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Nonlinear Processes
in Geophysics
Discussions

Interactive comment on "Application of local attractor dimension to reduced space strongly coupled data assimilation for chaotic multiscale systems" *by* Courtney Quinn et al.

Anonymous Referee #1

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General comment

This manuscript is exploring the use of covariant Lyapunov vectors (CLVs) to build the error covariance matrix in ensemble Kalman filtring methods. The set of vectors is selected based on the computation of a local Kaplan-Yorke dimension based on the finite time Lyapunov exponents. This approach is implemented in the context of a muli-scale system mimicking the (coupled) dynamics of a coupled tropical ocean-atmosphere system and the extra-tropical atmosphere. Different strategies of observation are then evaluated. It is found that observation within the atmosphere is essential, and that the variable number of CLVs to be used in building the error covariance matrix

is a successful strategy for strongly coupled data assimilation. Very interesting results are also obtained with the observation sampling of a shadowing trajectory, leading to measurement correlations.

This is an interesting manuscript exploring many aspects of the strongly coupled data assimilation and I would in principle recommend publication of this work. I have however an important concern on the use of the local Kaplan Yorke dimension and the CLVs that should be addressed before publication. It seems to me that the use of both is inconsistent. Let me clarify my point. When computing the FTLEs, one can use either the QR (associated with the backward Lyapunov vectors) decomposition, the Forward Lyapunov vectors obtained with backward integration in time, or the local amplification along the CLVs. Although all these are giving the appropriate asymptotic Lyapunov exponents, they are not providing similar variability of these quantities as illustrated for instance in Vannitsem and Lucarini (2016) you quoted. So if you use the QR decomposition and then select the CLVs on that basis this is probably not optimal.

Alternatively, if you use an estimate of the FTLE using the amplification along the CLVs then the "dimension" of the subspace of instabilities is not the same and one can wonder what is the signification of this quantity. This is related to your comment at line 14 of page 8 indicating that higher dimension is associated with more important alignment of CLVs. Imagine for instance that several CLVs are pointing in almost the same direction with large amplifications, then the dimension would be large but intuitively (as they point all in the same direction) we would expect a low dimension. The specific way you compute the FTLEs and the local dimension should therefore be clarified, and probably I would not call it "dimension". Furthermore, if the KY dimension is computed with the local amplification rates of the backward Lyapunov exponents, a comparison should be made with the use of the backward Lyapunov vectors in building the error covariance matrix. It would have been my first choice in view of the fact this is much less costly than the CLVs.

More specific comments

1. Figure 3, you mentioned 2 neutral Lyapunov exponents. I am wondering why you have two such exponents. Is there any specific symmetries allowing for that? Is'nt it a numerical artifact?

2. Table 2. The RMSE for the extratropics are very close to each other whatever the experiments. Are the differences significant?

3. Also in Table 2. An average dimension is computed. This average is based on the QR decomposition or some estimate with the CLVs? This is related to my main point. Please clarify how it is computed.

4. FTLEs are computed for 4 time units. What is happening to your analysis when this window is changed? And why choosing this specific window?

5. At page 12, in the two first paragraphs of Section 5, you present how the experiment is done. As far as I understood, the CLVs are computed during a limited period of time during the assimilation period. Am I right? At first reading it was not very clear to me and it would be nice to improve the presentation of that part. In particular, a sketch of the whole process in a figure would be really useful.

6. In the partially observed CDA, you also compute a local dimension. I am wondering how the CLVs are computed there since the trajectory of the model is probably very far from reality. Moreover, it was not clear to me whether you are using the CLVs of the reality or of the model integration. Would you please clarify how you do this? It can maybe be incorporated in the general description of the experiments mentioned at point 5 above.

Minor points

1. Line 7, page 11. Please modify the notation on the brackets. It looks like a vertical rectangle.

2. Line 20, page 11. I suppose that ïĄň should be larger than 1.

3. Line 22, page 11. Please do not use the terminology “model error”. It is confusing as model error is used when there is structural uncertainty in the model.

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