Residence Time of Energy in the Atmosphere

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Abstract. In atmospheric chemistry, a parameter called residence time is defined for each gas as $T = M/F$, where M represents the mass of the gas in the atmosphere and F is the total average influx or outflux, which in time averages are equal. In this letter we extend this concept from matter to energy which is also a conservative quantity and estimate the average residence time of energy in the atmosphere which amounts to about ~~56~~ 58 days. A similar estimation for the residence time of energy in the Sun is of the order of $10^7\,\mathrm{yr}$, which agrees with the Kelvin-Helmholtz time scale.

In this version, we have introduced new paragraphs and references and omitted some others that we considered unnecessary. The modifications have been marked according to the norms of the journal. With these modifications we have tried to meet the criticism raised by the two referees, which we sincerely acknowledge. In our opinion, the new version has gained in clarity for the reader.

Several English corrections suggested by Referee 2 have been introduced in the new version of the paper.

1 Introduction

After computing the Residence Time of Energy in the Atmosphere (about two months) we were somewhat doubtful about its interpretation. In this sense, the same computation in the Sun was useful because we found that what we called time of residence of energy in the Sun (10^7 yr), solar physicists called it Kelvin-Helmholtz time scale. And this time scale represents the time needed by the Sun to settle in a new equilibrium after a global thermal perturbation (Kippenhan and Weigert 1994, Spruit 2000, Stix 2003).

Referee 2 suggested to derive the concept of residence time from the continuity equation of fluid mechanics. In our opinion, this simple concept can be maintained as we have introduced it in the paper, since textbooks introduce it similarly.

When the inflow, F, of any substance into a box is equal to the outflow, then the amount of that substance in the box,M, is constant. This constitutes an equilibrium or steady state. Then the ratio of the stock in the box to the flow rate (in or out) is called residence time and is a time scale for the transport of the substance in the box

$$\cancel{t}\,T = \frac{M}{F}. \tag{1}$$

We are referring to a substance measurable and conserved. A good example of this type is the parameter defined in atmospheric chemistry as the average residence time of each individual gas, defined as Eq. (1). M is the total average mass of that gas in the atmosphere and F the total average influx or outflux, which in time averages for the whole atmosphere are equal. See, for example (Hobbs, 2000).

An alternative way of considering Eq. (1), (t=M/F), was suggested by Referee 2. It is related to the first interpretation received by the K-H time scale. The K-H scale was originally proposed as an estimation of the life time of the Sun. This would correspond to interpreting t in Eq. (1) as the time of depletion of an amount of energy M of the box.

In this letter we want to extend the substance that flows from matter to energy, and estimate the average residence time of energy in the atmosphere. At the end of this letter we will briefly analyze this concept for the Sun. Obviously in Eq. (1), M and F will now represent the total amount of energy in these two systems and the energy flux -in or out- respectively.

Both cases correspond to steady state problems because the storage of energy in the Earth´s atmosphere and in the Sun are not systematically increasing or decreasing.

In Section 2, we consider the Earth´s atmosphere as a big box and using the appropriate energy data in Eq. (1) we compute the time of residence.

In Section 3, we estimate the residence time of energy in the Sun and note that this residence time agrees with the Kelvin-Helmholtz (K-H) time scale. In Section 4 we present a ~~brief discussion~~conclusion.

2 Forms of energy in the atmosphere and time of residence.

One of the modifications was suggested by Referee 1. In the first Version, the data of energy in the atmosphere were taken from Peixoto & Oort (ref. 4 in the first version). However, in that reference, the flux of infrared radiation from the surface to the atmosphere IR(S->A), and its inverse IR(A->S), are not explicitly specified. These authors only quote the difference of these two fluxes. For our computation of the Residence Time of Energy in the Atmosphere we need to know the individual value of these IR fluxes. Thus, in the new section 2, we have moved to Hartmann's (1994) as the reference that provides both the data of atmospheric energy and the data of energy inflow and outflow in the atmosphere.

Referee 1 also raised the following point: in section 2, why had we omitted the anthropogenic contribution to the rest of fluxes? The reason is that this contribution is very small. Using data cited by Houghton we illustrate this question.

