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Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "Data-driven versus self-similar parameterizations for Stochastic Advection by Lie Transport and Location Uncertainty" *by* Valentin Resseguier et al.

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Answer to reviewer #1

First of all, we would like to express our warm thanks to the reviewers and the editor for the evaluation work they did and for all the comments and suggestions the reviewers have provided on our study. Hereafter, we will answer in details the questions of the referee #1. Accordingly, we will propose some changes that we could do in a revised version, if the editor enables a revised submission.

"In this study, the authors focused on the common challenge of the stochastic subgrid parameterization schemes: the unresolved velocity construction. Two kinds of parameterizations, data-driven and self-similar parameterizations, were applied to LU and SALT frameworks. The results show that these two parameterizations can lead to high quality ensemble forecasts. In my opinion, the main innovation of this study is the proposal of the self-similar parameterization, which improves the work of Resseguier et al. (2017b)."

Thank you for this summary of our work.

"Although this manuscript may be suitable for publication in NPG, there are still some issues to be addressed."

1. **"Recently, many parameterizations are available. In this study, the authors proposed a new self-similar parameterization. I know that its advantage is tuning-free. However, more interestingly, when this parameterization is used to the numerical models, whether the improvements of the simulations or forecast are significantly enhanced, comparing to other parameterizations, especially for the one of Resseguier et al. (2017b)."**

   For stationary, fully-developed turbulence and after including a tuning stage to optimize the match, the self-similar parameterization and the method of Resseguier et al. (2017b) give approximately the same results. Therefore, in order to appreciate improvements of the simulations or of the forecasts, we must either work with non-stationary flows or with erroneous tuning. This latter scenario is of course the meaningful one for the application of such models to the real world, where turbulence is non-stationary, heterogeneous, and so region-by-region and season-by-season tuning is impossible to do with quality checks in place.

   We believe that non-stationarity may yield a fair, yet idealized, comparison. So, in a revised manuscript, we propose to compare the two parameterizations in a non-stationary case. The figure 1 below shows simulated buoyancy fields initialized with the “case 2” of Constantin et al. (1994) and corresponding errors. After two days of advection, there is no turbulence yet, and a 128x128 resolution is sufficient to correctly resolve every scale. Therefore, no stochastic subgrid parameterization is needed. The self-similar method automatically adapts to the situation, whereas the method of Resseguier et al. (2017b) introduces spurious buoyancy isolines roughness by randomly folding these isolines. Accordingly, the method of Resseguier et al. (2017b) introduces more errors.

2. **“Figure 7 shows that the patterns obtained by the data-driven and self-similar parameterizations are similar to that in the Low-resolution deterministic SQG model at day 110. This means that, for the short-term simulations, the stochastic subgrid parameterizations have very weak improvements on the low-resolution simulations?”**

   Yes, this is right. For the short-term simulations, our stochastic subgrid parameterizations have often weak improvements on the low-resolution simulations, even though, sometimes, the stochastic subgrid parameterization can improve the simulation. Indeed, Resseguier et al. (2017b) show that the LU dynamics at a resolution 128x128 can trigger filament instabilities by random destabilization, and hence obtain a more realistic proportion of eddies and filaments. This is confirmed by the figures 2 and 3 of our submitted draft, also at a resolution 128x128. In figure 7, the resolution is coarser (64x64). Therefore, the stabilizing deterministic subgrid tensor (hyper viscosity) is stronger. This may explain an inhibition of filament instabilities here, and hence less difference between deterministic and stochastic coarse simulations.

   We could add this discussion to the subsection "3.3 One realization" in a revised version of our draft.

   Nevertheless, our main goal is not improving a single simulation. Our main goal is improving the uncertainty quantification without deteriorating single simulations.

3. **“The authors tested the two parameterizations in the SQG model. This model is very simple. Please discuss how to apply these parameterizations to the complicated atmosphere and ocean models.”**

   It is true that SQG is a simple model, in particular because the vertical variations are described by analytic formulas and hence do not need to be simulated. The reason for the selection of this model was that it occurs identically in the SALT and LU formulations, not that this is an especially realistic model. For more complicated atmosphere and ocean models, such as hydrostatic Boussinesq models, simplicity is not to be found. The vertical dimension imposes anisotropy and heterogeneity of the small-scale velocity, at least breaking symmetric between the vertical and horizontal directions. Thus anisotropy and heterogeneity need to be parameterized–in the SQG system the anisotropy and vertical heterogeneity is prescribed. Moreover, complex boundary conditions and heterogeneity often prevent the direct use of Fourier transform. We develop these two points in the following.

