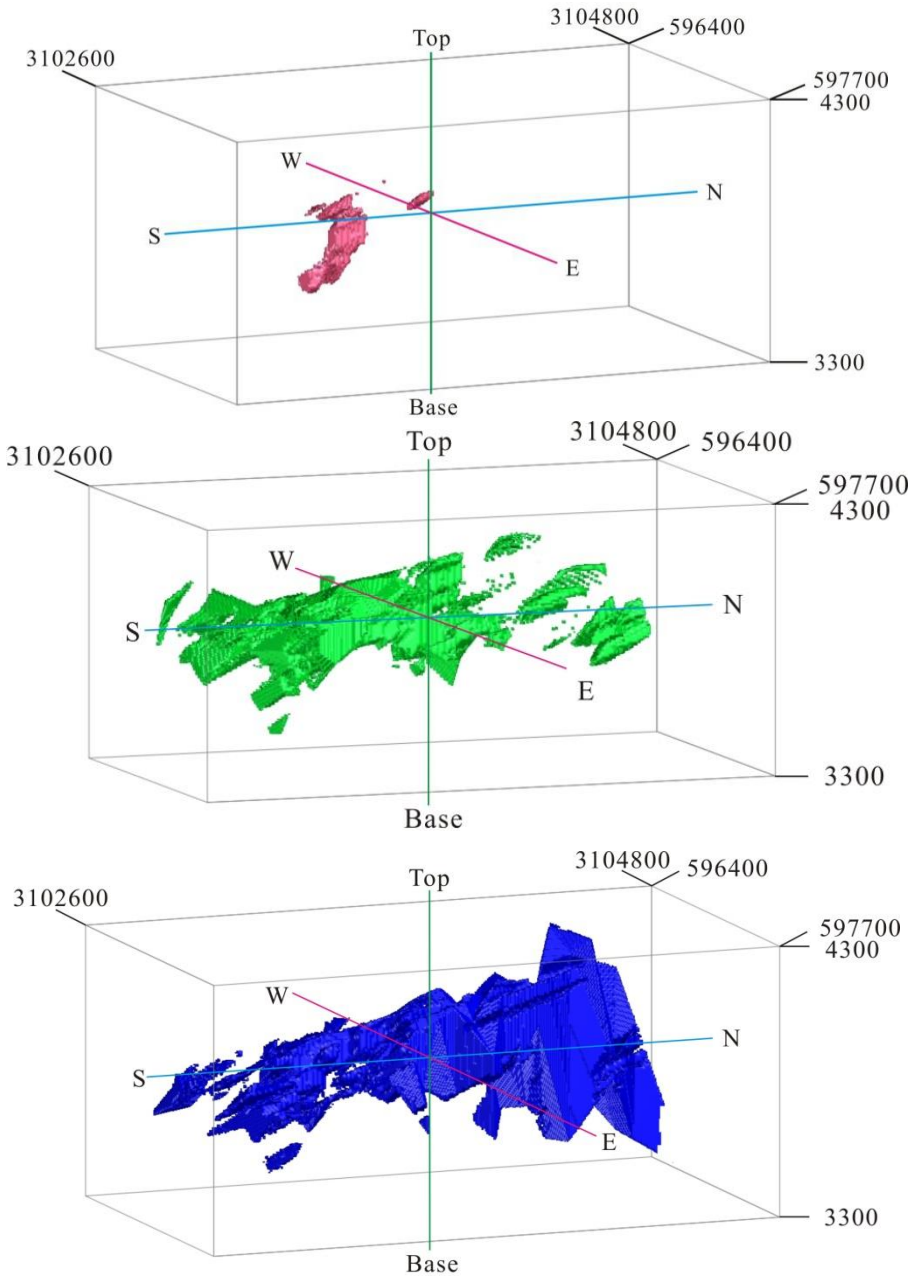
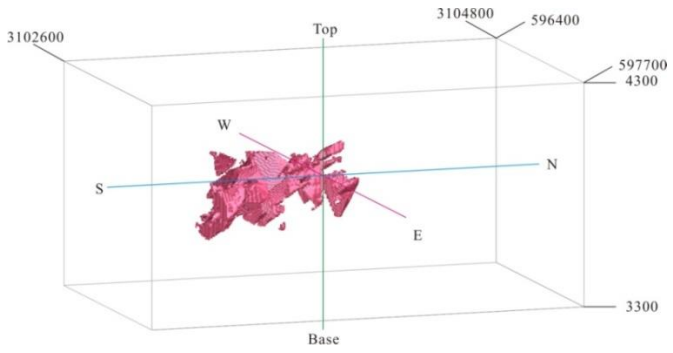


Fig.9. Zones in the Pulang deposit based on thresholds defined from the N–S fractal model of Cu data: (a) highly mineralized zones; (b) moderately mineralized zones; (c) weakly mineralized zones and barren host rocks. (Scale is in m^3.)

A comparison between highly mineralized zones based on the fractal models and potassic alteration zones resulting from the 3D geological model shows that there is a similarity among these fractal models. The overall accuracies for the C-V, N-S and S-V models are 0.50, 0.51 and 0.52, respectively (Table 6), which indicate that the S-V model gives better results for identifying highly mineralized zones in the deposit. The number of overlapped voxels (A) in the S-V model is higher than those in the N-S and C-V models. The correlation (from OA results) between highly mineralized zones obtained from S-V modeling and potassic alteration zones is better than that of the N-S and C-V model because of a strong proportional relationship between the extension and positions of voxels in the S-V model and the potassic alteration zones in the 3D geological model.

A comparison between phyllic alteration zones resulting from the 3D geological model and moderately and weakly mineralized zones from the fractal modeling shows that the overall accuracies of the C-V, N-S and S-V fractal models with respect to phyllic alteration zones of the geological model are 0.59, 0.54 and 0.56, respectively. The overall accuracy of moderately and weakly mineralized zones obtained from C-V modeling is higher than that of mineralized zones obtained from N-S and S-V modeling (Table 7). On the other hand, moderately mineralized zones defined by C-V modeling overlap with phyllic zones defined by the 3D geological model. However, the results of the C-V model are more accurate than those of the N-S and S-V models with respect to the phyllic zones defined by the 3D geological model.

(a)



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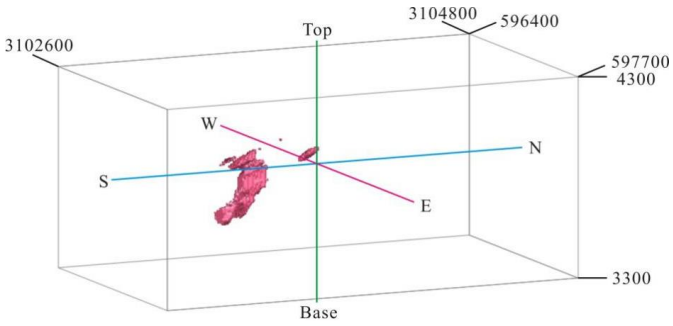
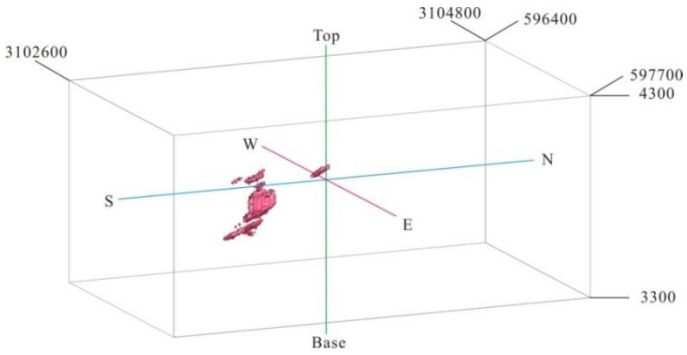


(c)



(d)

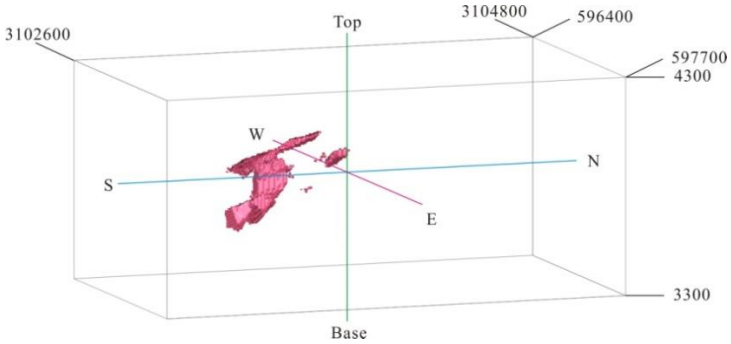
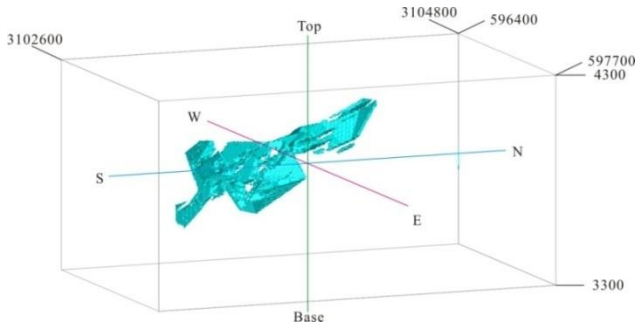
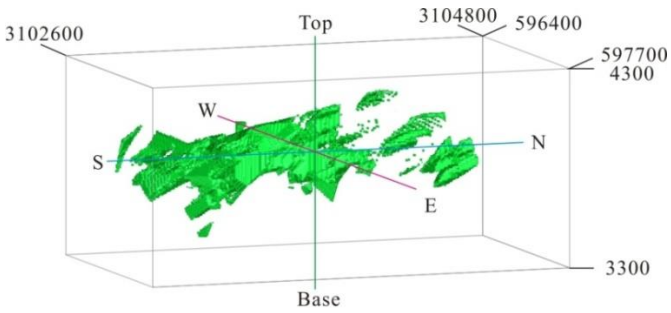


Fig.14. Highly mineralized zones in the Pulang deposit: (a) potassium-silicate zone resulted from the 3D geological model from drillhole geological data; (b) N–S modeling of Cu data; and (c) C–V modeling of Cu data; (d) S–V modeling of Cu data (Scale is in m^3.)

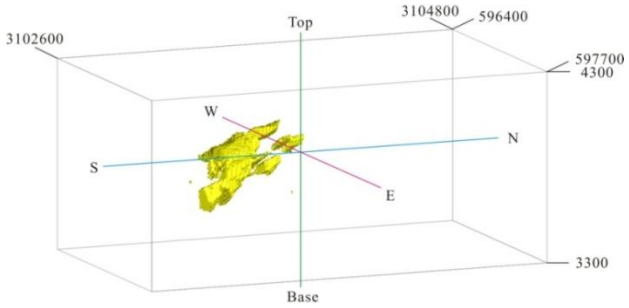
(a)



(b)



(c)



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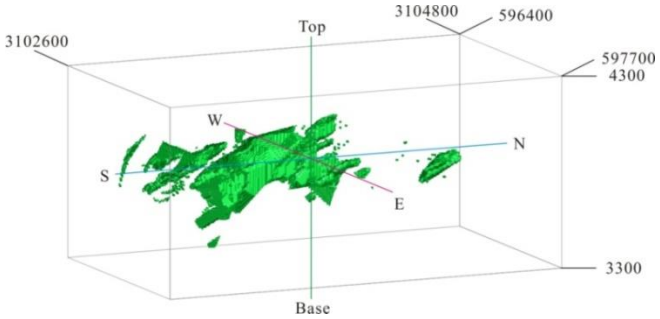

Fig.15. Moderately mineralized zones in the Pulang deposit:(a) quartz–sericite zones resulted from the 3D geological model from drillhole geological data; (b) N–S modeling of Cu data; and (c) C–V modeling of Cu data; (d) S–V modeling of Cu data (Scale is in m^3.)

Table 6 Overall accuracy (OA), Type I and Type II errors (T1E and T2E, respectively) with respect to potassic alteration zone resulted from geological model and threshold values of Cu obtained through C–V , N–S and S–V fractal modeling.