In this section we will use the energy data provided by ~~?~~Hartmann (1994). The most important forms of energy in the atmosphere are: the thermodynamic internal energy, U, the potential energy due to Earth´s gravity, P, the kinetic energy, K, ~~and~~ the latent energy, L, related to the phase transitions of water and E, the total energy. The values quoted by ~~these authors~~ this author of energy per unit surface in units of ~~$10^7\,\mathrm{J\,m^{-2}}$~~ $10^6\,\mathrm{J\,m^{-2}}$ are:

$$U = 180.311800, \quad P = 69.3700, \quad K = 0.1231.3, \quad L = 6.3870, \quad E = 256.12571 \tag{2}$$

For our purpose of computing the time of residence using Eq.(1), now we only need the inputs, and outputs, of energy in the atmosphere. ~~For this aim, we will use the data cited by ?.~~ It is common to express them in units of ~~$e = 3.45\,\mathrm{W\,m^{-2}}$~~ $e = 3.42\,\mathrm{W\,m^{-2}}$, where e is a percentage of the Solar Irradiance out of the atmosphere. The atmosphere absorbs ~~$25\,e$~~ $20\,e$ of solar energy, $29\,e$ are absorbed at the surface as sensible and latent heats, and finally it absorbs $100\,e$ as long-wave radiation emitted by surface (the radiation emitted by surface is ~~$104\,e$, but $4\,e$~~ $110\,e$, but $10\,e$ escapes to the space using the ~~called~~ so-called atmospheric window). Thus the total energy input in the atmosphere is:

$$F_i = 25e20e + 29e + 100e = 154e149e = 531509.6\,\mathrm{W\,m^{-2}}. \tag{3}$$

Regarding to the emitted energy flux, we identify two terms: the component emitted ~~towards the space, $66\,e$~~ spaceward, $60\,e$, and that emitted to the surface, commonly denoted ~~by~~ as the greenhouse effect, ~~$88\,e$~~ $89\,e$.

The sum of the outgoing terms coincides with that of the ingoing terms, $F_o = F_i = F$. Thus, using these values of E and F, the estimation of the residence time of energy in our atmosphere, t_a is:

$$t_a = \frac{E}{F} = \frac{256 \times 10^7\,\mathrm{J\,m^{-2}}}{531\,\mathrm{W\,m^{-2}}} = 4.82 \times 10^6\,\mathrm{s} \approx 56\,\mathrm{days}\frac{2571 \times 10^6\,\mathrm{J\,m^{-2}}}{509.6\,\mathrm{W\,m^{-2}}} = 5.05 \times 10^6\,\mathrm{s} \approx 58\,\mathrm{days} \tag{4}$$

We have not considered any anthropogenic contribution because it is negligible compared with the fluxes of solar origin mentioned above. In recent years, the consumption of fossil fuels has been about 10 Gtoe per year; this implies an energy flux of $0.08\,\mathrm{W\,m^{-2}}$ (Houghton, 2004)

3 Estimation of the solar energy and the time of residence of energy in the Sun.

In the paper we quote that in solar physics, Stix (2003) showed the agreement between the thermal adjustment time scale and the photon diffusion time scale, and then Referee 1 asked us to do a similar job in the atmosphere. We reply that we will try to do it shortly, but definitely this would be the matter of other paper. It is worth to notice that Stix's paper (2003) appeared more than 14 years after Bahcall's paper (1989).

In stars like the Sun, the total energy, E, is the sum of the gravitational energy, E_g, and the thermal energy, E_t,

$$E = E_g + E_t. \tag{5}$$

~~And the Virial theorem (Kippenhan and Weigert, 1994) links these~~ The Virial theorem (Kippenhan and Weigert, 1994) links the two energy reservoirs:

$$-E_g = 2E_t. \tag{6}$$

Therefore

$$E = \frac{1}{2}E_g. \tag{7}$$

~~But~~ Using the data from (Zombeck, 1990), the Sun's gravitational energy can be easily estimated

$$E_g \approx -\frac{GM_\odot^2}{2R_\odot} = -1.89 \times 10^{41}\,\mathrm{J}. \tag{8}$$

