A review of "In-depth analysis of a discrete p model " by U. Saint-Mont

General comment

The author largely motivates his paper by foreseen applications to cascade and multifractal processes, at least to the "p-model": "It is the aim of this contribution to introduce original concepts that shed new light on the latter paradigmatic cascade and allow key features to be derived in a rather elementary fashion". This goal is in agreement with the scope of NPG and the interests of its readership. But, this goal does not seem to be achieved in the present manuscript that furthermore introduces often complexity instead of claimed simplicity, e.g., introducing variables that are finally scarcely used. The algebra is often tedious, in particular demonstrations are not always as straightforward as they could be, definitions are not always precise and from time to time missing. The introduction of non obvious jargon terms does not help the reader who tries to decipher the present manuscript. For instance, there is a given uncertainty on which "p-model" is considered, whereas this model was claimed to be the main topic of this paper. In fact, it is not clear what is the added value of this paper with respect to earlier papers on multiplicative cascades: it seems to be concerned with much milder processes, sharing only superficial properties.

Overall, I consider that this manuscript is not publishable in its present form, but requires to be thoroughly clarified, including its goal; it thus requires a major revision that could be quite challenging.

Detailed comments

Which p-model?

In the introduction, the author presents the p-model as the iterative splitting "in proportion" $1-p$ and p respectively on the left and right subsegments of a (uniform) distribution over the initial segment ([0,1]). This is somewhat close to de Wijs (1951), who used the notations (1+d) and (1-d) for these proportions, where $d>0$ is the "dispersion coefficient". In the later case, the model is "micro-canonical" because it strictly preserves in a deterministic manner the content at each cascade step (simply because: $(1+d)/2+(1-d)/2=1$). Combining the two notations and respecting the positivity of the content "proportions" requires to identify p to 1+d (inverting left and right does not hurt!) and therefore $p>1$. One may note that the α-model corresponds to a stochastic generalisation of this model, with only a canonical conservation of the content, i.e., only on the statistical average.

However, in the rest of the paper p becomes a probability, with therefore the requirement $p\leq 1$, and the values $\mu(H_0)=0$ and $\mu(H_1)=1$ ("without real loss of generality"), with probability 1-p and p. This rather corresponds to the β-model, in fact a special case of the α-model, both being stochastic, canonical, multiplicative cascade models. The main difference is that the β-model is the exceptional case of mono-/uni- fractality, i.e., its support has a unique fractal dimension (in fact defined by $\beta=p$), whereas other α-models are multifractal models, with possible divergence of statistical moments.

Unfortunately, the precise reference to the page 329 of Mandelbrot (1974), which could have helped to clarify what the author has in mind, is outside of the page range (331-358) of this paper.

Unnecessary developments.

With respect to applications to cascade models, all the developments around the so-called "exponential sampling" seem rather useless, as well as the variables X_i. Indeed, what could be the

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foreseen advantage to introduce these variables which contain larger and larger numbers of identical replica of the same variable for increasing *i*? It is indeed much better to focus on the binary variable b_j (=0,1) that could be compacted into vectors $\mathbf{b}_n = (b_0, \ldots b_j \ldots, b_{n-1})$ or even better into dyadic expansions $\sum_{j=0}^{n-1} b_j 2^{j-n}$. The latter is certainly the most interesting one, because these expansions have been often used for the "coordinates" of subsegments of the cascade $\sum_{j=0}^{n-1} x_j 2^{j-n}$. Surprisingly, these subsegments have been only evoked in the introduction.

Missing information.

Very surprisingly there is no clear indication on how to proceed from step n to the step n+1, contrary to what is explicitly done for multiplicative cascades. The reader is rather invited to infer it from an "illustration" (bottom p.8, without a reference number and axis labels) and a table of a few examples (p.9, again without a reference number). Furthermore, Definition 1 is rather ambiguous: it could be understood that at each step is drawn independently of the previous one. In this case, the components of $\mathbf{b}_n$ are merely n independent p-Bernoulli variables, which render trivial many announced properties (e.g., theorem 2) but the relation with a multiplicative cascade has to be done. Due to missing informations, it is often difficult to have a definitive opinion what is really demonstrated. This is particularly the case of theorem 10, which seems to only state that the *Y* process and a *p*-model (still not perfectly defined) have a common type of probability distribution and therefore it does not shed any new light on the *p*-model.

Tedious algebra

In the framework of the previous hypothesis, not only the computations of the mean of Y_n (theorem 5) and its variance $\sigma^2(Y_n)$ (theorem 6) are trivial, as already pointed out by referee 1, because Y_n is then simply the sum mutually independent variables $b_j 2^j$, normalised by their number $2^n - 1$, but the resulting expression for $\sigma^2(Y_n)$ (Eq.1) can be further simplified, in particular to immediately obtain its asymptotic value for $n \to \infty$ (Lemma 8), without any recourse to induction. The same is true for the theorem 11 (including Lemma 12) that is particularly lengthy and awkward. In a general manner, NPG readers will not enjoy many algebraic developments that are too much elementary (e.g., on geometric series).

The "complete process"

All results of the first 5 sections were presented for the conditional variable *Y*, furthermore with $\mu(H_0) = 0$ and $\mu(H_1) = 1$ ("without real loss of generality"). The section 6 abruptly introduces some inner variability in the populations H_0 and H_1, and therefore considers the non-conditional variable $\bar{X}$. It seems that finite variances only introduce marginal fluctuations (theorem 11). This is interesting, although expected for a linear process. However, it should have been motivated, since it does not seem to help to be closer to a cascade process.

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