Interactive comment on "Application of Levy Processes in Modelling (Geodetic) Time Series With Mixed Spectra" *by* Jean-Philippe Montillet et al.

Anonymous Referee #1

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The paper discusses using Gaussian and non-Gaussian processes for geodetic time-series. The topic is very nice and timely, as there is a wide interest in community for this kind of processes, and hence all contributions are welcome. The current paper has a literature review on the models, and builds up a case study on specific synthetic and real datasets.

The drawback of the paper is that it does not provide a solid mathematical framework on which the time-series analysis is carried out. In my experience, this kind of papers should always have four key elements: Statistical model, estimation algorithm, synthetic examples and real data examples. Unfortunately the statistical model is not

properly build, and several mathematical errors are in the text. These could be alleviated by writing out formulas explicitly, so that the authors and readers could understand the model. The major issues of the paper are:

1) There are repeated fundamental mathematical mistakes and misunderstandings in the paper, including claim that wide sense stationary can have a temporally evolving mean, and that Brownian motion is stationary process (or that's at least how the reader understands the text). As the Lévy processes are mathematically extremely technical, the simple Gaussian process definitions cannot have such fundamental flaws. For example, random variable is used in many places where stochastic process should be used – Stochastic processes are collections of random variables.

2) For the parameter estimation algorithm, a 'Hector software' is used. We need to have more details than just the name of software and citation. We need to know whether this is optimisation or stochastic based estimation algorithm, and what kind of output it produces.

3) For the synthetic and real time-series analysis the authors do not show the typical time-series they are dealing with, but just some kind of summary statistics, which is totally insufficient. This makes it impossible to understand the benefit of the constructed method.

As the paper more or less uses widely known models, the paper is at best incremental from methodology point of view. The language used has lots of grammar problems. This is highlighted by the fact that authors use Levy instead of Lévy already in the paper title. There are lots of editorial problems, like exhaustive bracket using. I list below more several smaller and larger issues. Because of the underdeveloped results and presentation, I must, unfortunately, recommend rejection.

Some comments on the text:

Abstract: "Stable process is characterized by a large variance" — alpha-stable pro-

cesses do not have variances, rephrase.

line 14: bracket before e.g., but no closing bracket in the sentence.

line 16: extra hyphen before references?

line 17: What do you mean by functional models? Possibly parametric models?

line 25: flicker noise has power spectrum $1/f$, and thus it does not have a covariance function, and hence should be treated as a generalised stochastic process – simple mentioning of non-stationarity is not sufficient here.

line 27: Mandelbrodt –> Mandelbrot (also elsewhere)

line 58: cos and sin should not be in italic.

Equation (1) — c_j and e_j are not defined

Equation (2) — Use regular transpose notation, e.g. T and not dagger. Notation is sloppy, is $n0$ actually $n_0(t)$? ... and similarly $n1$!?

Line 67: What do you mean by "variance-covariance matrix" – define everything.

Paragraph starting line 71: If you make the assumption of wide-sense stationarity, you cannot assume a non-constant mean. Please check your definitions, and adjust.

Section 2.2: This section needs to rewritten by using mathematical formulas showing the relations between different objects – it is impossible now to follow this text.

Equation (4) 'sign' should not be in math format, there should be space after the second if, and ifs should be in text mode.

Line 122: fBm is self-similar, not all self-similar are fBm. Please be careful on the wording. Paragraph starting line 128: Please put the equations to the text – it is not enough to have some elementary formulas in the appendix. Also, do note that you cannot extend from fBm to fLsm by simple use of $H = 1/\alpha$, as you need to also change the probability measures. Please discuss this in detail, and discuss what is

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actually needed mathematically to have a properly defined fLsm.

Paragraph starting line 133: Please do write out the equations. This is too sloppy.

Line 143: You are dealing with stochastic processes, not random variables, adjust the language.

Line 144: a N-steps method —> an N-step method

Line 148: Space missing between model(Langbein

Enumerated list in page 6: Please use 1., 2. and 3. instead of 1-, 2- and 3-. Also item 1- has wrong properties, e.g. Brownian motion is not stationary. Also please check terminology in other parts, e.g. in 2- you talk about Gaussian distribution (one-dimensional) and then relate that to a stochastic process. Please see all these through.

Line 166: presents –> present

Section 3.2: Here in the beginning, you abandon all the non-Gaussian models discussed earlier, and return to a Gaussian case. Why?

Line 186: You should include the whole model in your model description, and not leave some parts in appendix – this includes also offset modelling, which is extremely important to include in the statistical analysis, as otherwise the methodology is not transparent.

Equation (5): Ok, here I am totally lost — you have an additive model, and you have somehow got the parameters $\widehat{\theta}_1$ and $\widehat{\theta}_2$, but how do you get them? What is your estimation algorithm, MCMC, optimisation, something else? Then you compute an extrapolation $[s_{L+1}],...,s_{L+N}]$. Why? What is N-th variation over here. You have two definitions for s here, and then you have a number of objects which you do not define, and use Δ^N sign ... is this N-th derivative?

Line 218: "Moreover, the estimation of the model parameters is carried out using the Hector software" — please open up this. Is it MCMC, optimisation or what kind of

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toolbox. I have never heard about it, so some basic background should be needed.

Line 234: defacto –> de facto

Section 3.3.1 – It is impossible to assess this one, as the time-series are not shown. Please plot all the necessary standard things for full understanding of the model. The current level is not adequate.

Section 3.3.2 — The same comments as for Section 3.3.1

Line 319: Very large variance for Lévy process?

Line 328: Please use $\langle \cdot \rangle$ instead of $< \cdot >$

Line 337: Please use $\gg$ not $>>$

line 356: long- memory –> long-memory

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