   (a) Third dimension, anisotropy and vertical heterogeneity :
   In order to impose the divergence-free condition ($\nabla \cdot \dot{B} = 0$) in 3D, one can build the small-scale velocity $\boldsymbol{\sigma}\dot{B}$ from a 3D curl $\nabla_{3D}\times$ and a 3D stream-function $\phi_\sigma \dot{B}$:

$$\boldsymbol{\sigma}\dot{\boldsymbol{B}} = \nabla_{3D} \times \left(\phi_\sigma \dot{\boldsymbol{B}} \right). \tag{1}$$

   This streamfunction can be filter along z in order to impose a given finite vertical correlation length H_σ related to the vertical resolution (e.g. $H_\sigma = 2\Delta z$) :

$$\phi_\sigma \dot{\boldsymbol{B}}(x,y,z) = \breve{\phi}_\sigma^z(z) *_z \left(\left(\phi_\sigma^{2D} \dot{\boldsymbol{B}} \right)(x,y,z) \right), \tag{2}$$

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Discussion paper

with $*_z$ a one-dimensional convolution along z and $\breve{\phi}_\sigma^z$ a one-dimensional vertical Gaussian filter:

$$\breve{\phi}_\sigma^z(z) = (2\pi)^{1/4} H_\sigma^{1/2} \exp\left(-\frac{z^2}{H_\sigma^2}\right). \tag{3}$$

For simplicity, the 3 components of the horizontal streamfunction $\phi_\sigma^{2D}\dot{\boldsymbol{B}}$ can be assumed to be statistically independent. Each of these z-dependent component can be defined – at each depth z independently – similarly to the horizontal streamfunction of our draft:

$$\left(\phi_\sigma^{2D}\dot{B}\right)_k (x,y,z) = \breve{\phi}_\sigma^{2D}(x,y,z) *_{(x,y)} \dot{\boldsymbol{B}}_k(x,y,z), \tag{4}$$

for $1 \leqslant k \leqslant 3$, where $*_{(x,y)}$ is a horizontal two-dimensional convolution, $\dot{\boldsymbol{B}}$ is 3D spatio-temporal white noise, $\breve{\phi}_\sigma^{2D}$ is a scalar horizontal spatial filter. If Fourier transforms are possible, that filter can be defined as in our draft at each depth z from the horizontal large-scale resolved velocity statistics. Otherwise, that filter can be a 2D Matérn covariance (Williams and Rasmussen, 2006; Lim and Teo, 2009; Lilly et al., 2017) of a correlation length $2\pi/\kappa_m$ related to the horizontal resolution (e.g. $\pi/\kappa_m = 2\sqrt{\Delta x \Delta y}$) and a regularity parameter $(r(z)+1)/2$ learned from the horizontal large-scale resolved velocity. Indeed, with such a parameterization, for large horizontal wave number $\kappa^{2D} = \sqrt{k_x^2 + k_y^2}$, the horizontal ADSD is a smooth-along-z version of :

$$cst. \left(\frac{\kappa^{2D}}{\kappa_m}\right)^{-r(z)}. \tag{5}$$

Therefore, self-similar arguments can be still be used to identify the regularity $(r(z)+1)/2$ at large and small scales.

(b) Boundary conditions and heterogeneity :
Non-periodic boundary conditions suggest small-scale velocity heterogeneity, at least in the variance. For instance, Dirichlet boundary conditions for the velocity suggest that the small-scale velocity variance should be zero on the boundary. Since this variance is non-zero inside the domain, there are variance heterogeneity.

A first solution is to ignore the small-scale velocity boundary conditions, as in (Chapron et al., 2018). Indeed, since that velocity appears only in a multiplicative way, the small-scale velocity homogeneity will be modulated by the transported resolved fields heterogeneity. Therefore, the erroneous small-scale velocity homogeneity is expected to have only weak impact.