		Potassic alteration of geological model	
		Inside zones	Outside zones
C–V fractal model of highly mineralized zones	Inside zones Outside zones	A 2850 C 77927 T1E 0.96 OA	B 1360 D 76913 T2E 0.02 0.50
N–S fractal model of highly mineralized zones	Inside zones Outside zones	A 3092 C 75025 T1E 0.96 OA	B 1570 D 75473 T2E 0.02 0.51
S–V fractal model of supergene enrichment zones	Inside zones Outside zones	A 4431 C 72985 T1E 0.94 OA	B 2318 D 75726 T2E 0.03 0.52

Table 7 Overall accuracy (OA), Type I and Type II errors (T1E and T2E, respectively) with respect to phyllic alteration zone resulted from geological model and threshold values of Cu obtained through C–V, N–S and S–V fractal modeling.

		Phyllic alteration of geological model	
		Inside zones	Outside zones
C–V fractal model of moderately and weakly mineralized zones	Inside zones Outside zones	A 36518 C 25461 T1E 0.41 OA	B 48027 D 69155 T2E 0.40 0.59
N–S fractal model of moderately mineralized zones	Inside zones Outside zones	A 35555 C 23955 T1E 0.40 OA	B 46943 D 48223 T2E 0.49 0.54
S–V fractal model of the hypogene zones	Inside zones Outside zones	A 40080 C 26899 T1E 0.40 OA	B 44943 D 54239 T2E 0.45 0.56

Application of fractal models to delineate mineralized zones in the Pulang porphyry copper deposit, Yunnan, Southwest China

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Abstract

The aim of this study is to delineate and identify various mineralized zones and barren host rocks based on surface and subsurface lithogeochemical data from the Pulang porphyry copper deposit, Southwest China, utilizing the number-size (N-S), concentration-volume (C-V) and power spectrum-volume (S-V) fractal models. The N-S model reveals three mineralized zones characterized by Cu thresholds of 0.28% and 1.45%: <0.28% Cu represents weakly mineralized zones and barren host rocks, 0.28%-1.45% Cu represents moderately mineralized zones, and >1.45% Cu represents highly mineralized zones. The results obtained by the C-V model depict four geochemical zones defined by Cu thresholds of 0.25%, 1.48% and 1.88%, representing nonmineralized wall rocks (Cu<0.25%), weakly mineralized zones (0.25%-1.48%), moderately mineralized zones (1.48%-1.88%), and highly mineralized zones (Cu>1.88%). The S-V model is used by performing a 3D fast Fourier transformation of assay data in the frequency domain. The S-V model reveals three mineralized zones characterized by Cu thresholds of 0.23% and 1.33%: <0.23% Cu represents leached zones and barren host rocks, 0.23%-1.33% Cu represents hypogene zones, and >1.33% Cu represents supergene enrichment zones. All the multifractal models indicate that high-grade mineralization occurs in the central and southern parts of the ore deposit. Their results are compared with the alteration and

mineralogical models resulting from the 3D geological model using a logratio matrix. The results show that the S-V model is best at identifying highly mineralized zones in the deposit. However, the results of the C-V model for moderately and weakly mineralized zones are more accurate than those obtained from the N-S and S-V models.

Keywords: Fractal; Concentration-volume (C-V) model; Number-size (N-S) model; Power spectrum-volume (S-V) model; Mineralized zone; the Pulang porphyry copper deposit

1. Introduction

The definition and delineation of different mineralized zones and non–mineralized wall rocks are the main goal in economic geology and mineral exploration. Investigation of ore mineralogy and paragenetic sequence provides useful data on ore-forming processes in deposits because typical characteristics of various types of ore deposits are reflected by their mineral assemblages (Craig and Vaughan, 1994; White and Hedenquist, 1995). Common methods generally use mineralography, petrography and alteration mineral assemblage analysis to delineate various mineralized zones in porphyry deposits (Beane, 1982; Schwartz, 1947; Sillitoe, 1997; Berger et al., 2008). Lowell (1968) first proposed a conceptual model of the lateral and vertical variations in mineralogy within alteration zones. Some similar models were developed for potassic alteration, which is usually situated in the center and deep parts of porphyry ore deposits, based on this conceptual model (Sillitoe and Gappe, 1984; Cox and Singer, 1986; Melfos et al., 2002). Fluid inclusion and stable isotope studies are other methods used to outline different mineralization phases based on thermometric and isotope element parameters and other geological parameters (e.g., Boyce et al., 2007; Faure et al., 2002; Wilson et al., 2007). Drillhole data and logging information, including mineralographical information, host rock changes and alterations are helpful in delineating mineralization zones. Different geological interpretations could be used to detect zone boundaries, which may also lead to different results because the elemental grade distribution may not be taken into

consideration.

Non-Euclidean fractal geometry (Mandelbrot, 1983) is an important branch of nonlinear mathematical sciences and has been applied in various research fields of the geosciences since the 1980s. The relationships between geology, geochemistry and mineralogical settings and spatial information can be researched by methods based on fractal geometry (Afzal et al., 2011; Carranza, 2008, 2009). Bolviken et al. (1992) and Cheng et al. (1994) have shown that geochemical patterns of various elements have fractal dimensions. The concentration-area (C-A) model was proposed by Cheng et al. (1994) to recognize geochemical anomalies from background concentrations and calculate elemental thresholds of different geochemical data. Furthermore, many other fractal models have been proposed and applied in geochemical exploration work, including the number-size (N-S) fractal model proposed by Mandelbrot (1983) and Agterberg (1995), the power spectrum-area (S-A) fractal model proposed by Cheng et al. (1999), the concentration-distance (C-D) fractal model proposed by Li et al. (2003), the concentration-volume (C-V) fractal model proposed by Afzal et al. (2011) and the power spectrum-volume (S-V) fractal model proposed by Afzal et al. (2012).

Methods of fractal analysis also illustrate the relationships between geological, geochemical and mineralogical settings and spatial information derived from the analysis of mineral deposit occurrence data (Carranza, 2008; Carranza et al., 2009; Goncalves et al., 2001). Various geochemical processes can be described based on the differences in fractal dimensions obtained from the analysis of relevant geochemical data. Afzal et al. (2011) considered that the log-log plots obtained by fractal methods are useful tools to delineate different geological populations of geochemical data, and the thresholds could be determined as some breakpoints in those plots.

The application of fractal models to delineate various grade mineralization zones was dependent on the relationships between the metal grades and volumes (Afzal et al., 2011; Agterberg et al., 1993; Cheng, 2007; Sim et al., 1999; Turcotte, 1986). Afzal et al. (2011 and 2012) proposed a concentration-volume (C-V) and power spectrum-volume (S-V) fractal model to delineate different porphyry-Cu mineralized

zones and barren host rocks. In this paper, N-S, C-V and S-V fractal models were applied to delineate various mineralized zones and barren host rocks in the Pulang porphyry copper deposit, Yunnan, Southwest China.

2. Fractal models

2.1. Number-size (N-S) fractal model

The number-size (N-S) method proposed by Mandelbrot (1983) can be utilized to describe the distribution of geochemical populations (Sadeghi et al., 2012). In this method, geochemical data does not undergo any preprocessing (Mao et al., 2004). This model shows a relationship between desirable attributes (e.g., Cu concentration in this study) and their cumulative number of samples (Sadeghi et al., 2012). A power-law frequency model has been proposed to explain the N-S relationship according to the frequency distribution of elemental concentrations and cumulative number of samples with those attributes (e.g., Li et al., 1994; Sadeghi et al., 2012; Sanderson et al., 1994; Shi and Wang, 1998; Turcotte, 1996; Zuo et al., 2009a).

The N-S model proposed by Mandelbrot (1983) can be expressed as follows:

$$N(\geq\rho)=F\rho^{-D} \quad (1)$$

where ρ denotes the element concentration, $N(\geq\rho)$ denotes the cumulative number of samples with concentrations greater than or equal to ρ, F is a constant and D is the scaling exponent or fractal dimension of the distribution of element concentrations. According to Mandelbrot (1983), log-log plots of $N(\geq\rho)$ versus ρ show linear segments with different slopes -D corresponding to different concentration intervals.

2.2. Concentration-volume (C-V) fractal model

Afzal et al. (2011) proposed a concentration-volume (C-V) fractal model based on the same principle of the concentration-area (C-A) model (Cheng et al., 1994) to analyze the relationship between the concentration of ore elements and accumulative volume with concentrations greater than or equal to a given value (Afzal et al., 2011; Zuo et al., 2016; Lin et al., 2013; Sadeghi et al., 2012; Soltani et al., 2014; Sun and Liu, 2014; Wang, G. et al., 2012). This model can be expressed as follows:

$$V(\rho \leq \upsilon) \propto \rho^{-a_1};\ V(\rho \geq \upsilon) \propto \rho^{-a_2} \quad (2)$$

$V(\rho \geq \upsilon)$ and $V(\rho \leq \upsilon)$ represent the occupied volumes with concentrations above or equal to and less than or equal to the contour value υ; υ indicates the threshold value of a zone; and a_1 and a_2 are the characteristic indexes. The thresholds obtained by this method indicate the boundaries between the different grade mineralization zones and barren host rocks of ore deposits. The drillhole data of the elemental concentrations were interpolated by using geostatistical estimation to compute $V(\rho \geq \upsilon)$ and $V(\rho \leq \upsilon)$, which are the volume values enclosed by a contour level ρ in a 3D model.

2.3. Power spectrum-volume (S-V) fractal model

Different geochemical patterns in the spatial domain could be seen as layered signals of various frequencies. Cheng et al. (1999) proposed the power spectrum-area (S-A) fractal model to recognize geochemical anomalies from backgrounds utilizing the method of spectrum analysis in the frequency domain according to this argument. This model is combined with a concentration-area (C-A) model (Cheng et al. 1994), offering a useful tool to determine an optimum threshold value between various patterns based on the scaling property.

Afzal et al. (2012) proposed the power spectrum-volume (S-V) fractal model to delineate different grade mineralization zones based on the same principle as the S-A model proposed by Cheng et al. (1999). The S-V model was utilized in the frequency domain by applying a fast Fourier transformation to the assay data. The straight lines obtained by log-log plotting indicate the relationships between the power spectra and relevant volumes of ore elements. These relationships were utilized to recognize the hypogene zones and supergene enrichment zones from the barren host rocks and the leached zone of the deposit. The recognition of various mineralization zones is based on the power-law relationships between the power spectra and occupied volumes. The formula is as follows:

$$V(\geq S) \propto S^{-2/\beta} \quad (3)$$

where the power-law relationships between the power spectra ($S=-||F(Wx, Wy, Wz)||$) and occupied volumes with power spectra greater than or equal to S can be

indicated by this form; F represents the fast Fourier transformation of the measurement μ(x, y, z); and Wx, Wy and Wz indicate wave numbers or angular frequencies in the X, Y and Z directions in a 3D model. The range of index β is $0<\beta\leq 2$ or $1\leq 2/\beta$ with the special cases of $\beta=2$ and $2/\beta=1$ corresponding to nonfractal and monofractal expressions, and $1<2/\beta$ corresponding to multifractals (Cheng, 2006).

By using the method of geostatistical estimation, the drillhole data of elemental concentration values were interpolated to construct a block model of ore element distribution. The power spectrum values can be obtained by 3D fast Fourier transformation of the ore element grades. The logarithm of all the power spectrum values and accumulative volume values were calculated. Additionally, the log-log plot between power spectrum and volume was drawn according to previously determined values. Then, the filters were constructed on the basis of threshold values obtained by the log-log plot of S-V. Finally, the power spectra were converted back to the space domain by utilizing inverse fast Fourier transformation.

3. Geological setting of the Pulang porphyry copper deposit

The Pulang porphyry copper deposit is situated in the southern end of the Yidun continental arc, Southwest China (Fig. 1). The continental arc was produced due to the westward subduction of Garze–Litang oceanic crust (Deng et al., 2014b, 2015; Wang et al., 2014). The Pulang ore deposit, one of the largest porphyry copper deposits in China (Deng et al., 2012, 2014a; Mao et al., 2012, 2014), is characterized by a typical porphyry-type alteration zone. The geological characteristics of the deposit, including the alteration types and their zonation, the geometry of the orebody, the metallogenic time and the geodynamic settings, have been systematically researched (Leng et al., 2012; Li et al., 2011, 2013). The deposit consists of five ore-bearing porphyry bodies covering an area of approximately 9 km^2, and the explored ore tonnage of Cu is estimated to be 6.50 Mt (Liu et al., 2013).

