Comments to the author:

Dear Shaun,

To speed up the review process, I had chosen the option that "the next review will be done only by the editor despite the expected thorough revisions". Unfortunately, I gradually became aware that there was no response to the referee and the track change copy is unusable, providing only a grey rectangular on the right side of the text. I initially thought that this could be due to bugs of the review system, but after some investigation it does not seem so. Therefore, please provide these documents, slightly updated if necessary.

I would have appreciated an acknowledgement of the substantial revisions that were made in the previous (fourth) version, they seem to have been undetected. In any case, with the exception of the first four referees which were answered in January 2020 (the first revision), the other comments I have received have been of the very general, very vague and slightly negative variety.

In other words, the reviews and comments since January 2020 seem to (barely) conceal a desire to kill the paper, they refrain from giving enough specifics to allow the paper to be improved or defended. We are both familiar with the genre! This contrasts with the attitude of the first four referees that were generally quite positive: the only one to suggest major changes had rather mild specifics that were addressed nearly two years ago. I went along with the three (post January 2020) major revisions because the process was so slow that in the meantime I was able to make my own improvements with virtually no guidance.

Certainly your latest round of comments will only allow me to add the three sentences indicated in the following. I will make these changes if you agree to publish the paper when they are done.

They may help to provide more complete answers to my own report that was primarily trying to highlight the main points of the referee's report, which is more detailed.

Until now, the only specific comments that I had were several that were addressed in the first round of responses and revisions in January 2020. My second round of changes were made while waiting for the new 5<sup>th</sup> referee who responded 13 months into the process. He noticed a sign error (an obvious typo with no consequence) and suggested Fourier techniques. By then I had already added the Fourier approach (without his suggestion), but I still kept the Mandelbrot approach in the main part of the paper with the Fourier approach in the appendix. Following your comments (25 months into the process), I put the Fourier approach into the main part of the paper and removed the Mandelbrot approach altogether. However, real space results are still needed for the important predictability part (section 4) that must be done in real space. For instance:

The reference made to physics for this Appendix A (instead of a rush to mathematical details) was misunderstood as a justification of the fractional Langevin equations, while the referee specifically challenged the Fourier technique chosen.

It would have been nice to see something specific in this direction since it isn't obvious what an alternative Fourier approach might consist of! At the moment I can't even guess!

However, I have accepted that the present paper could remain in the same approach despite the resulting complexity.

There is still a loss of symmetry along the early developments of Appendix A that should at least noted.

There is already a sentence explaining why both Fourier and real space results are needed, but I can add an appropriate sentence apologizing for the resulting lack of symmetry.

The strong reference to the Budyko-Sellers model is not justified, as its main physical process is absent.

This is a bit ridiculous! When the paper was originally submitted, I admit that there was only an "announcement" of the result that had been properly published. However over the 28 months since submission, there have been 4 publications with "FEBE" (or the h=1/2 case, HEBE) in the title! One of the latter- the application of the FEBE to temperature projections to 2100 (submitted in March 2020) is now waiting the referees after "minor revisions". Three additional published papers already reference the FEBE (a full list was also appended to the previous response). These publications outline two quite different derivations of the FEBE including one that is identical to Budyko-Sellers except that it uses the correct radiative-conductive surface boundary condition that Budyko-Sellers got wrong (see the two HEBE papers).

In other words, the strong reference to Budyko-Sellers is fully justified by the cited literature and needs no further support in the present paper!

The referee also strongly questioned several aspects of the Sect.4, including they considers it as limited to comparisons of prediction skill indicators, themselves discussed.

There are numerous skill indicators and I gave the most popular and easy to interpret one (the MSS), but there are clearly others (I will reference the extensive discussion and of these in the Del Rio Amador, L., and Lovejoy, S., Using regional scaling for temperature forecasts with the Stochastic Seasonal to Interannual Prediction System (StocSIPS), Clim. Dyn. doi: 10.1007/s00382-021-05737-5, 2021a which explicitly uses the high frequency FEBE limit for state of the art forecasting). In any case, it would have been nice to see specific comments explaining why the MSS is unacceptable (it might even help the GCM modellers that use it routinely)!

I apologise for miscounting the referees: they were 5, not 4 and my numbering must be increased by one and you can claim a record. Referee 5 can be thanked for his constructive comments, especially on the role of Fourier techniques, but more generally contributed to this paper being modified.

Unfortunately his comments were in fact not much help since they were so late that the transition to Fourier had already been done. I will nevertheless acknowledge him in a final version.

Best Daniel regards,