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Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "Application of Levy Processes in Modelling (Geodetic) Time Series With Mixed Spectra" *by* Jean-Philippe Montillet et al.

Jean-Philippe Montillet et al.

jean-philippe.montillet@unil.ch

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Dear Reviewer, thank you for taking time to review our manuscript. We would like to take the opportunity to discuss your arguments and to give some information which will also clarify and improve the manuscript.

1 - the authors should indicate clearly what is new in this manuscript with respect to these previous works Reply: We emphasize in line 32 (Âń This work discusses several statistical assumptions Âż . . .) that previous works has applied the FBm and the Levy Alpha stable distributions without or very few justifications about the underlining processes in the GNSS time series. For example, when can we use a family of distribution

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such as the Levy family with infinite variance and heavy tails? What does that imply for the kind of stochastic processes defining the GNSS time series ? So far the literature on geodetic time series analysis is missing this discussion (see Line 37 " Therefore this work aims at understanding when the Levy processes can be applied to model geodetic time series. "). Now, our methodology to investigate the use of these models, is based on the assumption of a third random variable to model the residual stochastic processes due to e.g. small transient signals, small jumps (coseismic offsets ..) . . . This third random variable is defined a Levy process. The definition of this Levy process falls in three specific cases Levy Gaussian, Fractional Levy and stable Levy. We then develop a N-step method which is based on the estimation of the stochastic models when varying the length of the time series. 2- " the authors do not introduced adequately the topic: what is GNSS data, why does it have non-stationary and stochastic components? " Reply: Section 2 is here to describe step-by-step what we call a geodetic time series and the specificities of GNSS data. In Line "50", "GNSS time series are generally regarded as a sum of geophysical signals (i.e. seasonal signal, tectonic rate) and stochastic processes (. . .)". With our experience in previous publications, we have summarized the modelling and processing of GNSS time series. We discuss some of the fundamental hypothesis such as the Gauss-Markov assumption and the WSS hypothesis of the coloured noise. This coloured noise is a power-law noise (P(f) = 1/fˆbeta). In Section 2.2, we underline the relationship between this power-law noise, the fBm and the FARIMA. Therefore, the stochastic noise of the GNSS time series includes short and long memory processes. This topic is large and can be discussed comprehensively, but due to space limitation and clarity of the manuscript, we needed to restrain our introduction to GNSS time series, their stochastic properties and associated models. We refer to Montillet and Bos 2019, Chapter 2 for a longer discussion of the stochastic properties of the GNSS time series. Note that various references to previous works in mathematical geodesy are included, but the paper requires a minimum knowledge of geodetic time series to grasp this whole discussion. We will add some sentences at the beginning of Section 2 such as: "Geodetic time series consists out of a set of observations at each epoch, containing noise which can be described as a set of multivariate random variables. The time series are modeled as a sum of a stochastic and functional (or trajectory) models. The functional model describes the geophysical processes intrinsic to the local and regional geodynamics of the area where the station is installed. The stochasticity originates from the unmodelled transient signals and the measurement errors (Williams et al., 2004; Bos et al., 2013; Montillet and Bos, 2019)."