The outcrop strata of the Pulang deposit are dominated by Upper Triassic Tumugou Formation clastic rocks and andesite and Quaternary sediments (Fig. 1c). The Triassic porphyry intrusions primarily comprise quartz diorite porphyry, quartz

monzonite porphyry, quartz diorite porphyrite and granodiorite porphyry. The Tumugou Formation strata were intruded by the quartz diorite porphyry with an age of 219.6 ± 3.5 Ma (zircon U-Pb dating) (Pang et al., 2009). Then, quartz monzonite porphyry with an age of 212.8 ± 1.9 Ma and granodiorite porphyry with an age of 206.3 ± 0.7 Ma (zircon U-Pb dating) (Liu et al., 2013) crosscut the quartz diorite porphyry. The quartz monzonite porphyry is considered to be associated with mineralization because its age is similar to the molybdenite Re-Os isochron age of 213 ± 3.8 Ma from the orebody (Zeng et al., 2004). Moreover, the Cu concentrations of the quartz monzonite porphyry are higher than those of the other porphyries.

The porphyry-type alteration zones transition from early potassium-silicate alteration through quartz-sericite alteration to propylitization, upward and outward from the core of the quartz monzonite porphyry (Fig. 4). The wall rocks near the porphyries were mostly changed into hornfels. Systematic drilling has demonstrated that the potassium-silicate and quartz-sericite zones host the main orebodies, constituting the core of mineralized zones. The propylitic zones and hornfels only develop weak mineralization. The orebodies occur mainly in potassium-silicate and quartz-sericite and occur as veins in the propylitic zones and hornfels. The major rock types in the deposit are quartz monzonite porphyry, quartz diorite porphyrite, granite diorite porphyry, quartz diorite porphyry and hornfels (Fig. 2). Metallic minerals mainly include pyrite, chalcopyrite with a small amount of molybdenite and pyrrhotite (Fig. 3).

4. Fractal modeling

Based on the geological data (which include the collar coordinates of each drillhole, azimuth and dip (orientation), lithology and mineralogy) recorded from 130 drillholes in the Pulang deposit, 20492 lithogeochemical samples were collected at 2 m intervals. The laboratory of the 3rd Geological Team of the Yunnan Bureau of Geology and Mineral Resources utilized the iodine-fluorine and oscillo polarographic method to analyze the concentrations of Cu and associated paragenetic elements, and its analytical uncertainty is less than 7% (Yunnan Diqing Nonferrous Metal Co. Ltd.,

2009). Only Cu concentrations were studied in this study. The histogram and Q-Q plot of the log-transformed Cu data indicate that the distribution of Cu data is log-normal (Fig. 5). The experimental semivariogram of the Cu data of the Pulang deposit indicates a range and nugget effect of 320.0 m and 0.25, respectively (Fig. 6). The spherical model is fitted with regard to the experimental semivariogram. The 3D model of the Cu concentration distribution of the Pulang deposit was produced with the ordinary kriging method using Geovia Surpac software on the basis of the semivariogram and anisotropic ellipsoid. Fundamentally, the accuracy of the interpolation results mainly depends on whether the interpolation model accurately fits the spatial distribution characteristics of the deposit. Ordinary kriging was used because it is compatible with a stationary model; it only requires a variogram, and it is the most commonly used form of kriging (Chilès and Delfiner, 1999). Goovaerts (1997) showed that the values in unsampled locations are estimated by the ordinary kriging method according to the moving average of the interest variables, satisfying various distribution forms of data. Ordinary kriging is a spatial estimation method in which the error variance is minimized. This error variance is based on the configuration of the data and its variogram (Yamamoto, 2005). The correct variogram in kriging interpolation can guarantee the accuracy of the interpolation results.

The accuracy of the spatial interpolation analysis is verified by comparing the difference between the measured values and the predicted values to select the best variogram model. To test the variogram model, the cross-validation method was used to determine whether the parameters of the variogram model were correct. The distribution of the residual is normal (Fig. 7), and the mean error between the actual and estimated Cu grades is equal to 0 (Table 1). This result indicates that this model is reasonable and that the variogram parameters used for estimating the Cu grade are unbiased.

The obtained block models were used as inputs to the fractal models. The Pulang deposit was modeled by 20 m×20 m×5 m voxels, and they were decided by the grid drilling dimensions and geometrical properties of the deposit (David, 1970). The Pulang deposit is totally modeled with 150973 voxels. The terms “highly”,

232 “moderately” and “weakly” have been used to classify mineralized zones based on
233 fractal modeling, in accordance with the classification of the ore grades in the deposit.

234 4.1. Number-size (N-S) fractal modeling

235 The N-S model was applied to the Cu data (Fig. 8). The selection of breakpoints
236 as threshold values is an objective decision because geochemical populations are
237 defined by different line segments in the N-S log-log plot. The straight fitted lines
238 were obtained based on least-square regression (Agterberg et al., 1996; Spalla et al.,
239 2010). In other words, the intensity of element enrichment is depicted by each slope
240 of the line segments in the N-S log-log plots (Afzal et al., 2010; Bai et al., 2010).

241 Based on the classification of the 3D model of Cu data and the thresholds obtained
242 from the N-S fractal model (Table 2), highly mineralized zones are situated in the
243 southern and central parts of the Pulang deposit and coincide with the
244 potassium-silicate alterations. However, small and highly mineralized zones are
245 located in the central parts of the Pulang deposit (Fig. 9). Moderately mineralized
246 zones occur along a northwest-southeast trend and correlate with the phyllic zones.
247 Weakly mineralized zones and barren host rocks are situated in the marginal parts of
248 the area.

249 4.2. Concentration-volume (C-V) fractal modeling

250 The occupied volumes corresponding to the Cu grades were computed to obtain
251 the concentration-volume model according to the 3D model of the Pulang deposit.
252 Through the obtained C-V log-log plot, the threshold values of the Cu grades were
253 determined (Fig. 10). These results indicate the power-law relationship between Cu
254 grade and volume. According to these results (Table 3), the low-concentration zones
255 exist in many parts of the deposit and occur along a northwest-southeast trend.
256 Moderately and highly mineralized zones are situated in several parts of the central
257 deposit and to the south of the deposit (Fig. 11).

258 4.3. Power spectrum-volume (S-V) fractal modeling

259 Based on the geological data (which include the collar coordinates of each
260 drillhole, azimuth and dip (orientation), lithology and mineralogy) recorded from 130

drillholes in the deposit, a 3D model and block model of the distribution of Cu in the Pulang deposit were constructed with ordinary kriging using Geovia Surpac software.

The power spectrum (S) was calculated for the 3D elemental distribution using 3D fast Fourier transformation in MATLAB (R2016a). The logarithmic values of the power spectra and relevant volumes were plotted against each other (Fig. 12). The straight lines fitted in the log-log plot indicate different relationships between the power spectra and occupied volumes. The thresholds of logS = 7.81 and logS = 8.70 were determined by the log-log S-V plot. The 3D filters were designed to separate different mineralization zones on the basis of these threshold values. Inverse fast Fourier transformation was used to convert the decomposed components back into the space domain by using MATLAB (R2016a). According to the results, the Cu concentrations of the hypogene zones range from 0.23% to 1.33% (Table 4), and values of >1.33% Cu correspond to the supergene enrichment zones, whereas values of <0.23% Cu correspond to the leached zone and barren host rocks (Fig. 13).

5. Comparison of the fractal models and geological model of the deposit

Alteration models have a key role in zone delineation and in presenting geological models, as described by Lowell and Guilbert (1970). The potassic and phyllic alterations control major mineralization within supergene enrichment and hypogene zones according to these models. Models of Cu mineralization zones derived via fractal models can be compared with geological data to validate the results of analysis in different porphyry Cu deposits. The results of the fractal modeling of the Pulang deposit were compared with the 3D geological model of the deposit constructed by using Geovia Surpac and drillhole data (Fig. 2). Moreover, the results obtained from these fractal models were controlled by mineralogical investigations.

Carranza (2011) has illustrated an analysis for the calculation of spatial correlations between two binary datasets, especially mathematical and geological models. An intersection operation between the mineralization zones obtained from fractal models and the different alteration zones in the geological model was

performed to derive the amount of voxels corresponding to each of the classes of overlap zones (Table 5). Using the obtained numbers of voxels, the Type I error (T1E), Type II error (T2E), and overall accuracy (OA) of the fractal model were estimated with respect to different alteration zones and the geological data (Carranza, 2011). The OAs of the fractal models of the mineralized zones were compared as follows.

A comparison between highly mineralized zones based on the fractal models and potassic alteration zones resulting from the 3D geological model shows that there is a similarity among these fractal models. The overall accuracies for the C-V, N-S and S-V models are 0.50, 0.51 and 0.52, respectively (Table 6), which indicate that the S-V model gives better results for identifying highly mineralized zones in the deposit. The number of overlapped voxels (A) in the S-V model is higher than those in the N-S and C-V models. The correlation (from OA results) between highly mineralized zones obtained from S-V modeling and potassic alteration zones is better than that of the N-S and C-V model because of a strong proportional relationship between the extension and positions of voxels in the S-V model and the potassic alteration zones in the 3D geological model.

A comparison between phyllic alteration zones resulting from the 3D geological model and moderately and weakly mineralized zones from the fractal modeling shows that the overall accuracies of the C-V, N-S and S-V fractal models with respect to phyllic alteration zones of the geological model are 0.59, 0.54 and 0.56, respectively. The overall accuracy of moderately and weakly mineralized zones obtained from C-V modeling is higher than that of mineralized zones obtained from N-S and S-V modeling (Table 7). On the other hand, moderately mineralized zones defined by C-V modeling overlap with phyllic zones defined by the 3D geological model. However, the results of the C-V model are more accurate than those of the N-S and S-V models with respect to the phyllic zones defined by the 3D geological model.

It could be considered that there are spatial correlations between different modeled Cu zones and geological features such as alterations and mineralogy. Several samples were collected from different drillholes in different grade mineralization zones of the Pulang deposit to validate the results of the fractal models. These

samples were analyzed by microscopic identification and XRF (X-ray fluorescence spectrometry). The PL-B82 sample was collected from the drillhole situated in a high-grade mineralization zone and includes a high chalcopyrite content and some molybdenite (Fig. 16a). The PL-B62 sample was collected from the drillhole situated in a moderate-grade mineralization zone and includes a low chalcopyrite content and some pyrrhotite in the polished section (Fig. 16b). The PL-B74 sample was collected from the drillhole located in a weakly mineralized zone with lower chalcopyrite content and some pyrrhotite (Fig. 16c and Fig. 16d). The results obtained from the mineralogy, microscopic identification and drillhole scanning by XRF of these samples indicate that the Cu concentrations are 1.80%, 1.32% and 0.41% in the PL-B82, PL-B62 and PL-B74 samples, respectively (Table 8).

6. Conclusions

In many cases, drillhole logging is dealing with the lack of proper diagnosis of geological phenomena, which can undermine the delineation of mineralized zones because it depends on the subjective interpretation of individual loggers, and no two loggers provide the same interpretations. However, conventional geological modeling based on drillhole data is fundamentally important for understanding the orebody spatial structure. Grades of ore elements are not determined by conventional methods of geological ore modeling, while the variation in ore grades in a mineral deposit is an obvious and salient feature. Given the problems mentioned above, using a series of newly established methods based on mathematical analyses such as fractal modeling seems to be inevitable.

