Residence Time of Energy in the Atmosphere

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Abstract. In atmospheric chemistry, a parameter called residence time is defined for each gas as $T = M/F$, where M represents the mass of the gas in the atmosphere and F is the total average influx or outflux, which in time averages are equal. In this letter we extend this concept from matter to energy which is also a conservative quantity and estimate the average residence time of energy in the atmosphere which amounts to about 58 days. A similar estimation for the residence time of energy in the Sun is of the order of $10^7\,\mathrm{yr}$, which agrees with the Kelvin-Helmholtz time scale.

1 Introduction

When the inflow, F, of any substance into a box is equal to the outflow, then the amount of that substance in the box, M, is constant. This constitutes an equilibrium or steady state. Then the ratio of the stock in the box to the flow rate (in or out) is called residence time and is a time scale for the transport of the substance in the box

$$\tau = \frac{M}{F}. \tag{1}$$

We are referring to a substance measurable and conserved. A good example of this type is the parameter defined in atmospheric chemistry as the average residence time of each individual gas, defined as Eq. (1). M is the total average mass of that gas in the atmosphere and F the total average influx or outflux, which in time averages for the whole atmosphere are equal. See, for example (Hobbs, 2000).

In this letter we want to extend the substance that flows from matter to energy, and estimate the average residence time of energy in the atmosphere. At the end of this letter we will briefly analyze this concept for the Sun. Obviously in Eq. (1), M and F will now represent the total amount of energy in these two systems and the energy flux -in or out- respectively.

Both cases correspond to steady state problems because the storage of energy in the Earth's atmosphere and in the Sun are not systematically increasing or decreasing.

In Section 2, we consider the Earth's atmosphere as a big box and using the appropriate energy data in Eq. (1) we compute the time of residence.

In Section 3, we estimate the residence time of energy in the Sun and note that this residence time agrees with the Kelvin-Helmholtz (K-H) time scale. In Section 4 we present a conclusion.

2 Forms of energy in the atmosphere and time of residence.

In this section we will use the energy data provided by Hartmann (1994). The most important forms of energy in the atmosphere are: the thermodynamic internal energy, U, the potential energy due to Earth's gravity, P, the kinetic energy, K, the latent energy, L, related to the phase transitions of water and E, the total energy. The values quoted by this author of energy per unit surface in units of $10^6\,\mathrm{J\,m^{-2}}$ are:

$$U = 1800,\ P = 700,\ K = 1.3,\ L = 70,\ E = 2571 \tag{2}$$

For our purpose of computing the time of residence using Eq.(1), now we only need the inputs, and outputs, of energy in the atmosphere. It is common to express them in units of $e = 3.42\,\mathrm{W\,m^{-2}}$, where e is a percentage of the Solar Irradiance out of the atmosphere. The atmosphere absorbs $20\,e$ of solar energy, $29\,e$ are absorbed at the surface as sensible and latent heats, and finally it absorbs $100\,e$ as long-wave radiation emitted by surface (the radiation emitted by surface is $110\,e$, but $10\,e$ escapes to the space using the so-called atmospheric window). Thus the total energy input in the atmosphere is:

$$F_i = 20e + 29e + 100e = 149e = 509.6\,\mathrm{W\,m^{-2}}. \tag{3}$$

Regarding to the emitted energy flux, we identify two terms: the component emitted spaceward, $60\,e$, and that emitted to the surface, commonly denoted as the greenhouse effect, $89\,e$.

We have not considered any anthropogenic contribution because it is negligible compared with the fluxes of solar origin mentioned above. In recent years, the consumption of fossil fuels has been about 10 Gtoe per year; this implies an energy flux of $0.08\,\mathrm{W\,m^{-2}}$ (Houghton, 2004). Then, the sum of the outgoing terms coincides with that of the ingoing terms, $F_o = F_i = F$. Thus, using these values of E and F, the estimation of the residence time of energy in our atmosphere, t_a is:

$$t_a = \frac{E}{F} = \frac{2571 \times 10^6 \,\mathrm{J\,m^{-2}}}{509.6\,\mathrm{W\,m^{-2}}} = 5.05 \times 10^6 \,\mathrm{s} \approx 58\,\mathrm{days}. \quad (4)$$

3 Estimation of the solar energy and the time of residence of energy in the Sun.

In stars like the Sun, the total energy, E, is the sum of the gravitational energy, E_g, and the thermal energy, E_t,

$$E = E_g + E_t. \quad (5)$$

The Virial theorem (Kippenhan and Weigert, 1994) links the two energy reservoirs:

$$-E_g = 2E_t. \quad (6)$$

Therefore

$$E = \frac{1}{2}E_g. \quad (7)$$

Using the data from (Zombeck, 1990), the Sun's gravitational energy can be easily estimated

$$E_g \approx -\frac{GM_\odot^2}{2R_\odot} = -1.89 \times 10^{41}\,\mathrm{J}. \quad (8)$$

Inserting (8) into (7) we obtain

$$\|E\| = 9.5 \times 10^{40}\,\mathrm{J}. \quad (9)$$

The ratio between $\|E\|$ and the solar luminosity, L (3.9×10^{26} W), which constitutes the outgoing energy flux, is our estimation for the energy residence time in the Sun, $t_\odot$

$$t_\odot = \frac{\|E\|}{L} \approx 2.6 \times 10^{14}\,\mathrm{s} \approx 0.83 \times 10^7 \mathrm{yr}. \quad (10)$$

The Kelvin-Helmholtz (K-H) time scale, for the Sun is:

$$t_{KH} \approx \frac{GM_\odot^2}{R_\odot L}\,\mathrm{yr} = 3 \times 10^7 \mathrm{yr} \approx \frac{E_g}{L}. \quad (11)$$

which is of the order of magnitude of the residence time of energy in the Sun $t_\odot$.

This time scale roughly predicts the time needed by the star to settle to equilibrium after a global thermal perturbation (Kippenhan and Weigert, 1994), (Spruit, 2000) and (Stix, 2003) [1].

4 Conclusion

In this letter, we have considered our atmosphere as a big box where energy is in equilibrium, and have estimated its residence time. It amounts to about 58 days. When the same idea is applied to the Sun, we obtain $t \approx 0.83 \times 10^7 \,\mathrm{yr}$.

In astrophysics, the question: "how long a photon might take to get from the core of the Sun to the surface" has been frequently put forward. The answer of several authors was $\approx 10^4 \,\mathrm{yr}$, see (Shu, 1982), (Bahcall, 1989), etc. In 1989, Mihalas and Sills (1992) pointed out that the average step length assumed for a photon diffusing through the Sun by the previous authors was too long. Correcting this step length, they obtained $1.7 \times 10^5 \,\mathrm{yr}$. Finally, Stix (2003), invoking the large heat capacity of the interior of the star, corrected the previous result up to a time scale of the order of $10^7 \,\mathrm{yr}$. Thus, this author showed the agreement between the thermal adjustment time scale and the photon diffusion time scale.

Bearing in mind what has been said, our conclusion for the residence time of energy in Earth's atmosphere ($t \approx 58$ days) is that it is the equivalent of what the K-H time scale is for the Sun. Therefore, after a global thermal perturbation, the atmosphere would need about a couple of months to come back to a new equilibrium.

References

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