REPORT ON
IN-DEPTH ANALYSIS OF A DISCRETE P MODEL
SUBMITTED TO
NONLINEAR PROCESSES IN GEOPHYSICS

According to the author, the paper introduces a discrete version of \( p \)-model based on a new sampling. Let us recall the model. The author is interested in

\[
S_n = \sum_{i=1}^{2^n-1} X_i, \quad \overline{X}_n = \frac{S_n}{2^n - 1}, \quad Y_n = \mathbb{E}(\overline{X}_n|B_n)
\]

where \( X_1, X_2, \ldots, X_{2^n - 1} \) represent the sampling, which is made as follows: the elements in the \( j \)-th group \( X_{2^j-1}, X_{2^j-1+1}, \ldots, X_{2^j-1} \) \((1 \leq j \leq n)\) are all (at the same time) selected to be 1 with probability \( p \) and 0 with probability \( 1 - p \); where \( B_n \) represents the state of selection. A realization of \( B_n \) can be identified with a binary vector \( (b_{n-1}, \ldots, b_1, b_0) \) \((b_{j-1} = 1 \text{ meaning that } 1 \text{ is selected for the } j\text{-th group})\), which can be identified with an integer between 0 and \( 2^n - 1 \) via the binary representation. The selections are assumed independent. Thus, let

\[
Z_j = \sum_{i=2^{j-1}}^{2^j-1} X_i.
\]

Then

\[
S_n = \sum_{j=1}^{n} Z_j.
\]

Observe that \( Z_j \)'s are independent and

\[
P(Z_j = 2^{j-1}) = p, \quad P(Z_j = 0) = 1 - p.
\]

It is then clear that \( S_n \) is a variant of binomial variable and it is rather direct to derive its properties like those stated in the paper. For example,

\[
\mathbb{E} S_n = \sum_{j=1}^{n} \mathbb{E} Z_j = p \sum_{j=1}^{n} 2^{j-1} = p(2^n - 1).
\]

Thus Theorem 5 stating that \( \mathbb{E} Y_n = p \) is trivial, because the expectation of \( Y_n \) is equal to that of \( S_n \) divided by \( 2^n - 1 \). Also notice that

\[
Y_n = \frac{1}{2^n - 1} \sum_{j=1}^{n} b_{j-1} 2^{j-1}
\]

where \( b_0, b_1, \ldots, b_{n-1} \) are independent and \( p \)-Bernoulli random variables. This allows us to quickly obtain Theorem 6. The representation (3) should be a key for obtaining other results, including that on the limit distribution of \( Y_n \) (Theorem 13).

I don’t understand why \( H_0 \) and \( H_1 \) are variables, and their expectations \( \mu(H_0) \) and \( \mu(H_1) \) and moments are mentioned.

Mathematically, things can be much simplified and easily obtained. The study of the limit law of \( Y_n \) has some interests.

It is a long paper with many discussions on different physical topics. But the relation between the studied model and these topics are not really discussed.