In this paper, the number-size (N-S), concentration-volume (C-V) and power spectrum-volume (S-V) fractal models were used to delineate and recognize various Cu mineralized zones of the Pulang porphyry copper deposit in the southern end of the Yidun continental arc, Southwest China. All these fractal models reveal that high-grade Cu mineralized zones are situated in the central and southern parts of the deposit. The Cu threshold values of highly mineralized zones are 1.45% and 1.88% based on the N-S and C-V fractal models. The Cu threshold of supergene enrichment

zones is 1.33% based on the S-V fractal model. The models of moderately mineralized zones contain 0.28-1.45% Cu according to the N-S model and 1.48-1.88% Cu according to the C-V model. The hypogene zones contain 0.23-1.33% Cu according to the S-V model. The N-S model reveals weakly mineralized zones and barren host rocks containing <0.28% Cu. In contrast, the C-V model reveals that the barren host rocks contain <0.25% and that the weakly mineralized zones contain 0.25-1.48% Cu. The S-V model reveals that the barren host rock and leached zone contain <0.23% Cu.

The comparison between highly mineralized zones based on the fractal models and potassic zones resulting from the 3D geological model illustrates that the S-V fractal model is better than the N-S and C-V model because the number of overlapped voxels (A) in the S-V model is higher than those in the N-S and C-V model. The overall accuracies for the C-V, N-S and S-V models are 0.50, 0.51 and 0.52, respectively (Table 6), which indicates that the S-V model gives the best results for identifying highly mineralized zones in the deposit. On the other hand, the correlation (from OA results) between the highly mineralized zones obtained from S-V modeling and the potassic alteration zones is better than those of the N-S and C-V models because of a strong proportional relationship between the extension and positions of the voxels in the S-V model and potassic alteration zones in the 3D geological model.

A comparison between phyllic alteration zones obtained from the 3D geological model and moderate grade mineralization zones obtained from the fractal models indicates that the OA values of the C-V, N-S and S-V fractal methods in reference to the phyllic alteration zones of the geological model are 0.59, 0.54 and 0.56, respectively. The overall accuracy of the moderately and weakly mineralized zones obtained from C-V modeling is higher than the mineralized zones obtained from N-S and S-V modeling (Table 7).

According to the correlation between the results driven by fractal modeling and geological logging from drillholes in the Pulang porphyry copper deposit, high-grade mineralization zones generated by fractal models, especially the S-V model, have a better correlation with potassic alteration zones resulting from the 3D geological

model than from the N-S and C-V models. The highly and moderately mineralized zones obtained from the fractal models are both situated in the southern and central parts of the Pulang deposit and coincide with potassic and phyllic alteration zones. There is a better relationship between the moderately and weakly mineralized zones derived by the C-V model and the phyllic alteration zones from the 3D geological model than those derived by the N-S and S-V models.

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Fig.1. Geological map of the Pulang porphyry copper deposit, SW China. Modified after Yunnan Diqing Nonferrous Metal Co. Ltd., 2009.

Fig.2. Geological 3D models including lithology, alteration and 3D drill hole plot with the legend of each in the Pulang porphyry copper deposit. (Scale is in m^3.)

Fig.3. Photographs of alteration and mineralization in the Pulang porphyry copper deposit, SW China. (a) Quartz monzonite porphyry with potassium-silicate alteration; (b) Quartz diorite porphyrite with quartz-sericite alteration; (c) Quartz diorite porphyrite with propylitic alteration; (d) Hornfels. Qtz=quartz; Pl=plagioclase; Kfs=K-feldspar; Bt=biotite; Ser=sericite; Chl=chlorite; Ep=epidote; Py=pyrite; Ccp=chalcopyrite; Mo=molybdenite; Po= pyrrhotite.

Fig.4. Cross section along exploration line 0 in the Pulang porphyry copper deposit, SW China. Modified after Wang et al., 2012.

Fig.5. Histograms of (a) the Cu raw and (b) logarithmic transformation data and (c) Q-Q plot of the log-transformed Cu data in the Pulang deposit.

Fig.6. The experimental semivariogram (omni-directional) of Cu data in Pulang deposit.

Fig.7. The cross-validation results: (a) residual VS Cu grade; (b) the residual d istribution histogram.

Fig.8. N–S log–log plot for Cu concentrations in the Pulang deposit.

Fig.9. Zones in the Pulang deposit based on thresholds defined from the N–S fractal model of Cu data: (a) highly mineralized zones; (b) moderately mineralized zones; (c) weakly mineralized zones and barren host rocks. (Scale is in m^3.)

Fig.10. C–V log–log plot for Cu concentrations in the Pulang deposit.

Fig.11. Zones in the Pulang deposit based on thresholds defined from the C–V fractal model of Cu data: (a) highly mineralized zones; (b) moderately mineralized zones; (c) weakly mineralized zones; (d) barren host rock.(Scale is in m^3.)

Fig.12. S–V log–log plot for Cu concentrations in the Pulang deposit.

Fig.13. Zones in the Pulang deposit based on thresholds defined from the S–V fractal model of Cu data: (a) the supergene enrichment zones; (b) the hypogene zones; (c) the leached zone and barren host rock (Scale is in m^3.)

Fig.14. Highly mineralized zones in the Pulang deposit: (a) potassium-silicate zone resulted from the 3D geological model from drillhole geological data; (b) N–S modeling of Cu data; and (c) C–V modeling of Cu data; (d) S–V modeling of Cu data (Scale is in m^3.)

Fig.15. Moderately mineralized zones in the Pulang deposit:(a) quartz–sericite zones resulted from the 3D geological model from drillhole geological data; (b) N–S modeling of Cu data; and (c) C–V modeling of Cu data; (d) S–V modeling of Cu data (Scale is in m^3.)

Fig.16. Chalcopyrite content in several samples based on mineralographical study: (a) PL-B82 sample was collected from the drillhole situated in the high grade mineralization zones.; (b) PL-B62 sample was collected from the drillhole situated in the moderately grade mineralization zones.; (c) and (d) PL-B74 sample was collected from the drillhole located at the weakly mineralized zones.

Table 1 The results of statistical characteristics of the residual.

Table 2 Thresholds concentrations obtained by using N-S model based on Cu% in Pulang deposit.

Table 3 Thresholds concentrations obtained by using C-V model based on Cu% in Pulang deposit.

Table 4 Ranges of power spectrum (S) for different mineralization zones in Pulang deposit.

Table 5 Matrix for comparing performance of fractal modeling results with geological model. A, B, C, and D represent number of voxels in overlaps between classes in the binary geological model and the binary results of fractal models (Carranza, 2011).

Table 6 Overall accuracy (OA), Type I and Type II errors (T1E and T2E, respectively) with respect to potassic alteration zone resulted from geological model and threshold values of Cu obtained through C–V , N–S and S–V fractal modeling.

Table 7 Overall accuracy (OA), Type I and Type II errors (T1E and T2E, respectively) with respect to phyllic alteration zone resulted from geological model and threshold values of Cu obtained through C–V, N–S and S–V fractal modeling.

Table 8 Results of XRF analysis of samples collected from different mineralized zones in the Pulang porphyry copper deposit.

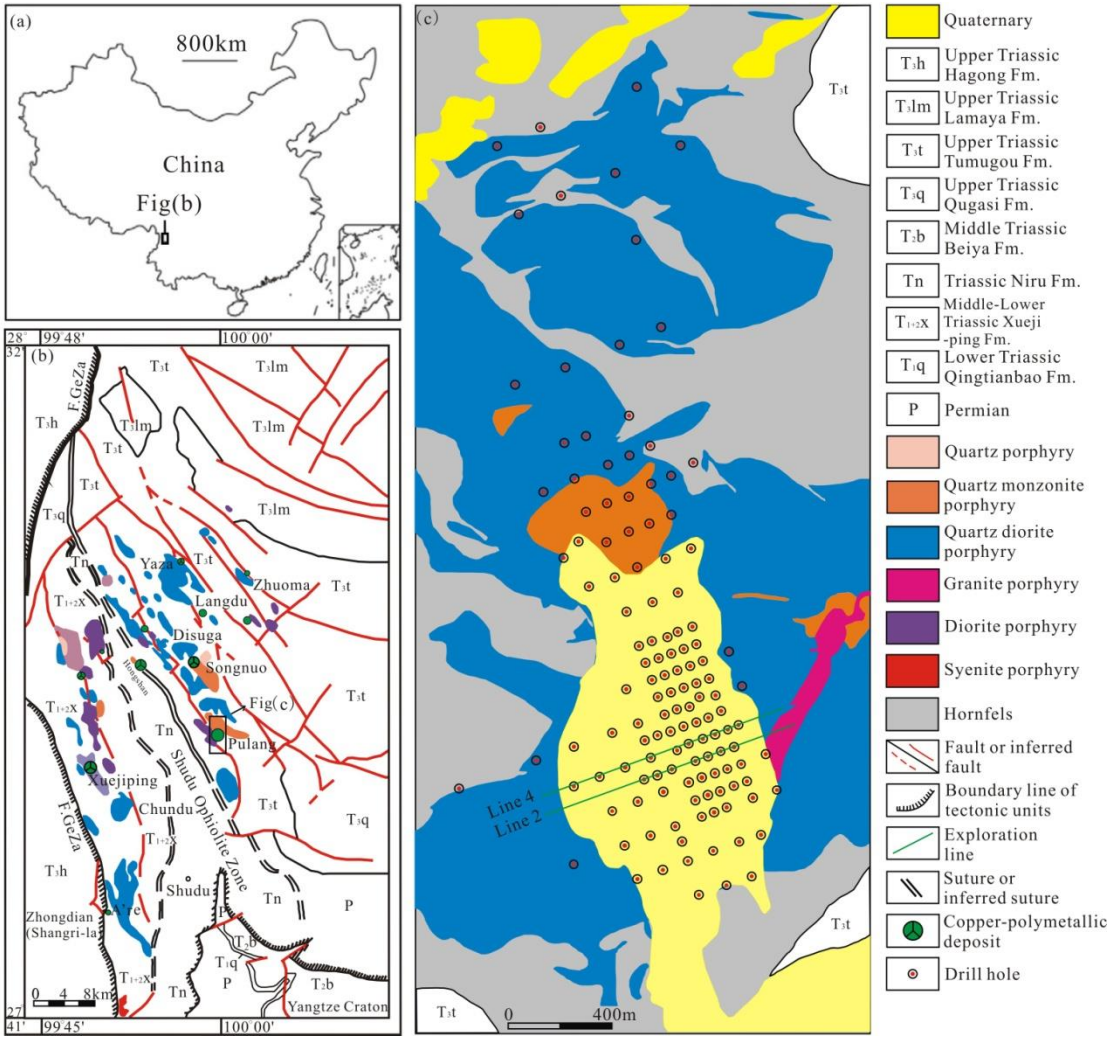

(a)
800km
China
Fig(b)
(b)
(c)
Quaternary
Upper Triassic Hagong Fm.
Upper Triassic Lamaya Fm.
Upper Triassic Tumugou Fm.
Upper Triassic Qugasi Fm.
Middle Triassic Beiya Fm.
Triassic Niru Fm.
Middle-Lower Triassic Xueji -ping Fm.
Lower Triassic Qingtianbao Fm.
Permian
Quartz porphyry
Quartz monzonite porphyry
Quartz diorite porphyry
Granite porphyry
Diorite porphyry
Syenite porphyry
Hornfels
Fault or inferred fault
Boundary line of tectonic units
Exploration line
Suture or inferred suture
Copper-polymetallic deposit
Drill hole
F.GeZa
Yaza
Langdu
Zhuoma
Disuga
Songnuo
Fig(c)
Pulang
Xuejiping
Chundu
Shudu Ophiolite Zone
Shudu
Zhongdian (Shangri-la)
Yangtze Craton
Line 4
Line 2
0 4 8km
0 400m

Fig. 1.

