Comments on NPG-2019-39: Fractional relaxation noises, motions and fractional energy balance equation

The manuscript is devoted to a stochastic fractional model for the Earth's energy balance. The author developed a framework for handling fractional equations driven by white noise forcing, and analytically determined both the small and large scale limits for fractional relaxation motions (fRm) and fractional relaxation noises (fRn). He derived the main statistical properties of both fRm and fRn, including Green function, autocorrelation function, Haar fluctuations, spectra and sample process. These are extensions of fractional Gaussian noise and fractional Brownian motion. Furthermore, he examined the prediction of fRn, fRm with a past value problem by the minimum square skill score, which are needed for forecasting the Earth's temperature.

The main points of the manuscript is expressed concisely and the paper is overall well organized. I recommend acceptance by NPG, after a revision taking into account the following comments and suggestions.

1. The main case studied in this manuscript is $a = -\infty$, i.e., Weyl fractional derivative. It would be nice to give more explanation on how to understand the $a = -\infty$ in the fractional derivative operator ${}_aD_t^H$, and what is the significance of the application in the model?

2. The caption of Fig.2 is not clear, such as " H increasing in units of 1/10 starting at a value 1/20 above the plot minimum". Perhaps it is better to say like this " H increasing in units of 1/10 starting at a value 1/20(upper left) to 39/20(bottom right)" ?

3. In Fig. 5a, 5b, 6a, 6b about fRn and fRm simulations, it would be nice to add the sample path simulation of fractional Gaussian noise and fractional Brownian motion for comparison, which will help reader to better understand the fractional relaxation noise.

4. Fig. 7 and Fig.8 present the simulation for fGn and fRn forecasts. Is it possible to say or predict the Earth temperature T based on fractional energy balanced equation?

5. It would be nice to give a summary of the main differences and links between fRn (fRm) and fGn (fGm) in conclusions.