3 – "the structure of the manuscript is very confusing, with many different models proposed from time to time, with no justification from the data." Reply: We think that there are two different issues in this comment. First, it seems that there is a confusion based on the definition of the GNSS time series. Therefore, we will clarify section 2 (including the few sentences written in the previous comment. Now, there are various stochastic noise models in GNSS time series analysis, because of the small geophysical signals (transients) and small offsets (due to local geodynamics or far away large magnitude Earthquake). We discuss that when formulating the assumptions about the Levy processes in Section 3.1 (see the assumptions behind Levy Gaussian, Fractional Levy and Stable Levy). In a nutshell, it has been formulated that the stochastic noise models of the GNSS time series are a sum of two random variables (r.v.) (also called stochastic processes as suggested by R1), modelling the white noise and coloured noise. The white noise is Gaussian distributed and the coloured noise is either a Flicker noise or a Power-law noise. Both are modeled via their covariance function as explained in Section 2 – eq. 2. Recently, it has been discussed the use of a third r.v. such as the random-walk (see discussions in He et al. 2017 and He et al. 2019). That is to model various transient signals (coseismic offsets, post-seismic relaxations ...) which can or cannot be geophysically related. Note that one needs also to take into account the processing of GNSS time series which can generate outliers and spurious signals. However, the definition of this third r.v. is generally related to the local geophysical activity (e.g. postseimic events and small tremors generating a random-walk in stations located in Cascadia mountains – He et al. 2019, Montillet et al. 2018). Here, we propose to define this third r.v. using the family of Levy processes. The family of Levy

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processes can model short and long memory processes and random jumps (Levy jump processes). However, it is not easy to model every time series with 3 r.v., because each time series is a unique sum of geophysical and stochastic processes. Therefore, we need to separate (as much as we can) the known geophysical signals (tectonic rate, seasonal signal) to the stochastic processes. Therefore, we have created this N-steps algorithm. By iterating the estimation of the stochastic and functional model, we can formulate the assumptions to characterize this 3 r.v. as defined in Table 1. In each step, we produce a residual time series which is the GNSS time series ÂÍminus" the estimated geophysical signals (e.g. tectonic rates and seasonal signal). Also between two steps, we increase the length of the time series. Note that the maximum increase in length is 1 year, because over a much longer time period we can introduce more small amplitude transient signals. For example, if there is no change (or negligible changes) in the estimated parameters of the stochastic noise models after N iterations, then the 3 r.v. is assumed to be a Gaussian Levy process following the properties of a pure Brownian motion. The processes are then assumed as short memory processes and are modeled as a Gaussian white noise. Thus, in this case we have a sum of two white Gaussian noises and a low-amplitude coloured noise. We postulate that the ARMA model should model the stochastic processes. The second case is when we have a noticeable change in the stochastic noise and functional models. That is when we have long-memory processes and high-amplitude coloured noise. The third r.v. is then chosen as a Fractional Levy process. In this case, the high-amplitude coloured noise produces long-memory processes and the FARIMA model should be used to model the stochastic processes. The last case is a special case when we have a large variance due to outliers or unmodelled signals of large amplitude, therefore there is anxiety in the chosen functional and stochastic models. We define the third r.v. as a stable Levy process which is directly related to the alpha stable Levy distribution.

Beyond that, there is the selection of the optimal noise model. This is another hot topic. Here, we have restricted the choice between the FL+WN and the PL+WN models. The choice is based on the maximization of the Akaike criterion. It is a pre-processing step before the N-step algorithm. We justify our strategy based on the results in He et al. 2017 and He et al. 2019. A comprehensive discussion is in Montillet and Bos 2019 – Chapter 1 and 2. It will be confusing to integrate a full discussion on the optimal choice of the stochastic noise model, therefore for a matter of clarity we have restricted this part to line 220-225 (The optimal choice of the stochastic model . . .).

4 –"But no data is shown to justify this / Then show some plots of such data, with power spectrum and pdf. " Reply: Below (see Figure 1), we attach the GNSS time series of ASCO station, together with their power spectrum. These figures will be added in the annexes of the paper.

5- "explain how to estimate the parameters. " Reply: The stochastic noise model of the residual time series (ARMA, FARIMA, power-law + white noise . . .) are estimated via maximum-likelihood using Hector software as discussed in Line 225-230. We have not emphasized the technique here, because it is also a long topic described in Montillet and Bos, 2019 (see the first 6 chapters). The best estimator (maximum likelihood, Monte Carlo Markov Chain . . .) or the statistical "strategy" is out of the scope of this paper. Note that we have also discussed this point in the previous discussion during Review 1 (see point 4). In the final version of the manuscript, we will add a short appendix to discuss the estimation of the stochastic model jointly with the geophysical model using the log-likelihood.