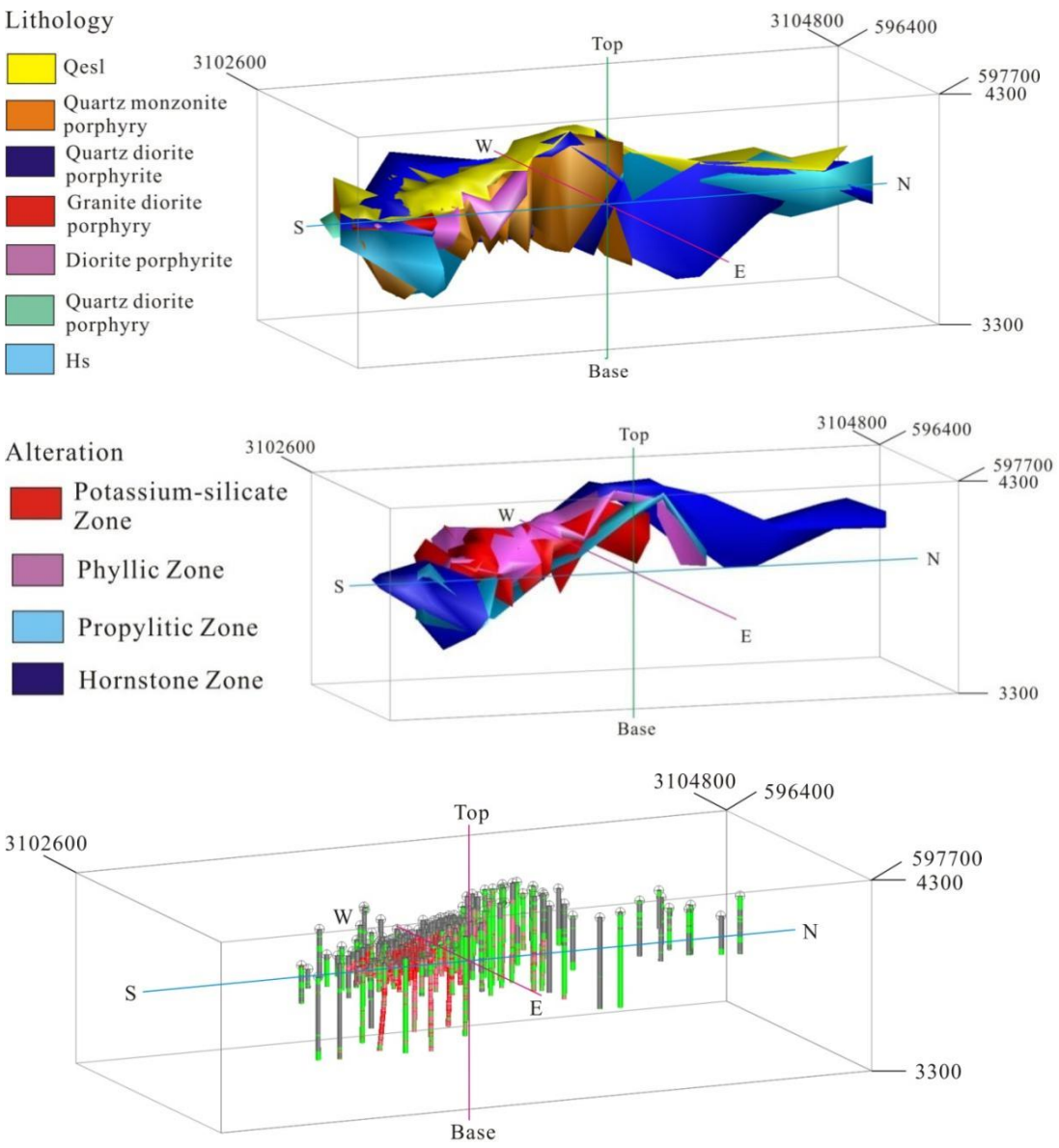

Lithology
Qesl
Quartz monzonite porphyry
Quartz diorite porphyrite
Granite diorite porphyry
Diorite porphyrite
Quartz diorite porphyry
Hs
3102600
3104800
596400
597700
4300
3300
Top
Base
W
S
N
E
Alteration
Potassium-silicate Zone
Phyllic Zone
Propylitic Zone
Hornstone Zone

Fig. 2.

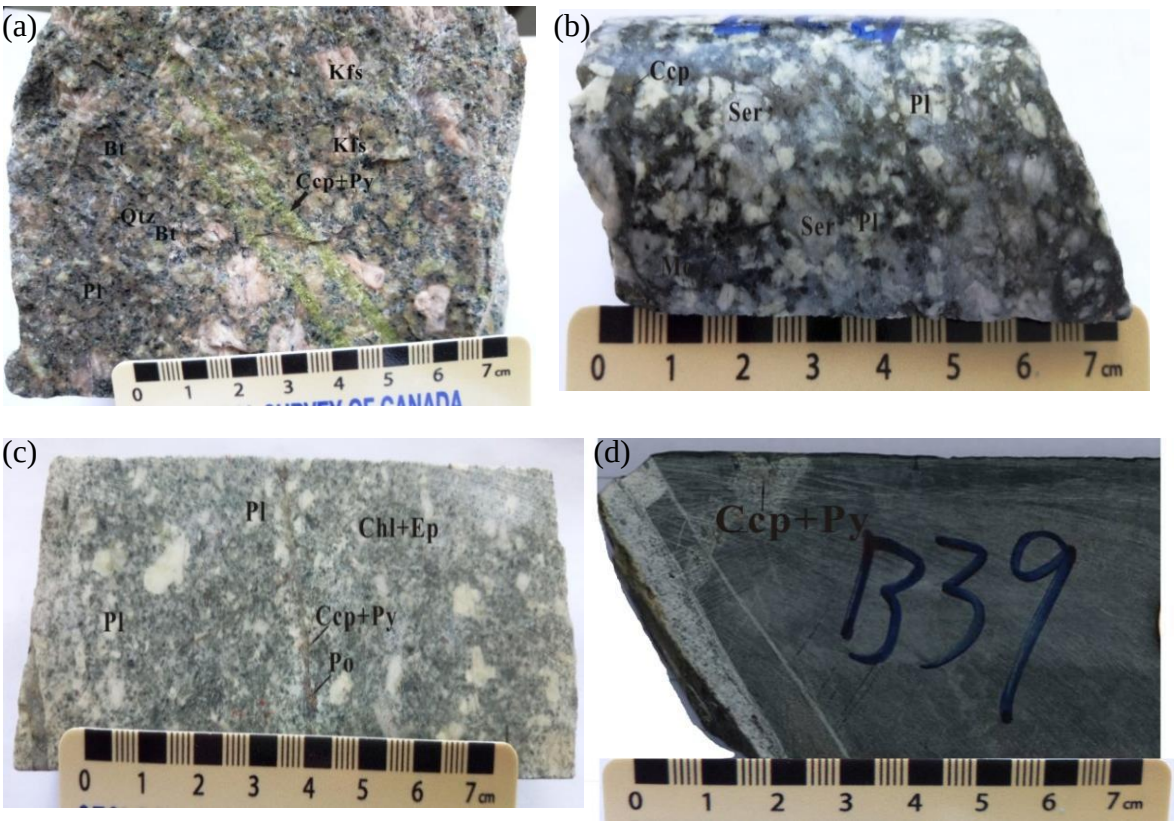

(a)
Kfs
Bt
Kfs
Ccp+Py
Qtz
Bt
Pl
(b)
Ccp
Ser
Pl
Ser
Pl
(c)
Pl
Chl+Ep
Pl
Ccp+Py
Po
(d)
Ccp+Py
B39

Fig. 3.

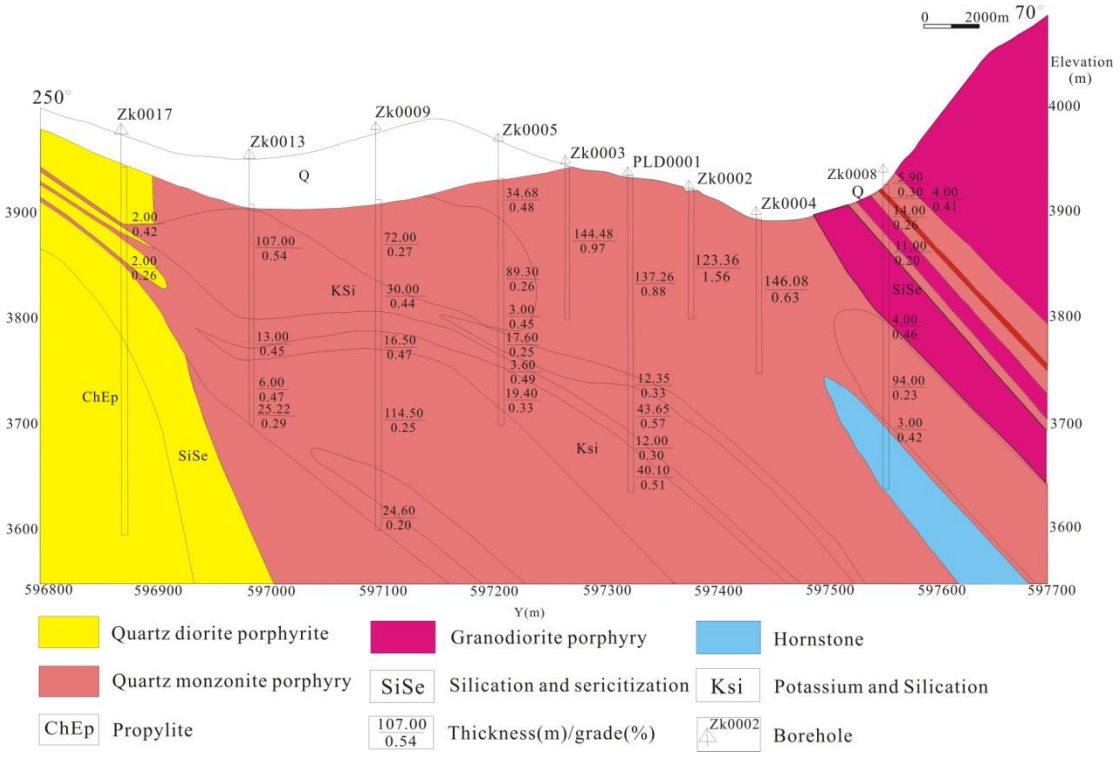

0 2000m
250°
70°
Elevation (m)
Zk0017
Zk0013
Zk0009
Zk0005
Zk0003
PLD0001
Zk0002
Zk0004
Zk0008
Q
KSi
ChEp
SiSe
Ksi
Y(m)
Quartz diorite porphyrite
Granodiorite porphyry
Hornstone
Quartz monzonite porphyry
SiSe
Silication and sericitization
Ksi
Potassium and Silication
ChEp
Propylite
Thickness(m)/grade(%)
Zk0002
Borehole

Fig. 4.

