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Interactive comment on "Application of Levy Processes in Modelling (Geodetic) Time Series With Mixed Spectra" *by* Jean-Philippe Montillet et al.

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Dear Reviewer,
Thank you for taking time to comment the manuscript. A few comments may be a general criticism due to the "jargon" used specifically in geodetic time series which sometimes is not as precise as in pure statistics. Thus, we would like to discuss deeply the major comments.

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Discussion paper

1 General comment "The drawback of the paper is that it does not provide a solid mathematical framework on which the time-series analysis is carried out."

We would like to emphasise that this manuscript focuses on modelling the stochastic noise processes of the geodetic time series using 3 stochastic processes, instead of 2 as commonly assumed. We justify our approach considering that recent papers (i.e. He et al., 2019; Book Montillet $\&$ Bos 2019; Langbein $\&$ Vrac 2019) have introduced the model Flicker Noise + White Noise + Random Walk ($FN + WN + RW$) to model these stochastic processes. Note that we will correct the manuscript by rephrasing the misuse of "random variable" instead of "stochastic process" as suggested by the reviewer.

2 "Unfortunately the statistical model is not properly build ..."

The approach taken in this paper follows the fitting of a trajectory/functional model together with a stochastic noise model. That is a comparable approach as the "parametric" approach defined in applied statistics. In Montillet et Bos (2019) (chapter 1), this topic is comprehensively discussed. Unfortunately, due to a lack of space, we have only reminded in the introduction to the reader the basic elements of this approach. Of course, there are different alternatives to this approach. One of them is the general "non-parametric" approach (e.g., MIDAS, $doi.org/10.1002/2015JB012552$). In the final manuscript, corrections will be made to the manuscript in order to avoid any misunderstanding.

Now, in order to model the stochastic noise properties of the geodetic time series, we study the stochastic noise properties of the residual time series, that is the time series after subtracting the estimated functional/trajectory model (e.g., tectonic rate and seasonal signal). That is why we have developed this N-step algorithm which is basically the iterative estimation of the functional and stochastic noise models when increasing the time series length. This algorithm allows investigating the variations of the estimated stochastic parameters, and thus to conclude on the proposed assumptions of a third stochastic process.

Therefore, we have selected three cases for the 3rd stochastic process: Gaussian, Fractional Levy and Stable Levy.

1- The Gaussian case is obvious, which is the case that the only important changes in the stochastic noise properties are in the parameters related to the white noise or slight variations of the coloured noise amplitude.

2-The Fractional Levy is related to the fractional Levy stable motion (fLsm) which is related to the fBm based on Weron et al. 2005. The relationship with the fBm is important, because several works on modelling geodetic time series are using the fBm, which then justifies this assumption. That is generally the variations of the coloured noise properties. If so, it is generally due to unmodeled transient signals, or small offsets buried in the noise floor. A particular case is the use of the Random-walk in the stochastic noise model as previously discussed.

The last case is the alpha stable process which is related to model the residual time series with the family of alpha-stable Levy distributions. That is a very specific case, because we underline in the conclusions that this case happens when residual time series are modelled by heavy tailed distributions.

3 "As the Lévy processes are mathematically extremely technical, the simple Gaussian process definitions cannot have such fundamental flaws."

In the light of the above explanations, we have restricted the use of the Levy processes to these 3 cases. Therefore, a full discussion on the Levy processes is out of the scope of this paper. In the supplementary materials, we will add a specific section on general definition of Levy processes, but it should remain outside of the main body of the manuscript to avoid any confusions.

4 "For the parameter estimation algorithm, a 'Hector software' is used" (This comment also refers to the use as Hector without its statistical basis)

The Hector software is based on maximum-likelihood such as:

$$\ln(L) = -1/2[N\ln(2\pi) + \ln(det(\mathbf{C})) + (\mathbf{y} - \mathbf{A}\mathbf{x})^T\mathbf{C}^{-1}(\mathbf{y} - \mathbf{A}\mathbf{x})] \quad (1)$$

With $(\mathbf{y} - \mathbf{A}\mathbf{x})$ represent the observations minus the fitted model and are normally called the residuals. The function $ln\ (L)$ must be maximised assuming that the covariance matrix $\mathbf{C}$ is known. There has been an important effort produced by geodesists to define $\mathbf{C}$ which is expressed by a long literature from the early works (Williams et al., 2004) to current discussions (see Montillet and Bos, 2019, Chapter 2 for a full summary). Now, the current definition of the covariance matrix $\mathbf{C}$ related to the sum of white and coloured noise is as expressed in equation 2

$$\mathbf{C} = \sigma_{n_0}^2\mathbf{I} + \sigma_{n_1}^2\mathbf{J} \quad (2)$$

Where I is the identity matrix, and J is the approximation of the covariance matrix of the selected coloured noise. This approximation depends on the software used to do the maximisation of the function $\ln(L)$. Generally, the main assumption is to use an approximation of power-law noises. Granger and Joyeux (1980) and following studies (see Montillet and Bos, 2019 Chapter 2 for more information) demonstrated that power-law noise can be achieved using fractional differencing of Gaussian noise. Several approximations can then be derived in order to simplify to perform the estimation of the inverse of $\mathbf{C}$, which quickly becomes a large matrix for long geodetic time series (> 5 years $= 1825$ samples), and taking into account the large amount of data when dealing with large network of stations (> 100 stations). In the specific case of Hector, Bos et al. (2013) made several assumptions on the coloured noise properties in order to get a Toeplix matrix to define J. Note that in the case of a third stochastic process, a third covariance matrix is added to equation 2. In order to improve the manuscript, we will add this paragraph to section 2.1.

