

Authors would like to thank the anonymous reviewer for his/her very careful reading of the paper and suggestions to improve it. Hopefully the modifications implemented will satisfy him/her.

Interactive comment on “Joint multifractal analysis: further developments and implementation on rainfall data” by Auguste Gires et al.

Anonymous Referee #2

Received and published: 24 November 2019

This work studies the behaviour of fields which are composed of a product of two universal multifractal (UM) fields. First, the properties of UM fields are briefly reviewed. Then the properties of multiplicative combinations of UM are discussed and it is shown how approximate UM parameters can be derived from products of UM fields. The authors warn for the possible confusion between the phase transition causing diverging scaling moment functions $K(q)$ and the combined nature of the field, both of which give rise to $K(q)$ which are higher than predicted by UM theory. The authors then perform a numerical experiment with the discussed set-up of one UM field φ and one combined field ϵ . They estimate the parameters of the underlying fields using their newly developed methodology, and demonstrate the use of a simplified correlation indicator. The validity of the approach seems to be constrained to UM fields with sufficiently similar values of α in this symmetric case.

The technique is then applied to observational rainfall data from a disdrometer to infer correlations between different properties such as rain rate, liquid water content, drop concentration and mass weighted diameter. For these fields the validity ranges of the parameters seem to be well respected. The result of such an analysis can be used to simulate one of these quantities, based on another known quantity and a random one.

General comments:

This paper shows a new technique to infer the properties of multiplicative fields, which could be useful to investigate correlations between UM fields and simulate a field based on a given one, if the correlation is known. The application to rainfall data nicely highlights the potential of this method.

Thank you for your positive feedback.

The title does not capture the subject of the paper, that is the analysis of correlation between approximate UM fields. "Further developments" is very vague for a title. I would also say "application to" instead of "implementation on".

Following your comment, the title was changed to : “Approximate multifractal correlation and products of Universal Multifractal fields, with application to rainfall data”

The structure of the manuscript is fine, the formalism is explained clearly and the results are shown in a logical way. The figures could be improved somewhat (see specific comments below). The equations, however, contain errors. I hope these are merely typographical, but to remove any doubts on the correctness of the results I suggest the authors provide their code and/or data as supplementary material or through a citable repository (e.g. Zenodo). This would also be in accordance with the best practices of this journal.

As suggested by the referee, code and/or data will be made available on a citable repository.

Finally, there are many grammatical and spelling errors throughout the manuscript (e.g. "betwen", "dash line", ...). Articles seem to be missing, e.g. p.2 l.44: Similar formalism -> A similar formalism. Please check the whole manuscript carefully for spelling and grammar; the list below is not complete.

These corrections as well as the ones below were implemented. The manuscript was also carefully checked for spelling and grammar.

Specific comments:

p.1 l.2: across wide -> across a wide

p.1 l.9: to retrieved -> to retrieve

p.2 l.24: Reader is -> The reader is

p.2 l.42: of define -> to define

p.2 l.50: relying this -> relying on this

p.3 l.68: an homogeneous -> a homogeneous

p.3 l.59: Please specify the "outer scale" more clearly.

p.4 l.88: as follow -> as follows

This was corrected, thank you for your careful reading

p.4 Eq. (40): I think the RHS should read

$$\lambda^{S(h,q)-K_\epsilon(q)-K_\phi(h)} \approx \lambda^{r(h,q)}$$

This was corrected (it was simply a typographical error)

Fig. 1: Spurious "=" in the caption.

p.5 Eq. (7): in the second line, the second term should start with $b \alpha Y$, not $a \alpha Y$.

p.7 l.156 Please mention the meaning of TM again here for clarity

l.157 Please mention the meaning of DTM again here for clarity

This was corrected

p.7 l.161: The fact that the empirical $K(q)$ in section 3.4 are lower than expected seems in contradiction with earlier remarks that the empirical $K(q)$ would in both cases be higher than expected: please clarify this or clearly disentangle the two kinds of phase transition that can occur. Indeed the two kind of multifractal phase transitions discussed result in different behaviour of the empirical $K(q)$ with regards to the theoretical one. Following the reviewer's comment, this was clarified in the section 2.1.

p.7 l.158, 163 and 172: "inputted" does not exist

This was corrected and changed to "input".

Fig. 4: It would be helpful to visualize the line $h_a + q = q_s$ on the surface (mentioned in p.8 l.177). $h_a + q = q_s$ would actually be another surface. So authors have the feeling it would not improve visualization to add another surface on the figure.

Fig. 5: It would be helpful to visualize the intersection between the two planes.

The orientation of the figure has been changed to improve visualisation.

p.10. Eqns. (12) and (13) are not consistent with each other. For the first line of Eq. (13), for example, I obtain:

$$\frac{\langle \phi_{\lambda}^{aq+h} \rangle}{\langle \phi_{\lambda}^{aq} \rangle \langle \phi_{\lambda}^h \rangle}.$$

For the third line I obtain

$$\frac{\langle \epsilon_{\lambda}^{a'h+q} \rangle}{\langle \epsilon_{\lambda}^{a'h} \rangle \langle \epsilon_{\lambda}^q \rangle}$$

and likewise for Eq. (14) and what follows. Please check carefully whether this affects the presented results. Also verify whether a and a' are not swapped in the rest of the manuscript (e.g. Eq. (16))

Indeed q and h were reversed in the mentioned equations. This was corrected. It means the the axis legend were reversed for Fig. 5 and 6 which are updated. It does not affect the other results since estimates are obtained with q=h=0.7

Fig. 6: It would be helpful to visualize the intersection between the two planes. Also it seems that the blue plane is covering the red plane where I would expect the red plane to be visible. Please improve this figure and mention the meaning of the different colours in the caption.

The figure was updated (see previous comment) and two views are now provided to improve visualization. Indeed the meaning of the colours was missing and is now in the caption.

Table 2 caption: "using the notations of 12" -> "using the notations of Eq. (12), "; "line" -> "row"

p.16 l.286: the one obtain -> the ones obtained

p.17 l.298: "the two the" -> "the two"

This was corrected.

p.18 l.315: "The characteristic parameters [...] as long as the power law exponents [...] can the obtained through [...] of the studied fields." I don't understand this sentence, please correct.

It should have been "as well as" and not "as long as". This has been corrected.