Inserting (8) into (7) we obtain

$$\|E\| = 9.5 \times 10^{40}\,\mathrm{J}. \tag{9}$$

The ratio between $\|E\|$ and the solar luminosity, L (3.9×10^{26} W), which constitutes the energy outgoing flux, is our estimation for the energy residence time in the Sun, $t_\odot$

$$t_\odot = \frac{\|E\|}{L} \approx 2.6 \times 10^{14}\,\mathrm{s} \approx 0.83 \times 10^7\mathrm{yr}. \tag{10}$$

The Kelvin-Helmholtz (K-H) time scale, ~~see for example (Kippenhan and Weigert, 1994),~~ for the Sun is:

$$t_{KH} \approx \frac{GM_\odot^2}{R_\odot L}\,\mathrm{yr} = 3 \times 10^7\mathrm{yr} \approx \frac{E_g}{L}. \tag{11}$$

which is of the order of magnitude of the residence time of energy in the Sun $t_\odot$.

This time scale roughly predicts the time needed by the star to settle to equilibrium after a global thermal perturbation ~~. As the~~ (Kippenhan and Weigert, 1994), (Spruit, 2000) and (Stix, 2003).

The K-H ~~time scale is nothing but~~

$$\cancel{t_{KH} \approx \frac{\|E_g\|}{L},}$$

~~the time of residence of energy computed for the Sun~~scale was originally, proposed as an estimation of the life time of the Sun. This would correspond to interpreting τ in Eq. (~~10) is basically the same thing that the old K-H time scale. Stix in (Stix, 2003), for the time scale of energy transport in the Sun, also estimated 3×10^7yr as the correct order of magnitude.~~ 1) as the time of depletion of an amount of energy M of the box.

4 ~~Final discussion~~Conclusion

With respect to the comment #5 of Referee 2 about the amount of time needed to return to the equilibrium after a global thermal perturbation, we reply that the time of residence of energy in the Sun is the time needed to settle to equilibrium after a global thermal perturbation; we recall the numerical experiment that Stix reports in his paper (Stix 2003). In a model of the Sun's structure, he increased the proton-proton cross section in a 2% and ran the model until a new equilibrium state was reached. The time scale elapsed was about 10^7 yr.

In this letter, we have considered our atmosphere as a big box where energy is in equilibrium, and have estimated its residence time. It amounts to about ~~56~~ 58 days. When the same idea is applied to the Sun, we obtain ~~$t \approx 0.87 \times 10^7$ yr~~ $t \approx 0.83 \times 10^7$ yr .

In astrophysics, the question: "how long a photon might take to get from the core of the Sun to the surface" has been frequently put forward. The answer of several authors was $\approx 10^4$ yr, see (Shu, 1982), (Bahcall, 1989), etc. In 1989, ~~?~~ Mihalas and Sills (1992) pointed out that the average step length assumed for a photon diffusing through the Sun by the previous authors was too long. Correcting this step length, they obtained ~~1.7 10^5 yr~~ 1.7×10^5 yr. Finally, Stix (2003), invoking the large heat capacity of the interior of the star, corrected the previous result up to a time scale of the order of 10^7 yr. Thus, this author showed the agreement between the thermal adjustment time scale and the photon diffusion time scale.

Bearing in mind what ~~is saidin Section 3~~has been said, our conclusion for the residence time of energy in Earth´s atmosphere (~~$t \approx 56$~~ $t \approx 58$ days) is that it is the equivalent of what the K-H time scale is for the Sun. Therefore, after a global thermal perturbation, the atmosphere would need about a couple of months to come back to a new equilibrium.

Data availability. The data used for the estimation of residence time in the Earth's atmosphere were extracted from (Hartmann, 1994). The data used in the estimation of the residence time in the Sun were obtained from (Zombeck, 1990).

Author contributions. Amalio Fernández-Pacheco conceived the idea; Carlos Osácar and Amalio Fernádez-Pacheco wrote the paper; Manuel Membrado contributed in the solar part of the letter.

Competing interests. The authors declare no conflict of interests.

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