Authors would like to thank the anonymous reviewer for his/her very careful reading of the paper and suggestions to improve it. Hopefully the modifications implemented will satisfy him/her.

This paper about joint multifractal analysis applied to rainfall data, has a theoretical and technical part, and an application on some rainfall data. I have several comments below. Some parts present confusions on the notations.

Line 25. I suggest to modify the sentence. It is not proved that all multifractal processes converge to UM (universal multifractals). There are many multifractal models that do not belong to UM.
Following your comment, the sentence was updated to "In the large class of Universal Multifractals (UM) which are the stable and attractive limits of non-linearly interacting multifractal processes and correspond to a broad, multiplicative generalization of the central limit theorem; Schertzer and Lovejoy, 1987, 1997)"

Line 81. For divergence of moments cite also Mandelbrot (1974) and Kahane (1985).
I guess that you are referring to these two papers:

- Kahane, J.P., Sur le Chaos Multiplicatif, Ann. Sci. Math. Que., 9, 435-444, 1985.
- Mandelbrot, B. Intermittent turbulence in self-similar cascades: Divergence of high moments and dimension of the carrier, J. Fluid 1036-1038, 1987.

These papers are cited in Schertzer and Lovejoy (1987), which specifically describes this effect in the framework of UM. Anyway, we included citations of these papers. In addition, we will mention that they did not address the quantification of the spurious statistical estimates on finite samples and their dependence on their size (Schertzer and Lovejoy 1992).

:Equation (4). There is a mixture between p, q and h. Please double check this, and also in other parts of the manuscript, to have consistent notations.
This was updated, thank you for your careful reading

Equation (5). Do the authors restrict to a > 0 and b > 0?
Yes and this was clarified in the text.

Figure 1. I recommend to plot X λ and epsilon λ since one does not understand what is the blue field.
Since we assumed that phi_lambda = X_lamdba, this is actually what is plotted. It was clarified in the caption and reference to phi was removed as well (see answer to previous comment).

Section 3. Why not indicate from the beginning that the aim is to study the relation between X λ and epsilon λ . I do not understand the use of φ here, and also I do not believe in the sentence line 96 "without loss of generality". It is here an hypothesis, it is not a general situation.
As suggested by the reviewer, to improve clarity, references to phi were removed throughout the section.

Equation (7). Second line, the prefactor of the second term is not correct (b α y and not a α x)
Indeed, this was corrected.

Line 128. Where the strange value q D = 91 comes from? This is much too large.
Values of q_D are obtained by solving this equation K(q_D)=(q_D-1)D using the pseudo UM parameters of epsilon_lambda. This was clarified in text and values were computed for each panel

of Fig. 2. We find values equal to 5.96, 4.68 and 119 (meaning that a wrong value was written in the first version of the paper).

Equation (9). Some mistakes: insert two minus signs and last term is K(q) and not K(a)
Thank you for your careful reading, this was corrected.

Equation (10). Equation (8) is given for the field epsilon, not for Y . Explain better how this equation is used to obtain α y . Indeed in equation (8) α y is nonlinearly related to other variables and it does not seem easy to isolate its expression. Same for equation (11). Where does this come from?
Indeed some clarifications were missing and they have been added:
- For Eq. 10 this parenthesis has been added : (noting that $\alpha_{\epsilon}C_{1,\epsilon} = C_{1,X} a^{\alpha_X}\alpha_X+C_{1,Y} b^{\alpha_Y}\alpha_Y$, and that the term $C_{1,Y} b^{\alpha_Y}$ is simply equal to $C_{1,\epsilon}-C_{1,X} a^{\alpha_X}$, which enables to remove the non linear part of the equation)
- For Eq. 11 this parenthesis has been added : (noting that $C_{1,Y} b^{\alpha_Y}=C_{1,\epsilon}-C_{1,X} a^{\alpha_X}$ and that we have $C_{1,Y}=C_{1,X}$)

Section 3.4:
Line 151. Why the use of discrete cascades? The term is not explained. Why not continuous cascades?
The following sentences were added to explain discrete cascades.
“The approach presented above is tested on numerical simulations obtained with discrete in scale cascades.
It consists in iteratively repeating a cascade step with a non infinetisimal scale ratio in which a 'parent' structure is divided into 'daughter' structures whose affected value is the one of the 'parent' structure multiplied by a random factor ensuring that Eqs. 1 and 2 remain valid. Such simple field generation process is sufficient for the purposes of this paper. The recent introduction of multifractal operators and vectors paves the way for physically-based, continuous (in scale) multivariate analysis of multifractal fields or measures (Schertzer and Tchiguirinskaia 2015, 2019)”

- Schertzer, D. and Tchiguirinskaia, I. (2015) ‘Multifractal vector fields and stochastic Clifford algebra’, *Chaos*, 25(12). doi: 10.1063/1.4937364.
- Schertzer, D. and Tchiguirinskaia, I. (2019) 'A century of turbulent cascades and the emergence of multifractal operators', Earth and Space Science (invited paper under review)

Line 157. What is DTM analysis ?
It is actually defined in section 1: “Double Trace Moment (DTM), specifically designed for UM fields, is commonly used to estimate UM parameters (Lavallée et al.,1993)”. Authors thinks that this description is sufficient for the purposes of this paper, but if the reviewer still thinks it should be completed, it can obviously be done.

Line 166 and further. Explain better the objectives and hypotheses of the numerical work. I understand that X and Y are simulated, epsilon is built with some values of a and b.
Then the exercise is (i) to find the approximate values of α_epsilon and C1_epsilon and (ii) to assume that epsilon and X are known, and try to find a, b and α y . Is this correct? If yes it should be clearly stated in the text.
You are indeed correct. Following your comment, clarifications were added:
“Before starting, let us clarify the objective of this section. X_{λ} and Y_{λ} are first simulated and then ϵ_{λ} is build with some values of a and b. The

purpose is after to retrieve the values of a, b and α_Y by simply analysing X_{λ} and ϵ_{λ} which are assumed to be known."

Lines 175-179. A quantification of the error is needed.
It is actually displayed by Fig. 4.

Section 4.1. This is very technical and of poor interest. It could be moved to an appendix.
It is indeed quite technical. But authors believe it might be better to keep it in the main part of the paper because it highlights the limitations of the developed framework and enables to introduce the simplified indicator.

Section 5:
Line 276 and further. Explain better the hypothesis of joint multifractal analysis. What is assumed to be known, what is the objective of the work, what is assumed, what is known and unknown.
The following sentence was added to clarify the study "The purpose is to check if the scale invariant analysis of correlations is relevant for these fields and then to quantify their correlations in this framework (i.e. write the fields as in Eq. 13-top- and estimate a, b and α_Y from simply the two fields)."

Line 280 is not "multiplying" but "raising to the power"
The sentence was re-written to insert this correction.

Line 280 and further. Do you obtain $N_t = R^{1/3} X^{3/4}$? Where X is an unknown field? If yes the equation should be written down and more interpretation should be given to this proposed relation.
Yes it is indeed correct, and it is now written down. And comments added on the implications, notably in terms of numerical simulations.

References: why some references have a web reference, some have two web references, and some have none. There is a text in capital letters in the second reference, that should be removed.
The capital letters were removed. With regards to the web references, it was done automatically from my bibtext library which could need to the updated for some web references. This can be done at the editorial stage.