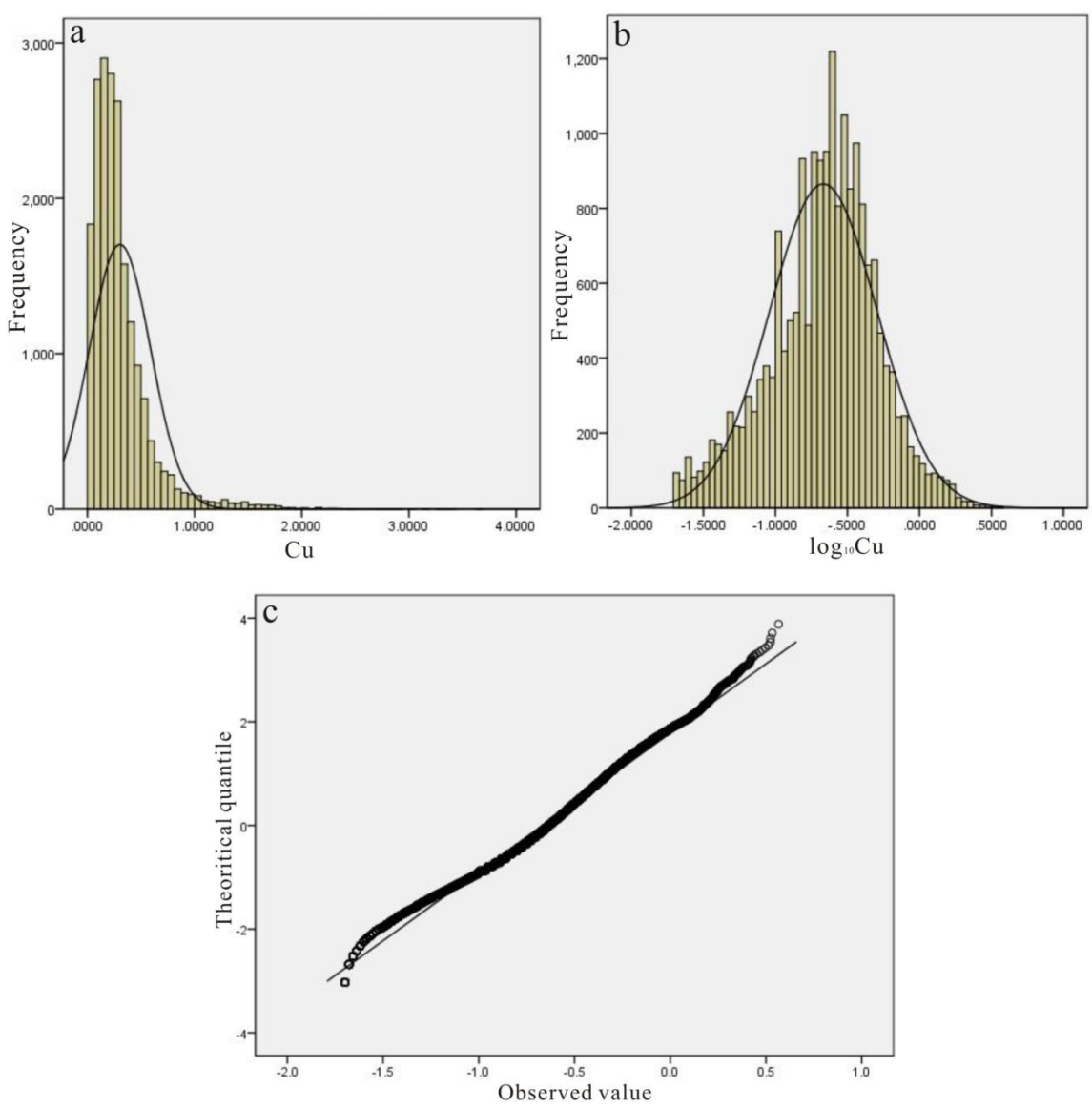

a
Frequency
Cu
b
Frequency
log10Cu
c
Theoritical quantile
Observed value

Fig. 5.

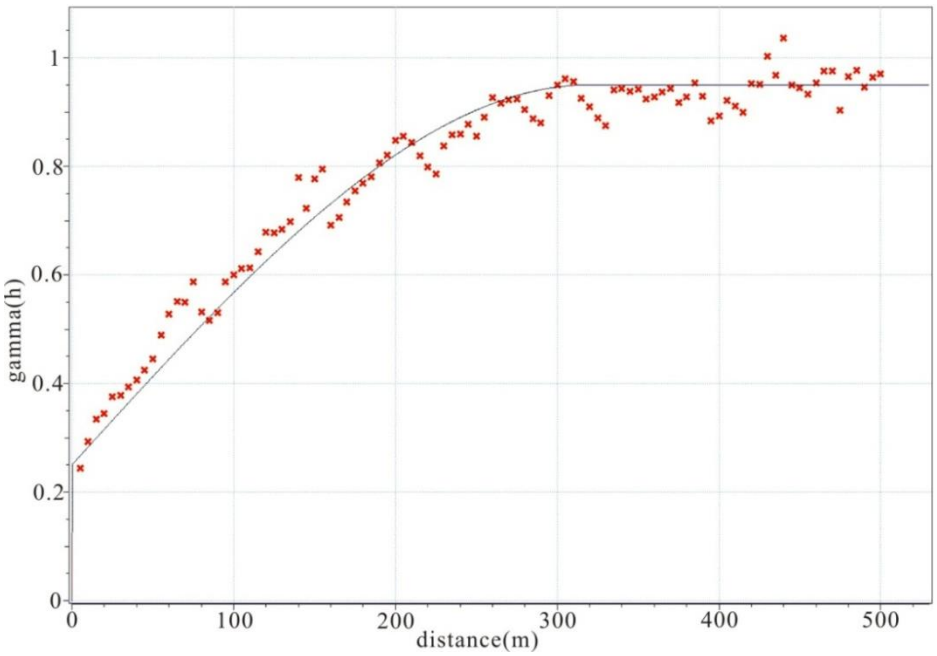

gamma(h)
distance(m)
0
0.2
0.4
0.6
0.8
1
0
100
200
300
400
500

Fig. 6.

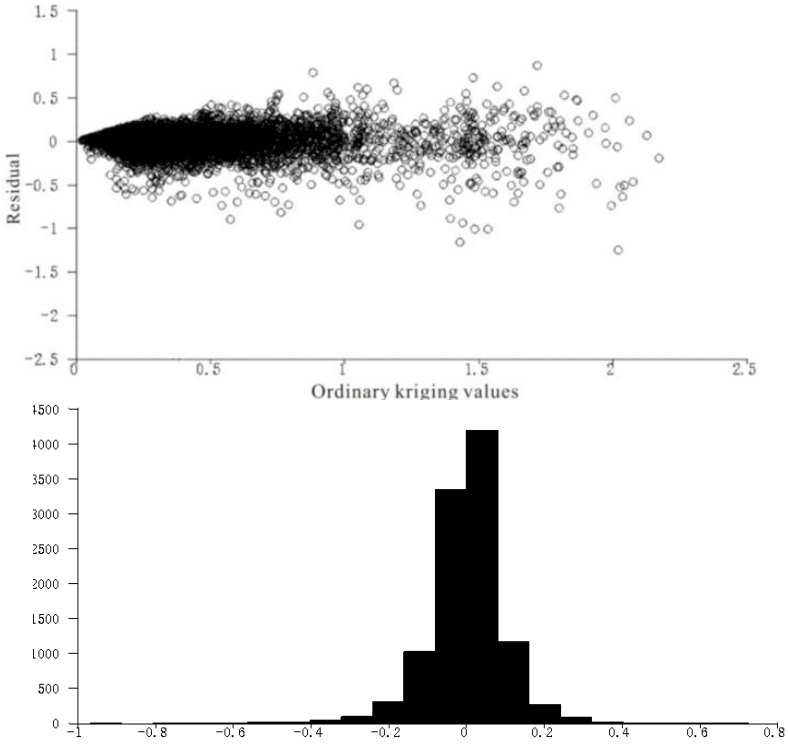
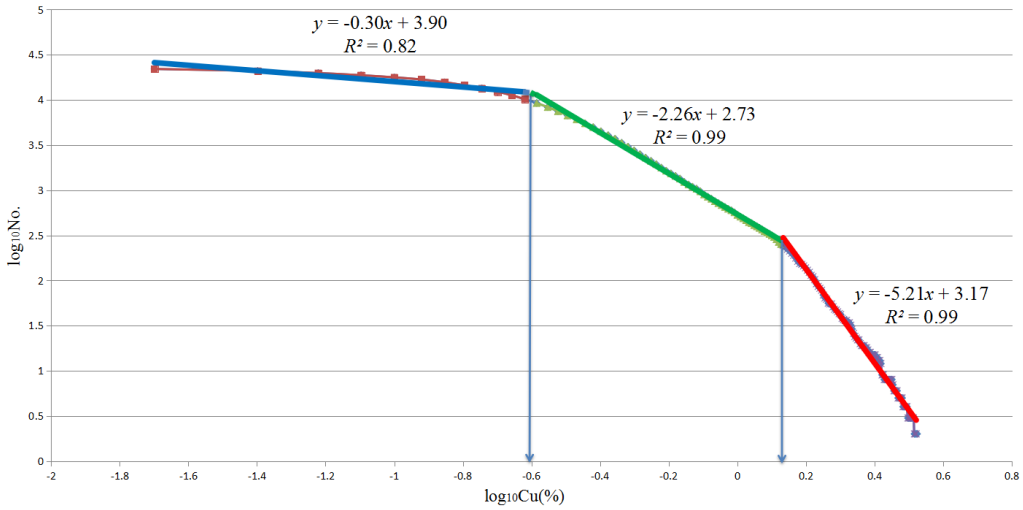

Residual
Ordinary kriging values
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
4500
4000
3500
3000
2500
2000
1500
1000
500
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

Fig. 7.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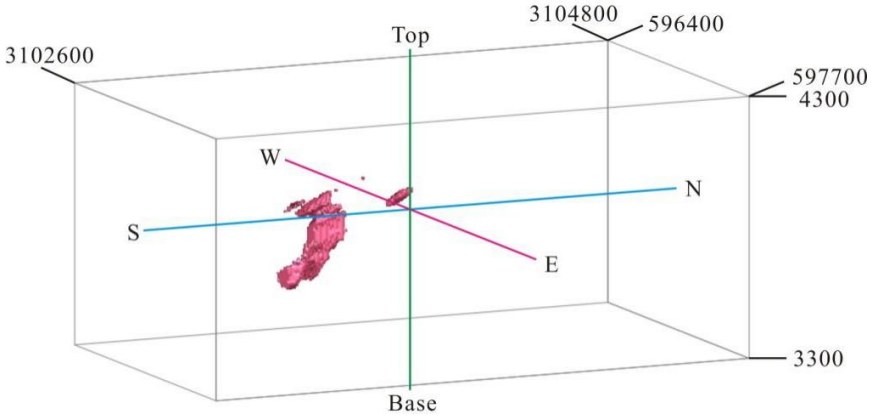
R² = 0.82
y = -2.26x + 2.73
R² = 0.99
y = -5.21x + 3.17
R² = 0.99
log10No.
log10Cu(%)

Fig. 8.

(a)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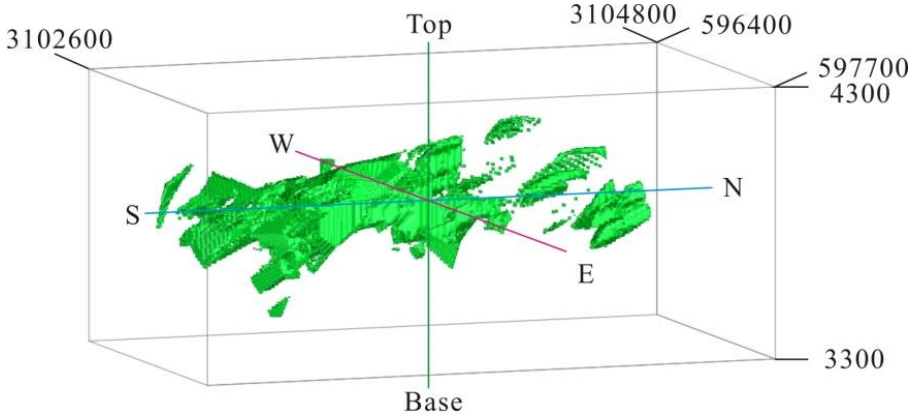
Top
3104800
596400
597700
4300
W
N
S
E
3300
Base

(b)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Top
3104800
596400
597700
4300
W
N
S
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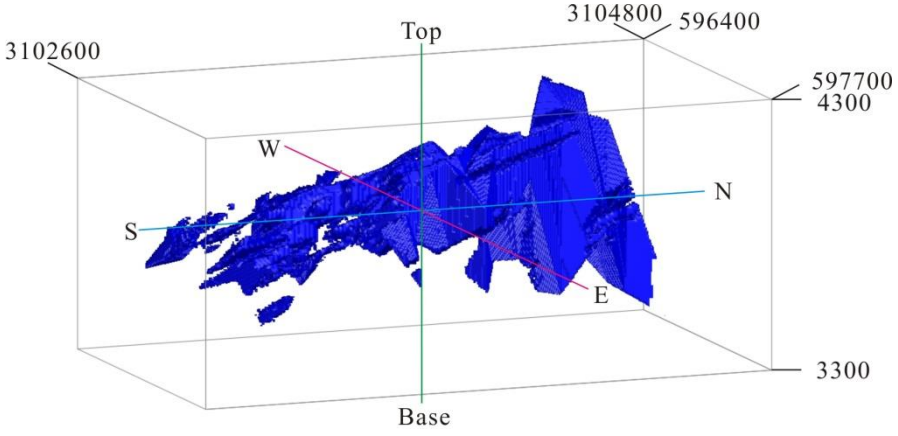
3300
Base

(c)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Top
3104800
596400
597700
4300
W
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S
E
3300
Base