6 – "when discussing Levy stable processes, a reference to a web page (Nolan) is not the correct citation. There are many works that can be cited, such as Samorodnitsky and Taqqu, Stable non-Gaussian random processes, 1994. " Reply: Thank you for this remark. We will add this reference in the final version.

7- "the authors mention FARIMA models, but these models are discrete. When discussing stochastic processes possessing scaling properties, no need to go to the discrete models. " Reply: The discrete models have been used in geodetic time series analysis (GNSS time series, tide gauges, see Chapter 2 in Montillet and Bos 2019). In our present work, we use them to formulate assumptions on the stochastic noise properties of the GNSS time series (short vs long range dependencies, ..).The FARIMA model is interesting for our time series (and more widely for geodetic time series), because it has the ability to mode long-range dependencies due to the relation between the fractional index (d) and the Hurst parameter (H) (see Line 100-105). We discuss this ability over the less complex ARMA model in the case of high amplitude coloured noise (see discussion Line 135-140). Note that we will include this reference "Pipiras and Taqqu 2017"

8 – "equation 3 is not correct, the good relation is beta=1+2H. " Reply: It depends on how you define beta, following (https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3947294/) If the definition is based on fractional Gaussian noise, beta =2H-1; if it is based on the fBm, it is based on beta =2H+1. Now, I use the definition from the geodetic community which is based on fractional Gaussian noise – see He et al. 2017 However, it is a good point to precise in the manuscript. We will add a sentence to indicate which definition we follow. Thank you.

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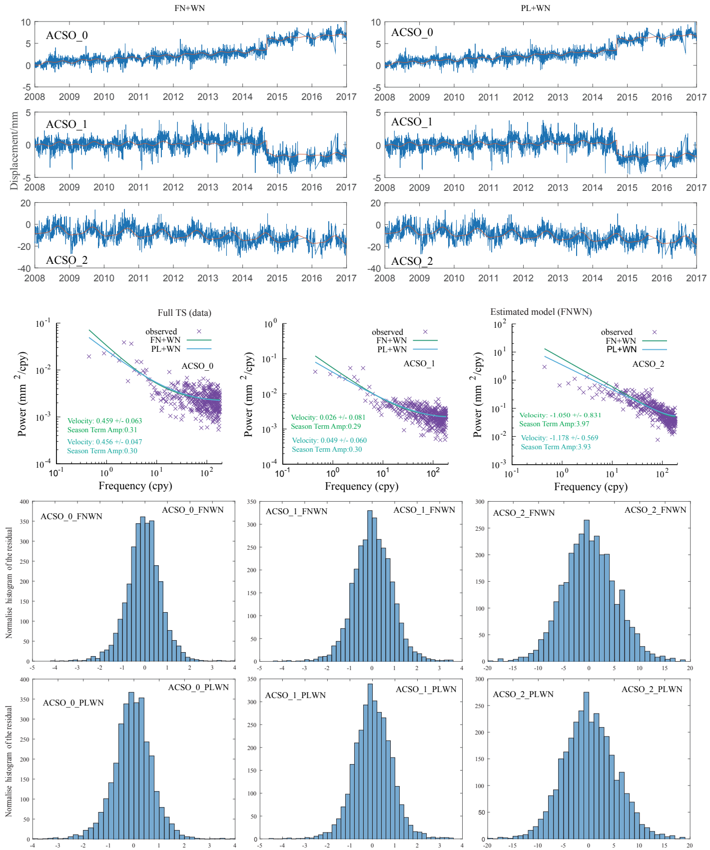

Fig. 1. : ASCO time series (with functional model on top - red) for two stochastic noise models (PL+WN, FN+WN); Power spectrum (East (0), North (1), Up (2)); histogram of the residual time series.