

Interactive comment on “Fractional relaxation noises, motions and the fractional energy balance equation” by Shaun Lovejoy

Anonymous Referee #2

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The paper introduces a novel class of stochastic processes, called "fractional relaxation noises and motions" (fR_n , fR_m) and discusses their particular application to a stochastic relaxation equation (Eq. 1), used to describe the Earth energy balance. The principal motivation behind the proposed processes is to synthesize the stochastic differential equation and scaling modeling approaches, each of which has been thoroughly explored (and shown efficient) in mathematics and applied literature.

The paper starts (Section 2) with introducing the fractional Langevin equation (Eq. 1) for the Earth energy balance, and uses it as a motivational example to develop the proposed theory, in parallel to what has been done for fractional Gaussian noises and Brownian motions (fG_n , fB_m). Section 3 discusses specific technical details, including derivations of process spectra and illustrating sample trajectories. This section also

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notices a practically important aspect of not being able to distinguish between fR_n , fR_m , and fB_m for specific ranges of parameters and for a finite range of observations. Section 4 examines the classical prediction problem, emphasizing that here one deal with the past value rather than initial value problem. Section 5 concludes.

The paper is clearly written (in particular, given its heavy math content). The materials is novel and of clear importance for the nonlinear geosciences community. I trust the paper will be of interest to the broad NPG readership and urge its publication.

I have a comment related to the paper organization. Given the discussed material and the publication venue, I expect there will be two main categories of readers (that of course overlap) – those who are more interested in the math details, and those who are mainly interested in qualitative findings and applications. I think the paper will benefit from reorganization that first will clearly list the main proven facts (process definition, statement of stationarity, correlation function, spectra, sample path, prediction), and then will present the underlying derivations. The current version is math heavy and makes it hard to clearly see the key points of the presented material. Also, it is very important to explicitly list the differences between the fR_n , fR_m and their counterparts fG_n , fB_m .

Other minor comments/typos: 1) l. 59-60: Rewrite to mention the authors (West et al.) outside of the reference. This is how the sentence was originally intended. 2) l. 68: "martingales" should not be capitalized. 3) l. 72 and everywhere: "Earth" should be capitalized. 4) Please use proper punctuation in all equations (commas, periods). 5) Eq. (1): Why not give refs to the classical EBM of Budyko and Sellers before diving into a fractional version? 6) Eq. (2): Define Gamma 7) l. 167: "standard Brownian motion" (instead of "usual Brownian motion")

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