Fig. 9

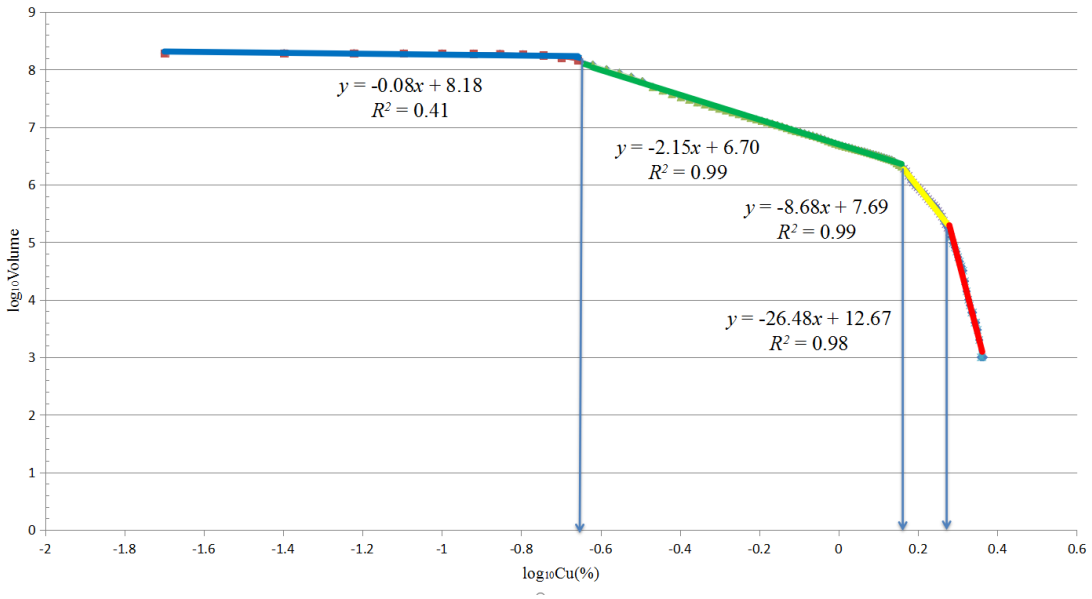
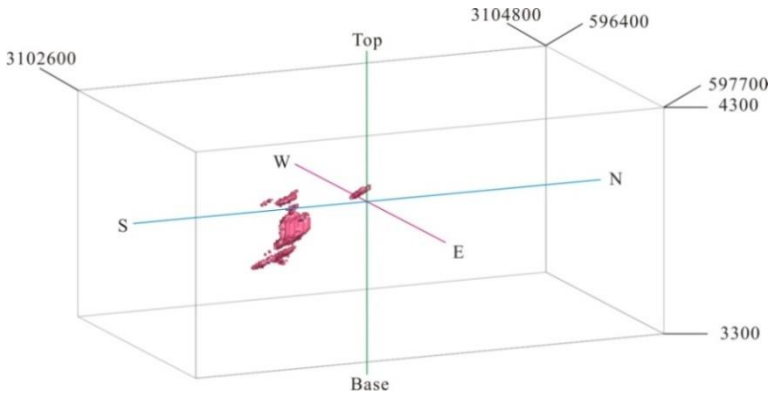

y = -0.08x + 8.18
R² = 0.41
y = -2.15x + 6.70
R² = 0.99
y = -8.68x + 7.69
R² = 0.99
y = -26.48x + 12.67
R² = 0.98
log10Volume
log10Cu(%)
9
8
7
6
5
4
3
2
1
0
-2
-1.8
-1.6
-1.4
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

Fig. 10.

(a)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3102600
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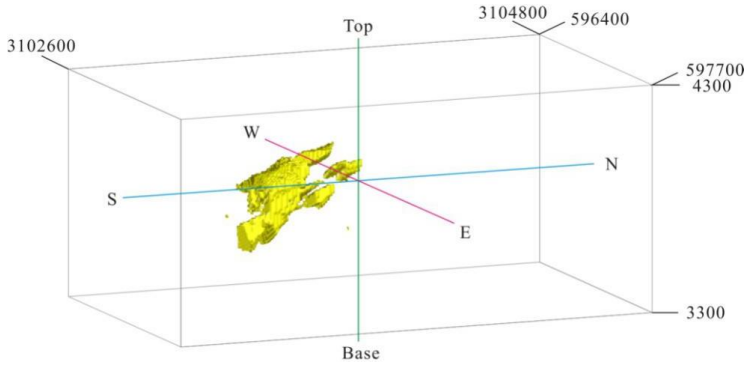
Base

(b)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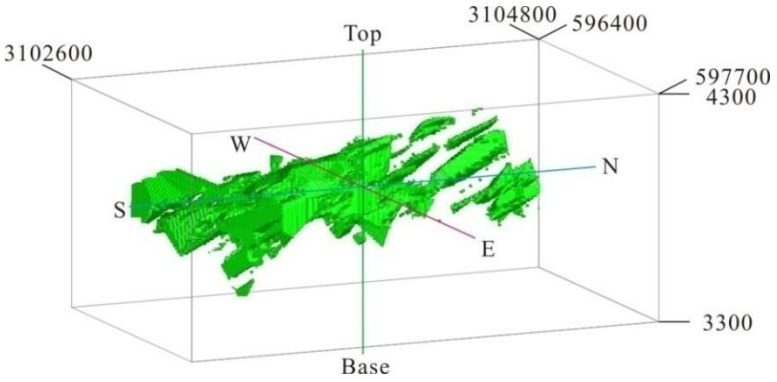
Base

(c)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Top
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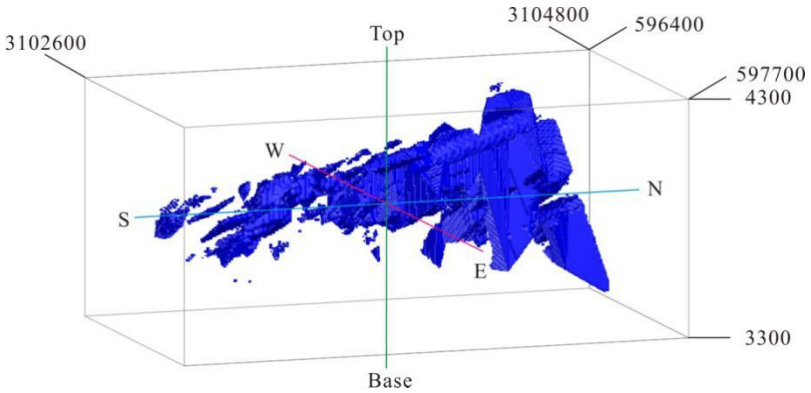
Base

(d)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Top
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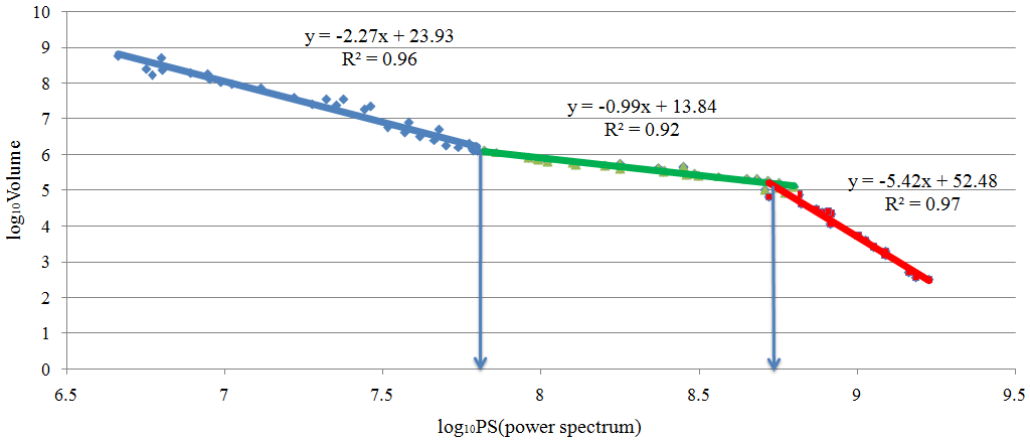
Base

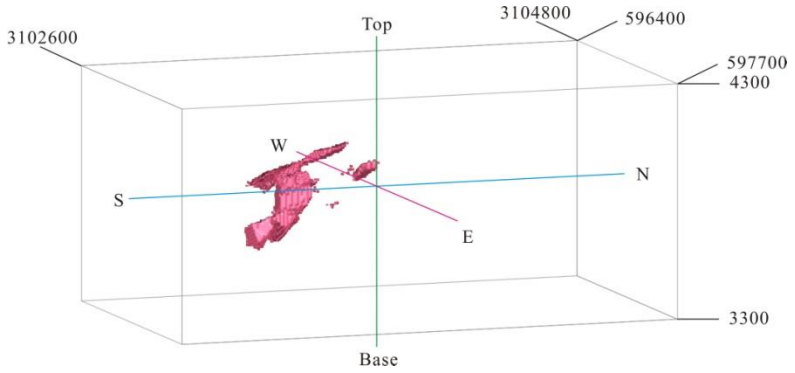
Fig. 11.



865

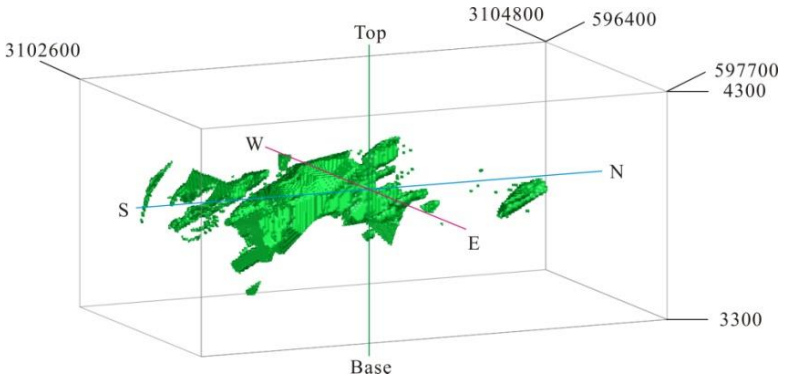
866 **Fig. 12.**

867 (a)



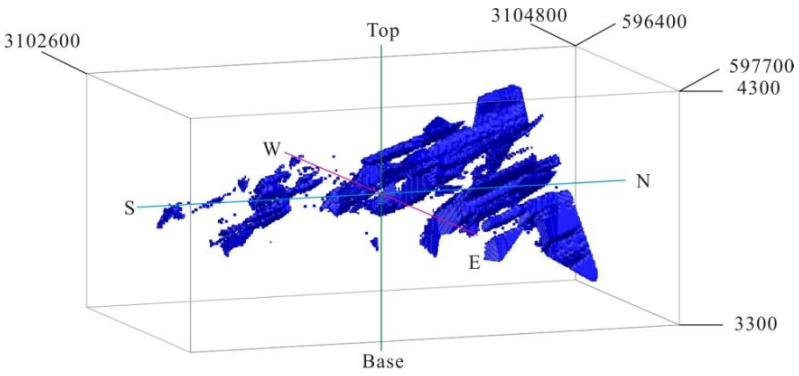
868

869 (b)



870

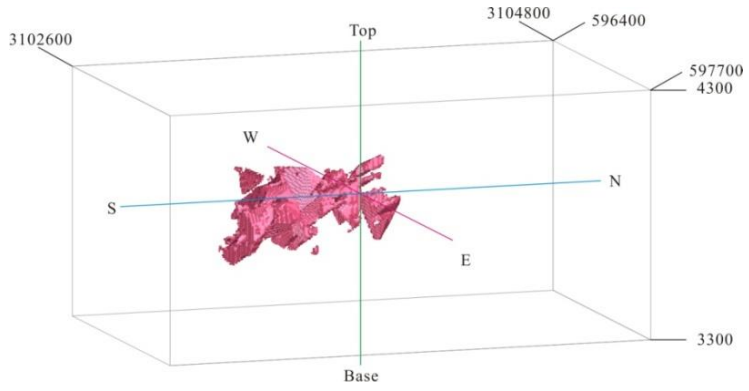
871 (c)



