REPORT ON
IN-DEPTH ANALYSIS OF A DISCRTE P MODEL
SUBMITTED TO
NONLINEAR PROCESSES IN GEOPHYSICS

According to the author, the paper introduces a discrete version of p-model based on a new sampling. Let us recall the model. The author is interested in

$$S_n = \sum_{i=1}^{2^n-1} X_i, \quad \overline{X}_n = \frac{S_n}{2^n-1}, \quad Y_n = \mathbb{E}(\overline{X}_n|B_n)$$

where $X_1, X_2, \cdots, X_{2^n-1}$ represent the sampling, which is made as follows: the elements in the j-th group

$$X_{2^{j-1}}, X_{2^{j-1}+1}, \cdots, X_{2^j-1} \qquad (1 \leq j \leq n)$$

are all (at the same time) selected to be 1 with probability p and 0 with probability $1-p$; where B_n represents the state of selection. A realization of B_n can be identified with a binary vector $(b_{n-1}, \cdots, b_1, b_0)$ ($b_{j-1} = 1$ meaning that 1 is selected for the j-th group), which can be identified with an integer betweet 0 and $2^n - 1$ via the binary representation. The selections are assumed independent. Thus, let

$$Z_j = \sum_{i=2^{j-1}}^{2^j-1} X_i.$$

Then

$$S_n = \sum_{j=1}^{n} Z_j. \tag{1}$$

Observe that Z_j's are independent and

$$P(Z_j = 2^{j-1}) = p, \quad P(Z_j = 0) = 1-p. \tag{2}$$

It is then clear that S_n is a variant of binomial variable and it is rather direct to derive its proprties like those stated in the paper. For example,

$$\mathbb{E}S_n = \sum_{j=1}^{n} \mathbb{E}Z_j = p\sum_{j=1}^{n} 2^{j-1} = p(2^n-1).$$

Thus Theorem 5 stating that $\mathbb{E}Y_n = p$ is trivial, because the expectation of Y_n is equal to that of S_n divided by $2^n - 1$. Also notice that

$$Y_n = \frac{1}{2^n-1}\sum_{j=1}^{n} b_{j-1}2^{j-1} \tag{3}$$

where $b_0, b_1, \cdots, b_{n-1}$ are independent and p-Bernoulli random variables. This allows us to quickly obtain Theorem 6. The representation (3) should be a key for obtaining other results, including that on the limit distribution of Y_n (Theorem 13).

I don't understand why H_0 and H_1 are variables, and their expectations $\mu(H_0)$ and $\mu(H_1)$ and monents are mentioned.

Mathematically, things can be much simplifed and easily obtained. The study of the limit law of Y_n has some interests.

It is a long paper with many discussions on different physical topics. But the relation between the studied model and these topics are not really discussed.