If one wants to impose small-scale boundary conditions anyway, the homogeneous small-scale velocity $v' = \boldsymbol{\sigma}\dot{\boldsymbol{B}}$ – defined above – can be "conditioned on" the values of the boundary conditions. As an example, we can consider a one-dimensional process $v'(x)$, with an (unconditioned) covariance $\gamma_{v'}$ and a correlation length (small compared to the domain size L). A velocity v'_{BC} which respect the boundary conditions $v'_{BC}(0) = v'_0$ and $v'_{BC}(L) = v'_L$ is the conditioning of velocity v' on the boundary conditions. That conditioned velocity can be simulated as:

$$v'_{BC}(x) = v'(x) - \frac{\gamma_{v'}(x-0)}{\gamma_{v'}(0)}\left(v'(0) - v'_0\right) - \frac{\gamma_{v'}(x-L)}{\gamma_{v'}(0)}\left(v'(L) - v'_L\right). \quad (6)$$

Nevertheless, we believe that the above discussion is overly complex and not mature enough to add it to our draft. Applying these parameterizations to the complicated atmosphere and ocean models would require future works.

4. **“In this study, the term “SALT-LU” appears frequently. In my opinion, this term may mislead readers. They can think that the authors aimed at combining the SALT and LU parameterizations.”**

In a revised version, we propose to use instead either only "LU" or "SALT and LU", depending on the readability of the sentence.

5. **“Lines 47 and 51. in (Gay-Balmaz and Holm, 2018) → in Gay-Balmaz and Holm (2018); in (Cotter et al., 2018b, a) → in Cotter et al. (2018b, a)"**

   We will correct this.

6. **"Line 126. Two vertical lines were not plotted in the left panel of figure 1."**

   We will correct this.

7. **"Line 401. Similar results UQ results → Similar UQ results?"**

   This is also a typo and we will correct it.

We thank again the reviewer for all these useful comments and questions.

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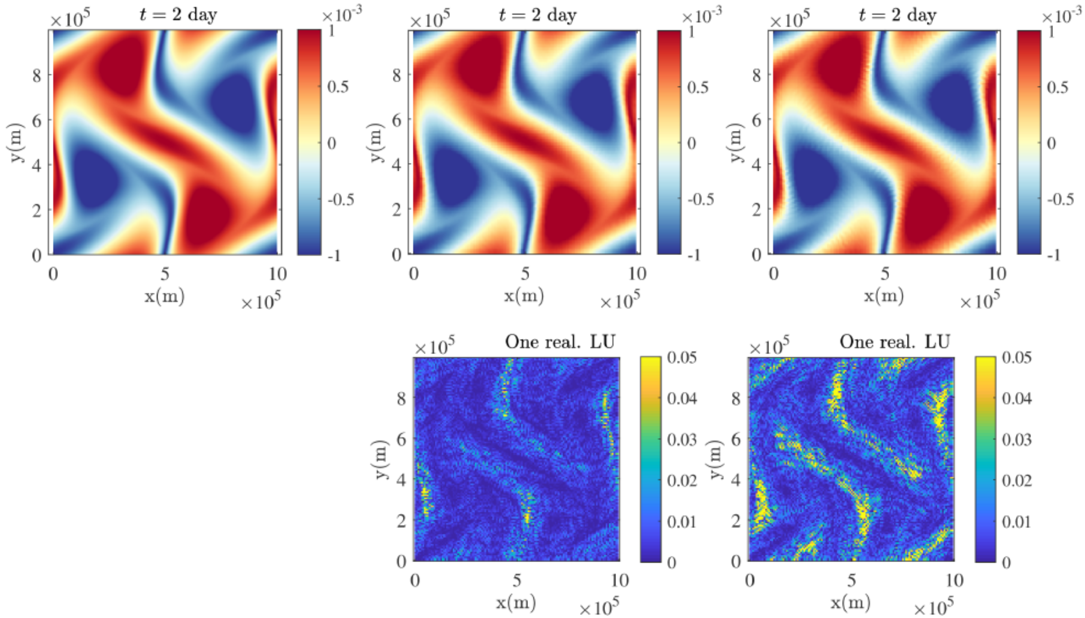

Figure 1: Buoyancy field ($m.s^{-2}$) (top) and corresponding normalized error (dimensionless) (bottom) after one day of advection for the 1024x1024 reference deterministic simulation (left), the 128x128 LU simulation with self-similar parameterization (middle), the 128x128 LU simulation with the method of Resseguier et al. (2017) (right).

Fig. 1.