872

873 **Fig. 13.**

(a)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W
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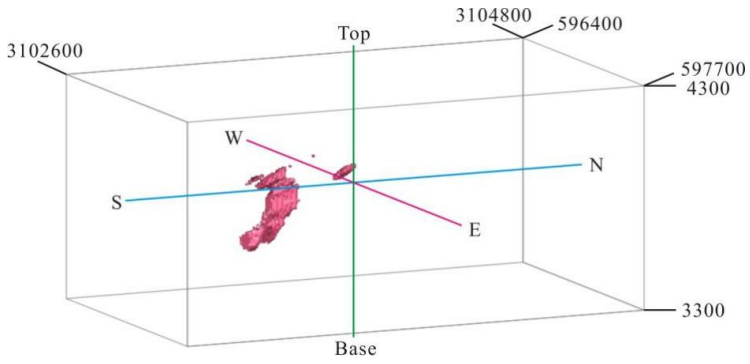
Base

(b)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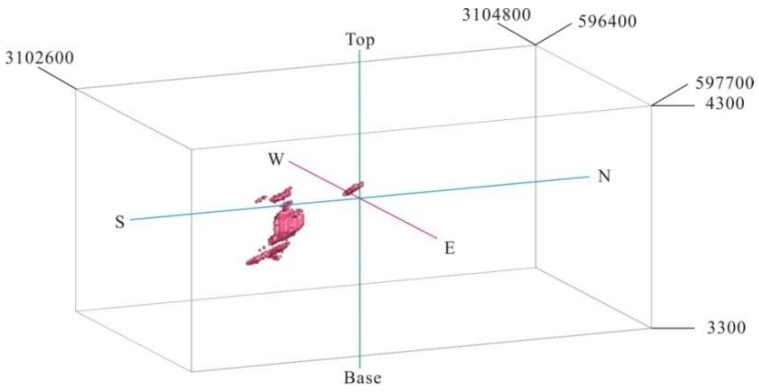
(c)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(d)

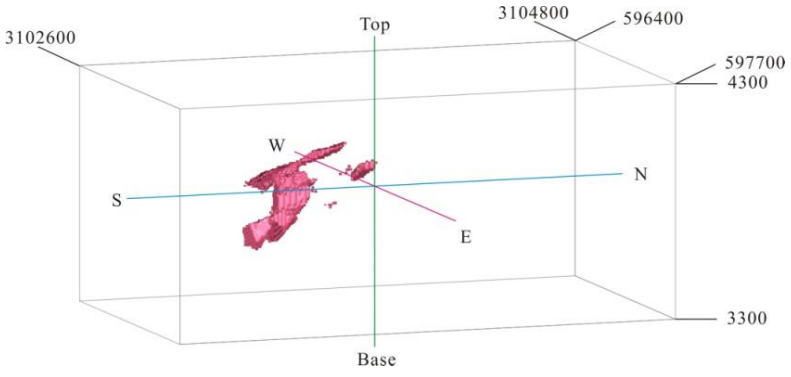
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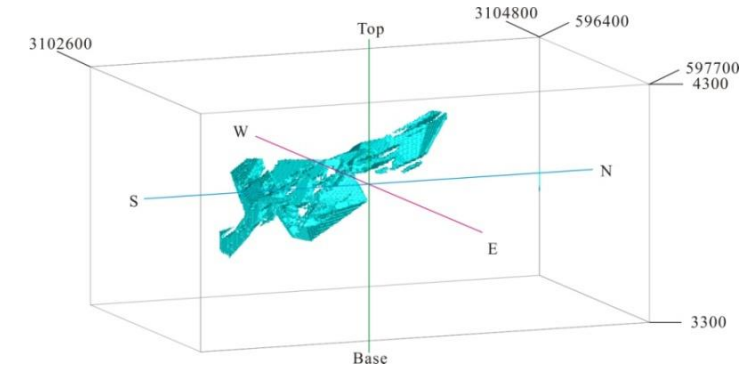
Fig. 14.

(a)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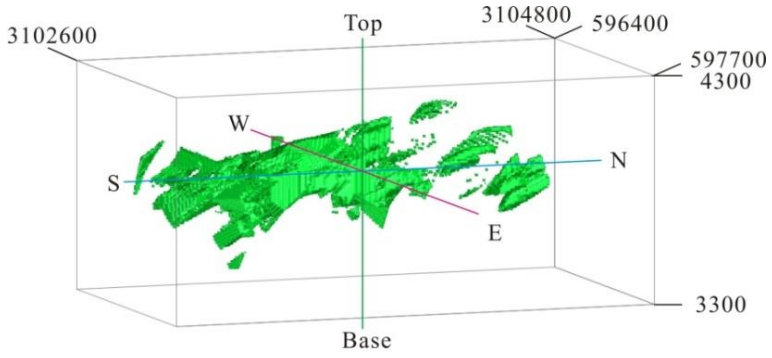
(b)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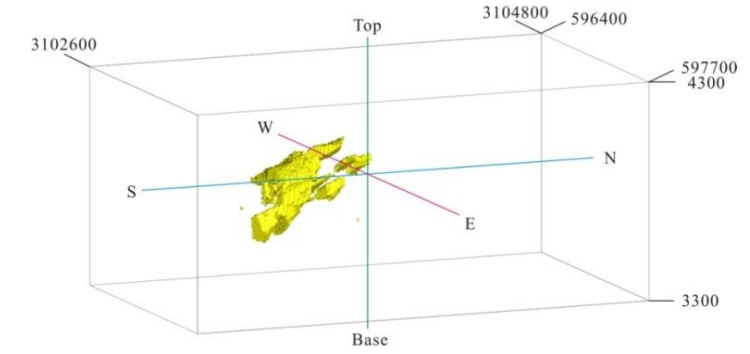
(c)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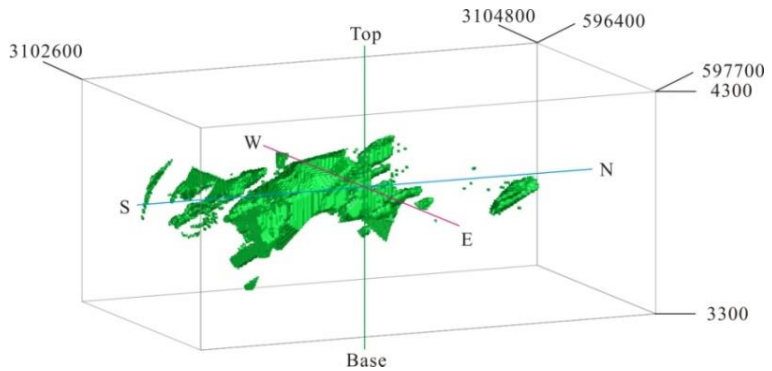
Base

(d)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597700
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Base

Fig. 15.

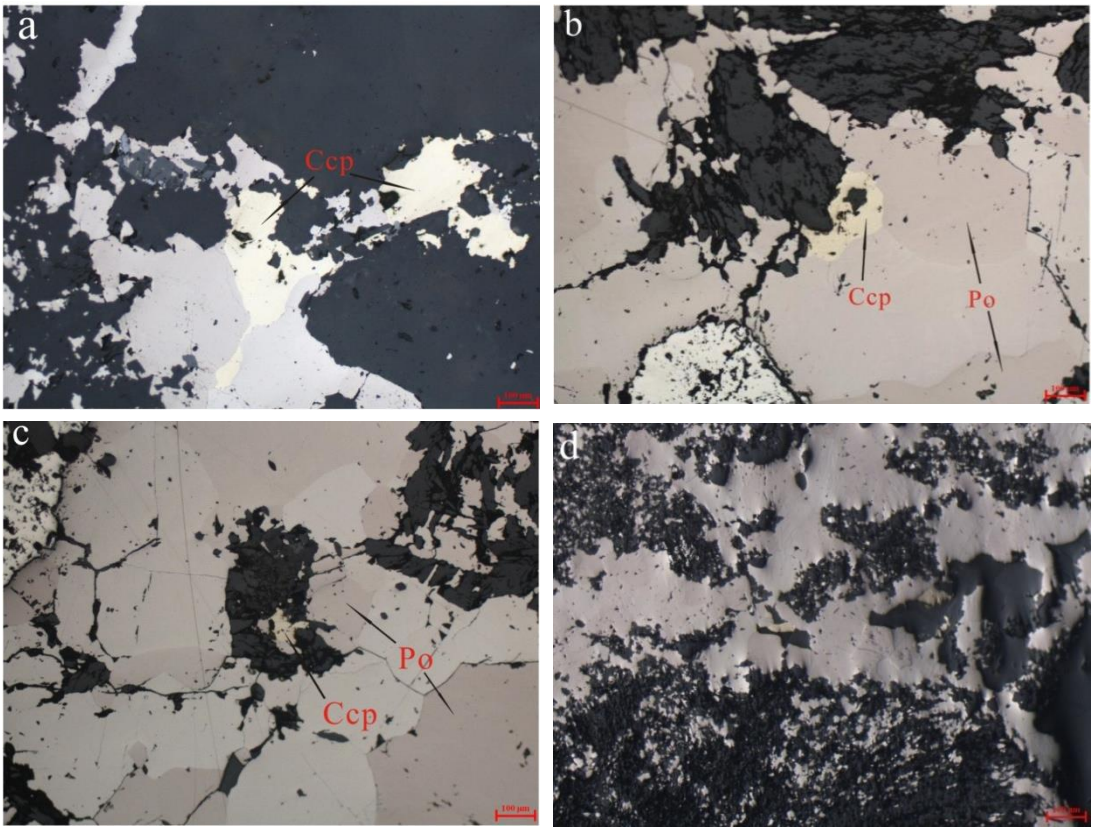

Fig. 16.

937 **Table 1**

Variables	Residual
Mean	0.000
Variance	0.016
Standard Deviation	0.127

938 **Table 2**

Mineralized zones	Thresholds (Cu%)	Range (Cu%)
Barren host rock and weakly mineralized		<0.28
Moderately mineralized	0.28	0.28-1.45
Highly mineralized	1.45	>1.45

939 **Table 3**

Mineralized zones	Thresholds (Cu%)	Range (Cu%)
Barren host rock		<0.25
Weakly mineralized	0.25	0.25–1.48
Moderately mineralized	1.48	1.48–1.88
Highly mineralized	1.88	>1.88

940 **Table 4**

Mineralized zones	PS threshold	Range of PS	Range (Cu%)
leached zone and barren host rock		<7.81	<0.23
hypogene zones	7.81	7.81-8.70	0.23-1.33
supergene enrichment zones	8.70	>8.70	>1.33

941 **Table 5**

		Geological model	
		Inside zone	Outside zone
Fractal model	Inside zone	True positive (A)	False positive (B)
	Outside zone	False negative (C)	True negative (D)
		TypeIerror=C/(A+C)	TypeIIerror=B/(B+D)
		Overallaccuracy=(A+D)/(A+B+C+D)	

942 **Table 6**

		Potassic alteration of geological model	
		Inside zones	Outside zones
C–V fractal model of highly mineralized zones	Inside zones	A 2850	B 1360
	Outside zones	C 77927	D 76913
		T1E 0.96	T2E 0.02
		OA	0.50
N–S fractal model of highly mineralized zones	Inside zones	A 3092	B 1570
	Outside zones	C 75025	D 75473
		T1E 0.96	T2E 0.02
		OA	0.51
S–V fractal model of supergene enrichment zones	Inside zones	A 4431	B 2318
	Outside zones	C 72985	D 75726
		T1E 0.94	T2E 0.03
		OA	0.52

943 **Table 7**

		Phyllic alteration of geological model	
		Inside zones	Outside zones
C–V fractal model of moderately and weakly mineralized zones	Inside zones	A 36518	B 48027
	Outside zones	C 25461	D 69155
		T1E 0.41	T2E 0.40
		OA	0.59
N–S fractal model of moderately mineralized zones	Inside zones	A 35555	B 46943
	Outside zones	C 23955	D 48223
		T1E 0.40	T2E 0.49
		OA	0.54
S–V fractal model of the hypogene zones	Inside zones	A 40080	B 44943
	Outside zones	C 26899	D 54239
		T1E 0.40	T2E 0.45
		OA	0.56

944 **Table 8**

Sample no.	Mineralized zones obtained by fractal models	Cu (%)
PL-B74	Weakly mineralized zones	0.41
PL-B62	Moderately mineralized zones	1.32
PL-B82	Highly mineralized zones	1.80

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