5 " For the synthetic and real time-series analysis the authors do not show the typical time-series they are dealing with, but just some kind of summary statistics, which is totally insufficient."

To recall the abstract of the paper, the interest of our development is supporting the approach of the three stochastic processes with different cases. Thus, we will add simulated time series in the appendices to give more details and support the simulations. We must emphasise that the Hector software includes a package to simulate geodetic time series (see Hector manual – see example 3 - $http://segal.ubi.pt/hector/manual_1.7.2.pdf$).

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6 “As the paper more or less uses widely known models, the paper is at best incremental from methodology point of view”

One must take into account that this discussion is part of a hot topic in geodetic time series analysis and a general effort to model more precisely longer and longer time series. For example, in He et al. (2019), the authors found for some time series that very long geodetic time series (over 10 years), the PSD experienced a flattening in the high frequency which makes the power-law model not the best choice anymore to model long-term stochastic processes. Thus, we believe that this research fits well into this general debate in geodesy, and the use of these Levy processes can shed a new light on the fundamental approach of modelling stochastic noise processes in geodetic time series.

7 “Section 3.2: Here in the beginning, you abandon all the non-Gaussian models discussed earlier, and return to a Gaussian case. Why?”

There is a confusion here. We do explain first the N-Step algorithm (see clarifications below). Then, we show how this algorithm allows to discriminate between the three cases for the third stochastic process. The Gaussian and non-Gaussian cases are explained together with Table 1.

8 “If you make the assumption of wide-sense stationarity, you cannot assume a non-constant mean”

A random process is said to be following the WSS assumption if its mean and auto-correlation function are time invariant (Haykins, 2000). Based on this definition, we

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cannot strictly introduce a slow varying mean. Therefore, we need to revise section 2.1 accordingly.

9 “ stable Levy process with very large (infinite) variance?”

Here, we assume that the stable Levy process is an alpha-stable Levy motion, selected as the third stochastic processes if the distribution of the residual time series displays heavy tails, therefore with a very large variance. We do not imply that every stable Levy process is characterised by a large variance. We will clarify this sentence in the manuscript.

10 Clarification on the algorithm – the N-step process

“Equation (5): Ok, here I am totally lost âĂŤ you have an additive model, and you have somehow got the parameters b_1 and b_2, but how do you get them? What is your estimation algorithm, MCMC, optimisation, something else? Then you compute an extrapolation $[sL + 1, \ldots, sL + N]$. Why? What is N-th variation over here. You have two definitions for s here, and then you have a number of objects which you do not define, and use $_N$ sign ... is this N-th derivative?”

The algorithm is an iteration of the estimation of the functional + stochastic noise model using Hector. The iteration is defined by taking longer and longer time series at each step, or in other words adding more data sample at each iteration. As mentioned earlier on, Hector is a software based on maximum likelihood. The first step estimates the time series with L data samples, whereas at the N-th iterations there are $L + N$ samples. The general idea is to see the variations in the estimated parameters of the stochastic noise model in order to select which type of processes (one of the 3 Levy processes) best models the stochastic noise properties.

In details, the algorithm is defined in a functional form. Thus, the time series is a sum of :

- $F(\theta_1)$: the trajectory model, which contains the functions describing various geophysical signals (seasonal signal, tectonic rate, offsets, post-seismic relaxations $\ldots$) - discussed in equation 1
- $G(\theta_2)$: the stochastic noise model (Flicker + white noise, Power-law + white noise $\ldots$)

θ_1 contains for example the amplitude of the seasonal signal, the amplitude of the tectonic rate, the time and amplitude of various offsets (i.e. related to geophysical phenomena or equipment changes), the estimation of the amplitude and time of a post-seismic relaxation. θ_2 contains the amplitude of the white noise, the amplitude of the coloured noise (depending on the selected model), the power-law index (if it is selected as an unknown variable). The parameters also vary with the stochastic model selected.

At the N-th iteration, we get the residual time series ΔS^N by subtracting the time series S with the estimated functional model $[F(\hat{\theta_1})]_N$. We then look at the estimated parameters ($\hat{\theta_2}$) in the stochastic noise model $[G(\hat{\theta_2})]_N$ and compared their variations with the previous estimates in previous iterations. Figure 1 and 2 show these variations using simulated and real time series. The $_N$ sign refers to the N-th iteration - the time series with $L + N$ samples.

Note there are no objects (no derivatives), just parameters described in Section 2. Also, we do not “extrapolate” the time series. Instead we use the time series by varying the length on the last year. We choose only one year (and no more), because we state

at Line 191 :
"(. . .) varying the length of the time series over several years is not realistic taking into account that real time series can record undetectable transient signals, undocumented offsets and other non-deterministic signals unlikely to be modelled precisely " The final version of the manuscript should include a clarification of the algorithm in the light of this discussion.

11 "Section 2.2: This section needs to rewritten by using mathematical formulas showing the relations between different objects – it is impossible now to follow this text."

We will revise this section by adding a proper definition of the ARMA and FARIMA. Note that the general idea is that the fitting of the FARIMA to the residual time series implies long-memory correlation. With the relationship with "d", parameter of the FARIMA and "H" (the Hurst parameter), there is a direct relationship between the FARIMA and the fBm (Montillet and Yu, 2015). Now, taking into account that fLsm is also defined with "H", we have also a direct relationship between FARIMA and fLsm. Also, we should clarify the relationship between the fBm and the fLsm as proposed by the reviewer.

Please also note the supplement to this comment:
https://www.nonlin-processes-geophys-discuss.net/npg-2019-48/npg-2019-48-AC1-supplement.pdf

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