Responses in Italics;

Dear Shaun,

First of all, I would like to clarify that there were only 4 referees and that the first 3 were not "uniformly positive".

My only information was that the referees were numbered 1 -5 and I responded to each, sometimes more than once. It would have been helpful to know that there were only four different people involved!

For example, referee 3 was very critical and referee 2 actually suggested a major revision. Referees 3 and 4 both complained bitterly about the lack of key references to the literature and actually helped to improve the state of the art considerably,

This was helpful but was addressed in the January 2020 revision.

although reviewer 4 is still not fully satisfied and believes that this remains a source of problems for your paper. I can also mention that I have received abrupt refusals to review.

I appreciate that it is not easy to find reviewers!

The referee 4 had strongly suggested to use the Fourier transform to derive the statistical characteristics of the fractional Langevin equation. The new Appendix A tries to respond to this and I am happy to acknowledge that it is the core of the present revision. However, the referee 4 raises several issues around the main question: how to do it in a very simple way? It has taken me some time to assess the extent to which this calls into question current algebraic developments. I am afraid this is the case since the beginning of the new Appendix A with its unfortunate rush to mathematical details of a given approach instead of trying to optimise the approach and to avoid cumbersome developments. Indeed, it should rely more on physics than a series of “convenient” equations.

The adoption of Fourier techniques had been done after January 2020 but before referee 4 suggested it: it was indeed a good idea to go beyond the old approaches used by Mandelbrot. Appendix A rushes to mathematical details because the physics is dealt with in the main text (and now in numerous publications, see below). Following your suggestion, this has now been augmented and the entire approach based on Fourier and Laplace methods. Unfortunately, the real space Green’s function approach is needed for the predictability results, so they are also given. They also allow an easier entry to the topic for geoscienctists.

Over the last years, the science has also advanced so that results on the fGn forced fractional relaxation process are also needed for applications. Therefore, the new version has been slightly extended to cover this possibility.

This may partly explain why Appendix B relies on a different technique than Appendix A.

Yes, now, appendix B is more clearly cast the half order case in the same Fourier/Laplace framework as the rest and has been (mildly) extended to the fGn forcing case.

But the main problem with Appendix B is that it actually addresses a distinct problem. This is an example of the inconsistencies briefly mentioned in the report, some of which I fear are generated by repeated use of incomplete notations, which, for example, do not make explicit the fact that only part of the solution is addressed.

The new version addressed this (see above).

The referee's report raises other important questions, such as the relationship between this fractional Langevin equation and the actual Budyko-Sellers model, in fact, the actual predictive capabilities of this equation.

Since this paper was submitted in July 2019, there have been many publications addressing this in great detail. Several new references have been included in the new version and more physical discussion has been given.

Theory:
Starting with the classical continuum equation (and then its fractional generalizations), the exact link with the Budyko-Sellers approach was given in a two part series: [Lovejoy, 2021a; b]. The FEBE was also derived phenomenologically in [Lovejoy et al., 2021]; this paper included a summary of the dimensional FEBE statistics (citing to the 2019 version of this paper).

Applications:
First, there was [Del Rio Amador and Lovejoy, 2019] (building on [Lovejoy, 2015], [Lovejoy et al., 2015]) who investigated the predictability of the global temperature using the high frequency fGn approximation to fRn. There was also [Hébert et al., 2021] who investigated the low frequency approximation for climate projections to 2100 with [Procyk et al., 2020], [Procyk, 2021] directly using the full FEBE to improve on this further (notably using several of the results of the NPG paper; the projections had much lower uncertainties than the GCMs). More recently, [Del Rio Amador and Lovejoy, 2021a; Del Rio Amador and Lovejoy, 2021b] addressed the regional application of the FEBE model for monthly and seasonal predictions (using the fGn approximation) showing that they were comparable to or better than the GCM alternatives.

Due to the above-mentioned inconsistencies, especially in the new materials, the paper still needs thorough revisions. The safest, simplest and most satisfactory way would be to modify the approach followed in Appendix A. But this would require a major rethink and an alternative might be to eliminate the inconsistencies in the current approach and make it more accessible despite its inherent complexity.

The need to quantify the statistics in real space while nevertheless using Fourier/Laplace techniques makes this paper complex. However, the new version eliminates the last traces of the Mandelbrot approach and is much more streamlined. Putting the technical part in appendices makes the main text lighter and more accessible. Thanks for the suggestions.

To speed up the process, I choose the option that the next review will be done only by the editor despite the expected thorough revisions.

I appreciate that it is not easy to find reviewers but the current version should be relatively glitch free!

Best regards,
Daniel

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